KOCHI UNIVERSITY OF TECHNOLOGY

Social Design Engineering Series *SDES-2021-5*

Robots and Unemployment

Noritaka Kudoh
Nagoya University

Hiroaki Miyamoto
Tokyo Metropolitan University

28th May, 2021

School of Economics and Management
Research Institute for Future Design
Kochi University of Technology

Robots and Unemployment

Noritaka Kudoh* Nagoya University

Hiroaki Miyamoto† Tokyo Metropolitan University

May 27, 2021

Abstract

This paper studies the impact of the robotics revolution on the labor market outcomes through the lens of capital-augmenting technological progress. We develop a tractable search-matching model with labor market segmentation and multi-factor production to find the condition under which the new technology harms the labor market in the long run. The robotics revolution hits the labor market for routine-task intensive jobs harder under a more generous unemployment policy. Automation of abstract tasks may cause a disaster for those who are reallocated to routine-task intensive occupations.

JEL classification: E32, J20, J64.

Keywords: robots, capital-augmenting technological progress, search-matching frictions, unemployment, routinization.

*E-mail: kudoh@soec.nagoya-u.ac.jp

†E-mail: hmiyamoto@tmu.ac.jp

1 Introduction

Recent advances in robots and artificial intelligence (AI) technologies create widespread concerns that the robotics revolution will eventually make human obsolete in the process of production (Brynjolfsson and McAfee, 2014; Ford, 2015; Harari, 2016; Frey and Osborne, 2017; Baldwin, 2019). In response to these concerns, there is a growing number of formal economic analyses regarding the impact of the robotics revolution on the labor market (Acemoglu and Restrepo, 2018; Berg et al., 2018; Caselli and Manning, 2019).

An important common element in the recent theoretical literature is that the firm's automation decision is explicitly modeled, making one of the shape parameters in the traditional production function endogenous (Acemoglu and Restrepo, 2018). While this approach helps us organize the likely channels through which the robotics revolution affects the labor demand, the existing models lack testable predictions or concrete conditions under which the new technology harms the labor market (Caselli and Manning, 2019). Further, task-based models with automation choice require new deep parameters to be estimated, and this imposes a serious restriction on making quantitative predictions.

To provide robust predictions, Berg et al. (2018) choose to study standard dynamic general equilibrium models with robots as equipment capital (Greenwood et al., 1997; Krusell et al., 2000; Autor et al., 2003; Autor and Dorn, 2013). While automation of tasks is not explicitly modeled, Berg et al. (2018) consider *capital-augmenting* technological progress (CATP). Berg et al. (2018) exploit their standard model structure to obtain a set of realistic parameters, and quantitatively show that if advances in robots decrease the marginal product of labor, then there is a series of short-run equilibria in which an increase in the productivity of robot causes the demand for labor to decline, and the period of short-run pain may take generations.

An important shortcoming of the existing models of CATP is that the labor market is assumed to be perfectly competitive, ruling out *unemployment* as a result of factor substitution. In this paper, we study the impact of robotics revolution on the labor market outcomes through the lens of CATP in the standard search-matching model of the labor market (Diamond, 1982; Mortensen and Pissarides, 1994; Pissarides, 2000), or the DMP model. This issue has not been fully explored in the literature and we aim to fill this gap.

As a preliminary analysis, we first study the impact of CATP in the context of the textbook search-matching model with *homogeneous* workers to ask whether the robotics revolution can cause joblessness for all workers. We show that CATP reduces unemployment and increases the wage rate in the long run because CATP cannot reduce the marginal product of labor when there are two inputs and the production technology exhibits constant returns to scale. Thus, robots cannot be a threat for all workers. This result is consistent with Caselli and Manning (2019).

We then present our main model, in which there are two types of workers, skilled and unskilled, and there is a distinct labor market for each skill level. Labor markets are segmented in that skilled workers participate in the labor market for *abstract* tasks whereas unskilled workers participate in the labor market for *routine* tasks. We show that, with worker heterogeneity and labor market segmentation, CATP can reduce the demand for routine labor input.

Further, we find and *quantify* a condition under which CATP increases unemployment for the unskilled in the long run. Namely, whether the robotics revolution results in technology optimism or technology unemployment depends on whether the elasticity of substitution between routine labor and capital is below or above some threshold level. Our quantitative model implies that this threshold is *3.91* for the United States.

In our quantitative analyses, we focus on a hypothetical scenario in which the productivity of robots doubles. In our optimistic case when the elasticity of substitution is 2.5, which is below the threshold, job creation and the wage rate increase for all types of workers and the overall unemployment rate decreases. However, the skill premium increases and the labor share decreases. In our pessimistic case when the elasticity of substitution between routine labor and capital is 5 or 10, routine workers face declines in both job-finding rate and wage rate, and the overall unemployment rate rises. Even in these pessimistic cases, the magnitude of job loss is quantitatively very small if robots and traditional capital are pooled as a single input of production.

We extend our basic model to include traditional capital as a distinct input. While capital heterogeneity itself does not cause technology unemployment, it *reduces the threshold elasticity* for technology optimism to arise because an investment surge absorbs part of the increase in the robot productivity. In this model, the robotics revolution increases the demand for routine labor if and only if the elasticity of substitution between routine labor and capital is below *2.52*. Further, the magnitudes of the impacts of the robotics revolution on the labor market variables are strikingly high. When the elasticity of substitution between routine labor and capital is 10, CATP increases the unskilled unemployment rate from 8.3% to 43.7%.

A policy implication obtained from our model is similar to those found in Mortensen and Pissarides (1999) and Hornstein et al. (2007). We find that the robotics revolution hits the labor market for routine-task intensive occupations harder under a more generous unemployment policy. When the elasticity of substitution between routine labor and capital is 10, the monthly job-finding rate for the unskilled falls from 0.42 to 0.05 under our baseline value of the unemployment benefit replacement rate of 0.25. However, when the unemployment benefit replacement rate is set 20% higher than our baseline level to be 0.30, it falls from 0.37 to 0.02.

Because our assumption that abstract tasks cannot be automated is controversial (Brynjolfsson and McAfee, 2014; Ford, 2015), we consider the possibility that some of the abstract occupations are subject to automation. In particular, we adopt Frey and Osborne's (2017) machine learning outcome that 47% of the occupations currently exist in the U.S. are in the category of high risk of being computerized to consider a scenario in which 47% of the skilled workers are reallocated to the labor market for routine jobs. We show that computerization of abstract tasks may cause a disaster in the routine labor market by reducing the monthly job-finding rate to 0.01 if the elasticity of substitution between routine labor and robots is 10.

In many scenarios we consider, we find asymmetry in the two labor markets. In the labor market for abstract-task intensive jobs, the adjustment is mainly through the wage rate: CATP significantly increases the wage rate while the job-find rate does not change much because the additional supply of skilled workers is limited as they are mostly on the job. On the other hand, in the labor market for routine jobs, the adjustment is mainly through job creation: CATP significantly reduces the job-finding rate while the wage rate does not decline much because workers have outside options in wage bargaining.

Cortes et al. (2017) and more recently Jaimovich et al. (2020) argue that workers with routine-task-intensive occupations are likely to exit from the labor market in response to the deterioration of the prospect of such occupations. To assess the importance of the labor market participation margin, we further extend our model in line with Tripier (2003) and Veracierto (2008). We find that part of the upward pressure on the unemployment rate goes to the participation margin, and its magnitude is up to one half of the overall change in the unskilled unemployment rate. However, if the household's income declines by automation, then the income effect operates to *increase* the labor force participation rate even when the job-finding rate for routine jobs becomes extremely low.

This paper is closely related with the literature on routinization and labor market polarization (Autor et al., 2003; Goos and Manning, 2007; Autor and Dorn, 2013; Cortes et al., 2017; vom Lehn, 2020). In particular, our analysis builds on their conceptual framework such as routine versus abstract tasks in production. While this literature focuses on explaining the decline in the middle of wage distribution, our model has only two skill types. This modeling choice is made for quantifying our model within a standard set of parameters and calibration targets. However, we note that our experiment on the decline in the proportion of the skilled can be interpreted as the decline in the middle, because the skilled workers in our model include those in the middle of income distribution.

The analytical framework of paper builds on the recent development in the field of dynamic general equilibrium with search-matching frictions in the labor market. One technical challenge in integrating search-matching frictions into our rich production technology is that a firm employing a large number of employees takes into account of its size when negotiating with its workers over the wage rates. To avoid the potential complication that is not the center of our investigation, we adopt a vertically integrated market structure commonly employed in the recent business cycle literature (Trigari, 2009; Christiano et al., 2016). This modeling device allows us to study the labor market equilibrium conditions for a given set of competitive factor prices.

The idea that technological progress may destroy jobs has been extensively studied in the creative destruction literature (Aghion and Howitt, 1994; Mortensen and Pissarides, 1998; Hornstein et al., 2007; Pissarides and Vallanti, 2007). In this literature, jobs are destroyed when the technology embodied in capital becomes obsolete and unemployment occurs because workers switch from one job to another with a newer technology. Because the production unit is a capital-worker pair, technological progress itself is accompanied with job creation. In contrast, we are more interested in the scenario in which technological progress permanently changes the composition of labor demand.

This paper is organized as follows. Section 2 presents a preliminary analysis based on a two-factor production function with search-matching frictions in the labor market to highlight the importance of worker heterogeneity. Section 3 presents our main model with worker heterogeneity and labor market segmentation. Section 4 conducts quantitative analyses, and we apply this quantitative model to obtain some policy implications in Section 5. Section 6 explores the possibility that abstract tasks are automated. Section 7 extends our main model to consider labor force participation. Section 8 concludes. Proofs and additional results are found in the Appendix.

2 Homogeneous Workers

This section presents a preliminary analysis to clarify the impact of advances in robotics on the aggregate labor market in a standard search-matching model. We introduce the following three (minimum) components into the textbook DMP model. First, production requires both labor and capital. Second, technological progress is capital-augmenting. Third, capital is supplied by the representative household. Within this simple framework, we ask whether CATP can cause persistently high unemployment.

2.1 Environment

To bypass the issue of intra-firm bargaining between a firm and a continuum of workers, we assume a vertical market structure commonly employed in the business cycle literature. There is a single consumption good, produced by the representative final good producer. Production of the final consumption good requires capital and intermediate inputs (or tasks) supplied by intermediate-good firms.[1] Technological progress is capital-augmenting: $y_t = F(x_t, a_t k_t)$, where y_t is output, x_t is the amount of tasks supplied by intermediate-good firms, k_t is the stock of capital, and a_t is the level of capital-augmenting technology. We assume that the production function satisfies the neoclassical properties: $F_1 > 0$, $F_2 > 0$, $F_{11} < 0$, $F_{22} < 0$, and the Inada conditions ($\lim_{x\to 0} F_1 = \infty$, $\lim_{ak\to 0} F_2 = \infty$, $\lim_{x\to\infty} F_1 = 0$, and $\lim_{ak\to\infty} F_2 = 0$). We also assume that F satisfies constant returns to scale (CRS). The assumptions of CRS and decreasing marginal products jointly imply $F_{12}(= F_{21}) > 0$.[2]

It is assumed that the final-good firm behaves competitively. Thus, the firm chooses its demand for x_t and k_t to maximize $F(x_t, a_t k_t) - p_t x_t - r_t k_t$, where p_t is the price of a unit of task (ultimately performed by workers) and r_t is the rental price of capital. Thus, the input prices are determined as their marginal products:

$$p_t = F_1\left(x_t, a_t k_t\right) = F_1\left(1, K_t\right), \tag{1}$$

$$r_t = F_2(x_t, a_t k_t) a_t = F_2(1, K_t) a_t, \tag{2}$$

[1]Each intermediate-good firm acts as an intermediary and plays two important roles. One is to find a worker. The other is to bargain with its employee over the wage rate.

[2]Because F is homogeneous of degree 1, F_1 is homogeneous of degree 0, which implies $F_{11}(x_1, x_2)x_1 + F_{12}(x_1, x_2)x_2 = 0$, from which $F_{12}(x_1, x_2)x_2 = -F_{11}(x_1, x_2)x_1 > 0$. By symmetry of partial derivatives, we also obtain $F_{21} > 0$. We thank Kenji Yamamoto for his comments on the properties of homogeneous functions.

where $K_t = a_t k_t / x_t$ is the effective capital-labor ratio. Note that F_1 and F_2 are homogeneous of degree zero.

Consider an infinite-horizon economy with a continuum of ax-ante homogeneous individuals. Total labor force is normalized to 1. Ex-post, individuals are either employed or unemployed. The unemployment rate is denoted by u_t. As in Merz (1995) and Andolfatto (1996), all individuals are members of a single representative household and there is perfect risk sharing among the members. The primary role of the household is to supply capital. The household's lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}, \tag{3}$$

where β is the household's subjective discount factor, C_t is the level of consumption, which is common for all individuals, and σ is the elasticity of intertemporal substitution (which corresponds to the relative risk aversion in a stochastic environment). The budget constraint is $C_t + \tau_t + k_{t+1} - (1-\delta)k_t = w_t(1-u_t) + \bar{b}u_t + r_t k_t + \pi_t$, where π_t is the profit from ownership of all firms, τ_t is lump sum taxes to the government, k_t is the stock of capital, δ is the capital depreciation rate, w_t is the wage rate, $\bar{b}$ is the unemployment insurance benefits provided by the government, and r_t is the rental rate of capital. The household chooses sequences of C_t and k_{t+1} to maximize (3) subject to the budget constraint. The first-order conditions imply $C_t^{-\sigma} = \Lambda_t$ and $\Lambda_t = \beta\Lambda_{t+1}(r_{t+1} + 1 - \delta)$, with the transversality condition, $\lim_{t\to\infty} \Lambda_t \beta^t k_t = 0$, where Λ_t is the Lagrange multiplier for this problem.

When making decentralized decisions, each individual discounts future payoffs by

$$B_t = \frac{1}{1 + r_{t+1} - \delta} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}, \tag{4}$$

which is a deterministic version of the stochastic discount factor. Note that $B = \beta$ holds in any steady state.

2.2 Labor Market

We assume search-matching frictions in the labor market. Total number of matches made in period t is given by a constant-returns-to-scale matching technology $m(u_t, v_t)$, where u_t is the number of job-seekers and v_t is the number of vacancies. To ensure the vacancy-filling probability and the job-finding probability to be in between 0 and 1, we adopt the specification of den Haan et al. (2000) and Hagedorn and Manovskii (2008):

$$m(u, v) = \frac{uv}{(u^\psi + v^\psi)^{1/\psi}}, \tag{5}$$

where ψ is a parameter.

The vacancy-filling rate, the probability at which a vacancy is filled in period t, is $q_t = m(u_t, v_t)/v_t = (1+\theta_t^{\psi})^{-1/\psi}$, where $\theta_t = v_t/u_t$ is labor market tightness. Similarly, the job-finding rate, the probability that a job-seeker is employed in the next period, is given by $Q_t = m(u_t, v_t)/u_t = \theta(1+\theta^{\psi})^{-1/\psi} = \theta_t q_t$. The matching elasticity with respect to unemployment is given by $\theta^{\psi}/(1+\theta^{\psi})$ and the matching elasticity with respect to vacancies is given by $1/(1+\theta^{\psi})$.

Separations are exogenous. At the end of each period, a current production unit separates with probability λ. Thus, the equilibrium unemployment rate evolves according to

$$u_{t+1} = u_t + \lambda(1-u_t) - Q_t u_t. \tag{6}$$

The values of employment (W_t), unemployment (U_t), a filled job (J_t), and a vacancy (V_t) are given by $W_t = w_t + \lambda B_t U_{t+1} + (1-\lambda) B_t W_{t+1}$, $U_t = b + Q_t B_t W_{t+1} + (1-Q_t) B_t U_{t+1}$, $J_t = p_t - w_t + \lambda B_t V_{t+1} + (1-\lambda) B_t J_{t+1}$, and $V_t = -c + q_t B_t J_{t+1} + (1-q_t) B_t V_{t+1}$, where c is the cost of posting a vacancy. The flow value of nonwork b is the sum of unemployment insurance benefits $\bar{b}$ and the saved disutility of work in units of consumption (Hall and Milgrom, 2008).

As in Pissarides (2000), we assume free entry of vacancies, which implies that the equilibrium number of vacancies is determined so that $V_t = 0$ holds for all t. This yields the following job creation condition:

$$\frac{c}{q_t} = B_t \left[p_{t+1} - w_{t+1} + \frac{(1-\lambda)\, c}{q_{t+1}} \right]. \tag{7}$$

The wage rate is determined through bilateral wage bargaining. As in Pissarides (2000), the wage rate is determined so that $\eta(J_t - V_t) = (1-\eta)(W_t - U_t)$ is satisfied for each period, where η is the worker's bargaining power. Substitute the value functions and the free entry condition into the bargaining outcome to yield the wage equation:

$$w_t = \eta p_t + (1-\eta) b + \eta c \theta_t. \tag{8}$$

2.3 Equilibrium

Because each intermediate firm can hire at most one employee and each employee supplies one unit of labor service, the amount of intermediate goods (or, tasks) supplied equals the number of employed individuals. Thus, $x_t = \ell_t = 1 - u_t$.

The aggregate profit of all firms is given by $\pi_t = y_t - w_t(1-u_t) - r_t k_t - cv_t$. The government finances unemployment insurance benefits by lump-sum taxes. Thus, $\tau_t = u_t \bar{b}$. Eliminate π_t and τ_t from the household's budget constraint to obtain the resource constraint as

$$C_t + k_{t+1} - (1-\delta)k_t = y_t - cv_t. \tag{9}$$

A perfect-foresight equilibrium is determined by (1), (2), (4), (6), (7), (8), (9), and $K_t = a_t k_t / \ell_t$. Given a long-run level of productivity $a_t = a$, we can determine a steady-state of this economy as the solution to the following system of equations:

$$r = \frac{1}{\beta} - 1 + \delta = F_2(1, K)a, \tag{10}$$

$$p = F_1(1, K), \tag{11}$$

$$\frac{\left(\beta^{-1} - 1 + \lambda\right) c}{q} = (1-\eta)\, p - (1-\eta)b - \eta c\theta, \tag{12}$$

$$w = \eta p + (1-\eta)b + \eta c\theta, \tag{13}$$

with $K = ak/\ell$ and $u = \lambda/(\lambda + Q)$. Given a level of a, (10) determines the steady-state level of K. Given K, (11) pins down the level of p. Thus, from (10) and (11), we can implicitly define $p = P\,(a)$. Given this, (12) and (13) determine θ and w.

Lemma 1 *$P'(a) > 0$.*

Proof. Totally differentiate (10) and (11) to obtain $F_2 da + aF_{22} dK = 0$ and $dp = F_{12} dK$, from which $dp/da = -(F_{12}F_2/aF_{22}) > 0$ because $F_{12} > 0$. ■

Proposition 1 *In any steady state, CATP decreases u and increases w.*

Proof. A steady-state is determined by

$$\frac{\left(\beta^{-1} - 1 + \lambda\right) c}{q} = (1-\eta)\, P\,(a) - (1-\eta)b - \eta c\theta, \tag{14}$$

$$w = \eta P\,(a) + (1-\eta)b + \eta c\theta, \tag{15}$$

$$u = \frac{\lambda}{\lambda + Q}. \tag{16}$$

Because $P'(a) > 0$, an increase in a increases both θ and w in the long run. ■

Thus, if workers are homogeneous and the robotics revolution can be expressed as capital-augmenting technological progress, then the robotics revolution unambiguously creates jobs and increases the wage rate in the long run. To put it differently, robots cannot be a threat for all workers in the long run.

Our next question is whether there is *short-run pain* for workers. The mechanism that potentially decreases job creation in the short run is through the discount factor,

$$B_t = \frac{1}{1 + F_2(1, K_{t+1}) a_{t+1} - \delta}, \tag{17}$$

which is the inverse of the real interest factor. If the discount factor decreases significantly in response to the robotics revolution, then it decreases job creation as it is interpreted as the reverse of the capitalization effect (Pissarides, 2000).

We argue that this effect should not be large enough to increase the unemployment rate. To see this, suppose for now that $\sigma = 0$ (utility is linear in consumption), in which case we obtain $B_t = \beta$ and $r_t = \beta^{-1} - 1 + \delta$ for all t. Then, the effect through the discount factor disappears and the equilibrium conditions become

$$\frac{c}{q_t} = \beta \left[P\left(a_{t+1}\right) - w_{t+1} + \frac{\left(1 - \lambda\right) c}{q_{t+1}} \right], \tag{18}$$

$$w_t = \eta P\left(a_t\right) + (1 - \eta) b + \eta c \theta_t, \tag{19}$$

$$u_{t+1} = u_t + \lambda \left(1 - u_t\right) - Q_t u_t. \tag{20}$$

In this case, u_t unambiguously *decreases* even in the short run. This result implies that for a short-run pain to arise, the curvature of the utility function should be very large. If this is the case, then through (the reverse of) the capitalization effect, the increase in the real interest rate reduces job creation because the opportunity cost of creating a vacancy is higher when the rate of returns on capital is higher.

3 Heterogeneous Workers

The previous section clarifies that in the textbook DMP model with homogeneous workers, CATP stimulates job creation and increases the wage rate. In this section, we explore the impact of the robotics revolution on the distribution of jobs and wages. For this purpose, we consider worker heterogeneity and labor market segmentation. The key ingredient is a class of constant elasticity of substitution (CES) production technology considered by Krusell et al. (2000) and in particular Berg et al. (2018).

3.1 Environment

There are two types of workers: type-A and type-R. Type-A workers have skills capable of performing *abstract tasks*, which are hard to be automated. In contrast, type-R workers have skills capable of performing *routine tasks*, which are relatively easy to be automated. The population share of type-A workers is ϕ and the remaining proportion $1-\phi$ is type-R. The number of employed workers of type i and the number of job-seekers of type i are denoted by ℓ_t^i and u_t^i, respectively. Thus, $\phi = \ell_t^A + u_t^A$ and $1-\phi = \ell_t^R + u_t^R$.

Consider a production function with three factors: $F(\ell_t^A, \ell_t^R, a_t k_t)$. We assume that the production function satisfies the neoclassical properties: $F_1 > 0$, $F_2 > 0$, $F_3 > 0$, $F_{11} < 0$, $F_{22} < 0$, $F_{33} < 0$, and the Inada conditions. We also assume that F satisfies constant returns to scale. When there are three factors of production, one of the three cross-partial derivatives can be *negative* under our maintained assumptions, as shown in Appendix A.

To obtain testable implications from our model, we follow Krusell et al. (2000) and in particular Berg et al. (2018) to adopt a nested CES function for the final good producer:

$$y_t = A[\mu_1 (x_t^A)^{\rho_1} + (1-\mu_1) z_t^{\rho_1}]^{\frac{1}{\rho_1}}, \tag{21}$$

where x_t^A is the input of *abstract* tasks (performed by type-A workers) that the producer purchases from type-A intermediate-good firms, z_t is the input of *routine* tasks, $1/(1-\rho_1)$ is the elasticity of substitution between abstract and routine inputs, and $0 < \mu_1 < 1$. The routine input z_t is a composite good, given by

$$z_t = [\mu_2 (x_t^R)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}]^{\frac{1}{\rho_2}}, \tag{22}$$

where x_t^R is the input of routine tasks (performed by type-R workers) that the producer purchases from type-R intermediate-good firms, $1/(1-\rho_2)$ is the elasticity of substitution between type-R labor input and capital. We assume $\rho_1 < \rho_2 < 1$.

Our specification is a natural extension of Autor et al. (2003), who explore the implication of computerization for wage inequality. If we set $\rho_1 = 0$ (so that (21) becomes Cobb-Douglas) and $\rho_2 = 1$ (so that x_t^R and $a_t k_t$ become perfect substitutes), then our production function reduces to that of Autor et al. (2003).[3] Type-A workers provide abstract inputs that are essential for production. On the other hand, because routine tasks can be supplied both by type-R workers and capital, a change in the composition of inputs in (22) toward capital can be interpreted as automation.

[3] Autor and Dorn (2013), Jaimovich et al. (2020), and vom Lehn (2020) employ similar specifications to study polarization of employment.

Factor markets are perfectly competitive. From (21) and (22), the firm's profit-maximization implies

$$p_t^A = A[\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1}{\rho_1}-1}\mu_1, \tag{23}$$

$$p_t^R = A[\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1}{\rho_1}-1}(1-\mu_1)Z_t^{\rho_1-1}[\mu_2 + (1-\mu_2)K_t^{\rho_2}]^{\frac{1}{\rho_2}-1}\mu_2, \tag{24}$$

$$r_t = A[\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1}{\rho_1}-1}(1-\mu_1)Z_t^{\rho_1-1}[\mu_2 + (1-\mu_2)K_t^{\rho_2}]^{\frac{1}{\rho_2}-1}(1-\mu_2)K_t^{\rho_2-1}a_t, \tag{25}$$

where $Z_t = z_t/x_t^A$ is the ratio of routine and abstract inputs and $K_t = a_t k_t/x_t^R$ is the ratio of machines and routine labor.

Proposition 2 *(i) $F_{13}(\ell^A, \ell^R, ak) \geq 0$; (ii) $F_{23}(\ell^A, \ell^R, ak) < 0$ holds if and only if*

$$\frac{1}{1-\rho_1}\frac{1}{s} < \frac{1}{1-\rho_2}, \tag{26}$$

where $s = p^A\ell^A/y \in (0,1)$.

Proof. In Appendix B. ■

Proposition 2-(ii) states that an increase in capital decreases the marginal product of type-R labor if and only if the elasticity of substitution between routine labor and capital is sufficiently high. Condition (26) appears in Berg et al. (2018) as the condition for short-run pain to arise. Our result clarifies how the nested CES specification restricts the cross-partial derivatives of the general production function F.

As in the preceding section, the representative household's lifetime utility is given by (3) and the budget constraint is now $C_t + \tau_t + k_{t+1} - (1-\delta)k_t = w_t^A l_t^A + w_t^R l_t^R + \bar{b}^A u_t^A + \bar{b}^R u_t^R + r_t k_t + \pi_t$. The household chooses the sequences of C_t and k_{t+1} to maximize (3) subject to the budget constraint. The first-order conditions imply $C_t^{-\sigma} = \Lambda_t$ and $\Lambda_t = \beta\Lambda_{t+1}(r_{t+1} + 1 - \delta)$, with the transversality condition, $\lim_{t\to\infty}\Lambda_t\beta^t k_t = 0$, where Λ_t is the Lagrange multiplier for this problem. Each individual discounts future payoffs by B_t defined in (4).

3.2 Labor Market

There are two *segmented* labor markets, one for abstract-task intensive jobs (type-A labor market) and the other for routine-task intensive jobs (type-R labor market). Type-A workers participate in type-A labor market and type-R workers participate in type-R labor market. There is no reallocation of workers across the sectors.[4] The assumption that the proportion of the skilled ϕ is unaffected by the robotics revolution is clearly too strong. We address this issue in Section 6.

The number of new employees in market i ($i = A, R$) is determined by

$$m(u^i, v^i) = \frac{u^i v^i}{[(u^i)^{\psi_i} + (v^i)^{\psi_i}]^{1/\psi_i}}, \tag{27}$$

where v^i is the number of type-i vacancies and ψ_i is the shape parameter for market i. Each type-i vacancy is matched to a type-i job-seeker with probability $q_t^i = [1 + (\theta_t^i)^{\psi_i}]^{-1/\psi_i}$, where $\theta_t^i = v_t^i / u_t^i$ is labor market tightness in market i. Similarly, the probability that a type-i job-seeker is matched with a type-i vacancy is given by $Q_t^i = \theta_t^i [1 + (\theta_t^i)^{\psi_i}]^{-1/\psi_i} = \theta_t^i q_t^i$. The matching elasticity with respect to unemployment is given by $\theta^{\psi_i}/(1 + \theta^{\psi_i})$.

Separations are exogenous and take place at the end of each period. The law of motion for the stock of type-i employment satisfies $\ell_{t+1}^i = (1 - \lambda^i)\ell_t^i + q_t^i v_t^i$, where λ^i is the separation rate in market i. Similarly, the law of motion for the stock of job seekers in market i is $u_{t+1}^i = (1 - q_t^i) u_t^i + \lambda^i \ell_t^i$.

The values of employment (W_t^i), unemployment (U_t^i), a filled job (J_t^i), and a vacancy (V_t^i) are given by $W_t^i = w_t^i + \lambda^i B_t U_{t+1}^i + (1 - \lambda^i) B_t W_{t+1}^i$, $U_t^i = b^i + Q_t^i B_t W_{t+1}^i + (1 - Q_t^i) B_t U_{t+1}^i$, $J_t^i = p_t^i - w_t^i + \lambda^i B_t V_{t+1}^i + (1 - \lambda^i) B_t J_{t+1}^i$, $V_t^i = -c^i + q_t^i B_t J_{t+1}^i + (1 - q_t^i) B_t V_{t+1}^i$, respectively. Free entry of jobs for each market ($V_t^i = 0$) yields the following job creation condition:

$$\frac{c^i}{q_t^i} = B_t \left[p_{t+1}^i - w_{t+1}^i + \frac{(1 - \lambda^i)\, c^i}{q_{t+1}^i} \right]. \tag{28}$$

Finally, Nash-bargained wage rates are determined by $\eta(J_t^i - V_t^i) = (1 - \eta)(W_t^i - U_t^i)$, from which we obtain the wage equation:

$$w_t^i = \eta p_t^i + (1 - \eta) b^i + \eta c^i \theta_t^i. \tag{29}$$

3.3 Equilibrium

A type-A intermediate-good firm employs a type-A worker to complete one unit of abstract task, and sells it to the final-good firm at the competitive price p_t^A. Similarly, a type-R intermediate-good firm employs a type-R worker to complete one unit of routine

[4] In this model, type-A workers have no incentive to participate in type-R labor market because the equilibrium job-finding rate and wage rate are both lower than those in type-A labor market. On the other hand, type-R workers have no ability to participate in type-A labor market.

task, and supplies it at price p_t^R. In any equilibrium, $x_t^A = \ell_t^A$ and $x_t^R = \ell_t^R$. The price of the final good is normalized to one.

The aggregate profit of all firms is given by $\pi_t = y_t - w_t^A \ell_t^A - w_t^R \ell_t^R - r_t k_t - c^A v_t^A - c^R v_t^R$. The government finances the unemployment benefits by lump-sum taxes. Thus, $\tau_t = u_t^A \bar{b}^A + u_t^R \bar{b}^R$. Substitute these equations into the household budget constraint to obtain the resource constraint as

$$y_t - c^A v_t^A - c^R v_t^R = C_t + k_{t+1} - (1-\delta) k_t. \tag{30}$$

Equilibrium conditions are (4), (21)–(25), $Z_t = z_t / x_t^A$, $K_t = a_t k_t / x_t^R$, (28), (29), (30), $x_t^A = \ell_t^A = \phi - u_t^A$, $x_t^R = \ell_t^R = 1 - \phi - u_t^R$, $u_{t+1}^i = u_t^i + \lambda^i \ell_t^i - Q_t^i u_t^i$, and $\theta_t^i = v_t^i / u_t^i$.

A steady-state equilibrium is determined by a set of variables $\{Z, K, p^A, p^R, \theta^A, \theta^R, u^A, u^R, z, k\}$ satisfying

$$\begin{aligned} r = {} & A \left[\mu_1 + (1-\mu_1) Z^{\rho_1}\right]^{\frac{1}{\rho_1}-1} (1-\mu_1) Z^{\rho_1 - 1} \\ & \times \left[\mu_2 + (1-\mu_2) K^{\rho_2}\right]^{\frac{1}{\rho_2}-1} (1-\mu_2) K^{\rho_2 - 1} a, \end{aligned} \tag{31}$$

$$p^A = A \left[\mu_1 + (1-\mu_1) Z^{\rho_1}\right]^{\frac{1}{\rho_1}-1} \mu_1, \tag{32}$$

$$\begin{aligned} p^R = {} & A \left[\mu_1 + (1-\mu_1) Z^{\rho_1}\right]^{\frac{1}{\rho_1}-1} (1-\mu_1) Z^{\rho_1 - 1} \\ & \times \left[\mu_2 + (1-\mu_2) K^{\rho_2}\right]^{\frac{1}{\rho_2}-1} \mu_2, \end{aligned} \tag{33}$$

$$\frac{(r - \delta + \lambda^i) c^i}{[1 + (\theta^i)^{\psi_i}]^{-1/\psi_i}} = (1-\eta)(p^i - b^i) - \eta c^i \theta^i, (i = A, R) \tag{34}$$

$$u^A = \frac{\phi \lambda^A}{\lambda^A + \theta^A [1 + (\theta^A)^{\psi_A}]^{-1/\psi_A}}, \; u^R = \frac{(1-\phi)\,\lambda^R}{\lambda^R + \theta^R [1 + (\theta^R)^{\psi_R}]^{-1/\psi_R}}, \tag{35}$$

$$z = \left[\mu_2 \left(1 - \phi - u^R\right)^{\rho_2} + (1-\mu_2)(ak)^{\rho_2}\right]^{\frac{1}{\rho_2}}, \tag{36}$$

$$z = \left(\phi - u^A\right) Z, \tag{37}$$

$$ak = \left(1 - \phi - u^R\right) K, \tag{38}$$

where $r = \frac{1}{\beta} - 1 + \delta$ is constant in any steady state. A steady-state equilibrium is summarized by two loci on the K-Z plane. One locus, defined implicitly by (31), describes the capital market equilibrium and is labeled as the CM curve. The other locus, defined implicitly by (32)–(38), describes the labor market equilibrium and is labeled as the LM curve. Lemma 2 below establishes the properties of the loci.

Lemma 2 *(i) (31) implies $dZ/dK < 0$ holds for each level of a; (ii) (31) implies $dZ/da > 0$ for each level of K. (iii) (32)–(38) are independent of a and jointly imply $dZ/dK > 0$.*

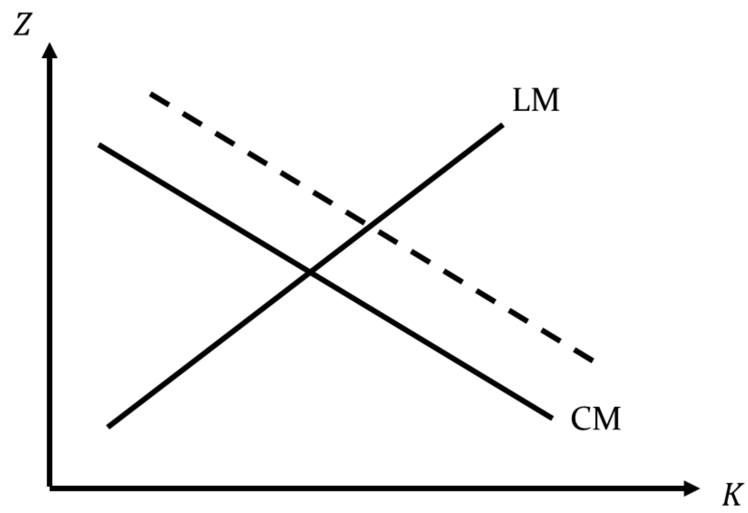

Figure 1: Steady-State Equilibrium

Proof. In Appendix C. ■

As shown in Figure 1, the CM locus is downward sloping and the LM locus is upward sloping. Because an increase in the level of technology shifts the CM curve up, CATP increases both Z and K, which in turn increases p^A. Thus, as expected, CATP increases the demand for abstract tasks and hence the demand for type-A workers. The impact of robotics revolution on the demand for type-R workers depends on the detail of the model.

Proposition 3 *CATP decreases p^R if and only if*

$$\frac{1}{1-\rho_1}\frac{1}{s} + \varepsilon^A \frac{1-s}{s} < \frac{1}{1-\rho_2}, \tag{39}$$

where

$$\varepsilon^A = \frac{dx^A}{dp^A}\frac{p^A}{x^A} > 0,\ s = \frac{p^A x^A}{y} = \frac{\mu_1}{\mu_1 + (1-\mu_1)Z^{\rho_1}} \in (0,1).$$

Proof. In Appendix D. ■

Thus, CATP decreases the demand for type-R labor input if and only if the elasticity of substitution between routine labor input and capital is sufficiently high. In this case, CATP causes joblessness for type-R *in the long run*. Note that condition (39) is more strict than (26), implying that $F_{23}(\ell^A, \ell^R, ak) < 0$ is not enough to generate long-run job losses for type-R workers.

Acemoglu and Restrepo (2018) question the usefulness of the CATP approach for investigating the robotics revolution by claiming that CATP cannot reduce the labor demand.[5] We have two objections to this claim. First, as we discussed in Section 2, the CATP approach in the model with homogeneous workers should be used to address whether CATP can reduce labor demand for *all* workers or not. For this specific question, this approach provides a clear answer, which is no.

Second, as Proposition 3 states, the CATP approach with worker heterogeneity and labor market segmentation can generate a steady-state decline in the demand for type-R workers. Further, Proposition 3 provides a *testable condition* under which the new technology harms the labor market for routine-task intensive occupations. Thus, while the two-factor model may have a serious limitation, the CATP approach, when applied to a multi-factor model, offers a rich framework for predicting the impact of the new technology.

3.4 Alternative CES Nesting

Our CES nesting implies that the elasticity of substitution between type-A labor and type-R labor is the same as that between type-A labor and capital. Krusell et al. (2000), however, recommend the use of an alternative nesting,

$$\begin{aligned} y &= A[\mu_1(\ell^R)^{\rho_1} + (1-\mu_1)z^{\rho_1}]^{\frac{1}{\rho_1}}, \\ z &= [\mu_2(\ell^A)^{\rho_2} + (1-\mu_2)(ak)^{\rho_2}]^{\frac{1}{\rho_2}}, \end{aligned} \tag{40}$$

which implies that the elasticity of substitution between type-R labor and type-A labor is the same as that between type-R labor and capital. According to this specification, with high complementarity between type-A labor and capital, the composite input z in (40) can be interpreted as broadly-defined capital in a two-factor production function. While this formulation is useful for exploring capital-skill complementarity (as in Krusell et al., 2000), it cannot describe the scenario in which the robotics revolution might cause technology

[5]They argue, "if automation were to be conceptualized as capital-augmenting technological change, it would never reduce labor demand or the equilibrium wage, and it would increase the labor share—two predictions that are neither intuitively appealing nor always consistent with the evidence" (Acemoglu and Restrepo, 2018, p.49).

unemployment because CATP with a two-factor production function necessarily increases both ℓ^R and z in (40), as we have shown in Section 2.

3.5 Importance of Worker Heterogeneity

Berg et al. (2018) emphasize the role of *heterogeneous capital* in reducing the (short run) demand for human labor. To assess whether heterogeneous capital is more important than heterogeneous labor, this section presents a model with two types of capital, traditional capital (denoted by k_t^T) and robot capital (denoted by k_t^R). We assume that CATP takes place only on robots. The rates of returns on traditional capital and robot capital are r_t^T and r_t^R, respectively. For brevity, their depreciation rates are assumed to be the same. We also assume that workers are *homogeneous* as in Section 2. Production technology is given by

$$y_t = F(k_t^T, x_t, a_t^R k_t^R) = A[\mu_1 (k_t^T)^{\rho_1} + (1-\mu_1) z_t^{\rho_1}]^{\frac{1}{\rho_1}},$$
$$z_t = [\mu_2 (x_t)^{\rho_2} + (1-\mu_2)(a_t^R k_t^R)^{\rho_2}]^{\frac{1}{\rho_2}},$$

which is identical to Model 1 of Berg et al. (2018). If we replace k_t^T with x_t^A, then this production function becomes (21).

Proposition 4 *In any steady state, CATP decreases u and increases w.*

Proof. In Appendix E. ■

In other words, the demand for human labor increases in the long run as the productivity of robot capital increases. We therefore conclude that worker heterogeneity, not capital heterogeneity, is essential for studying technology unemployment. What drives the result is rate-of-return equalization between robot capital and traditional capital. On the other hand, labor market segmentation prevents wage equalization between the two tasks.

3.6 Heterogeneous Capital

This section extends our main theoretical model presented in section 3.1 to distinguish robots and computers from traditional capital. While capital heterogeneity alone cannot generate a decline in the demand for human labor (as clarified in Section 3.5), drawing a line between the two types of capital allows us to work with a richer model in our quantitative studies.

The stock of traditional capital is denoted by k_t^T and depreciates at rate $\delta^T < 1$. The stock of computer capital such as robots, computers, and software is denoted by k_t^R and depreciates at rate $\delta^R < 1$. Both types of capital are owned by the representative household. The rates of returns on traditional capital and robot capital are denoted by r_t^T and r_t^R, respectively.

To maintain the structure of the model in section 3.1, we use the Cobb-Douglas function as in Krusell et al. (2000) to aggregate traditional capital and (21):

$$y_t = A(k_t^T)^\alpha [\mu_1 (x_t^A)^{\rho_1} + (1-\mu_1) z_t^{\rho_1}]^{\frac{1-\alpha}{\rho_1}}, \tag{41}$$

$$z_t = [\mu_2 (x_t^R)^{\rho_2} + (1-\mu_2)(a_t^R k_t^R)^{\rho_2}]^{\frac{1}{\rho_2}}, \tag{42}$$

where $0 < \alpha < 1$ and $\rho_1 < \rho_2 < 1$. The equilibrium conditions are derived in Appendix F.

With (41), the income share of abstract-task intensive labor satisfies

$$s = \frac{p^A x^A}{y} = (1-\alpha)\frac{\mu_1}{\mu_1 + (1-\mu_1)Z^{\rho_1}}.$$

Thus, Proposition 3 is modified accordingly as follows.

Proposition 5 *CATP decreases p^R if and only if*

$$\frac{1}{1-\rho_1}\frac{1-\alpha}{s} + \varepsilon^A \frac{1-\alpha-s}{s} < \frac{1}{1-\rho_2}. \tag{43}$$

4 Quantitative Analysis

This section provides quantitative analyses based on the model with heterogeneous workers presented in Section 3.1. We also study a richer quantitative model to allow for capital heterogeneity presented in Section 3.6. Following Berg et al. (2018), we consider a hypothetical scenario in which the productivity of robots doubles. While Berg et al. (2018) focus on the transition to a new steady state to highlight a series of short-run pain as their model exhibits long-run gains for workers, we focus on steady states to highlight the possibility of *long-run pain*.

4.1 Calibration

We calibrate our model so that the initial steady state with $a = 1$ matches the current U.S. economy. We set the model period to be a month and choose the subjective discount factor β to be 0.997, which corresponds to the annual real interest rate of approximately 4 percent. We set the capital depreciation rate δ to be 0.0083, or 10% annually, the standard value in the literature. We choose the value of the coefficient of relative risk aversion σ to be 2, which is within a plausible range in the literature. We set the proportion of type-A workers in labor force ϕ to be the proportion of individuals with college education or higher, which is 0.45 (Acemoglu and Autor, 2011).

According to Fallick and Fleischman (2004), the monthly transition rate from employment to unemployment equals 0.0097 for the skilled and 0.0378 for the unskilled. We therefore adopt their estimates and set $\lambda^A = 0.0097$ and $\lambda^R = 0.0378$.

Unfortunately, there is no consensus in the literature regarding the worker's bargaining power in wage determination. We thus focus on the symmetric case, $\eta = 1/2$.[6] Another controversial parameter in the literature is the value of nonwork b^i. We follow Hall and Milgrom (2008) to target the flow value of nonwork to be 71% of the wage rate. Thus, we set b^i to satisfy $b^i = 0.71w^i$. Because quantitative results of search-matching models are known to be highly sensitive to the target flow value of nonwork, we present a sensitivity analysis in Appendix G.

To pin down the shape parameters in the matching function ψ_i and vacancy costs c^i, we target the aggregate vacancy-unemployment ratio, job finding rates, and vacancy durations in the data. The mean value of the aggregate vacancy-unemployment ratio in the U.S. is 0.72 (Pissarides, 2009). Using the Current Population Survey, Hagedorn et al. (2016) estimate that the average monthly job finding rate is 0.3618 for the skilled and 0.4185 for the unskilled. They also find that a vacancy for the skilled lasts 2.128 times longer than one for the unskilled. Thus, $1/q^A = 2.128 \times 1/q^R$. Appendix H provides the results under the Cobb-Douglas specification.

The input share parameters μ_1 and μ_2 are chosen so as to match the wage premium of 1.68 (Acemoglu and Autor, 2011) and the labor share of 0.61 (Berg el al., 2018). The aggregate productivity parameter A is chosen to normalize the steady-state output y to be 1. The existing studies find that the elasticity of substitution between the skilled and the unskilled ranges between 1.4 and 2.0 (Katz and Murphy, 1992; Ciccone and Peri, 2005; Goldin and Katz, 2008). For our benchmark, we set $\rho_1 = 0.286$ so that the elasticity of substitution $1/(1-\rho_1)$ is 1.4.

As Berg et al. (2018) note, there is no econometric estimate available for ρ_2. We there-

[6]There are mainly two strategies in the literature. One is to set $\eta = 1/2$, and the other is to set η to match the estimated elasticity of the matching function with respect to unemployment.

fore consider the following three cases for the elasticity of substitution: Case 1 when it is moderately low ($1/(1-\rho_2) = 2.5$); Case 2 when it is moderately high ($1/(1-\rho_2) = 5$); and Case 3 when it is very high ($1/(1-\rho_2) = 10$). We set the initial level of technology to be $a = 1$ and set the value of A so that $y = 1.0$ for each case. Under our calibration strategy, the initial steady-state values are invariant to the value of ρ_2.

The calibrated parameter values are summarized in Table 1[7].

Table 1: Calibrated Parameter Values: Homogeneous Capital

c^A	c^R	ψ_A	ψ_R	b^A	b^R	μ_1	μ_2	A
0.205	0.211	0.644	1.169	0.587	0.349	0.467	0.891	0.967

The equilibrium job finding rates and equilibrium input prices match their target values. The model generates the aggregate unemployment rate of 5.7 percent, which is close to the observed average unemployment rate between 1948 and 2019. The equilibrium numbers of skilled and unskilled job seekers are 0.012 and 0.046, respectively. These numbers imply that the unemployment rate for the skilled u^A/ϕ is 2.6 percent and that for the unskilled $u^R/(1-\phi)$ is 8.3 percent. The equilibrium wage premium matches its calibration target, which is 1.68.

The implied matching elasticity with respect to unemployment in the type-A labor market is 0.52, which is within the plausible range of 0.5-0.7 reported by Petronglo and Pissarides (2001). The implied matching elasticity with respect to unemployment in the type-R labor market is 0.36. While this is off the range suggested by Petronglo and Pissarides (2001), this value is consistent with the estimate by Coles and Smith (1996), which is 0.4.

The model with heterogeneous capital presented in Section 3.6 has robot capital k^R and traditional capital k^T as distinct factors of production. To pin down the value of α in (41), we follow Berg et al. (2018) to target the income share of robot capital to be 0.04 and that of traditional capital to be 0.35. We set the depreciation rates, δ^R and δ^T, to be the same level at 0.0083 as in the homogeneous capital model.[8] As Table 1 and Table 2 show, the calibrated parameters regarding the labor market are the same for the two models and for all cases. Only parameters for the production function are slightly different because the

[7]The parameter values of μ_1, μ_2, and A presented in the table are those for Case 1. For Case 2, $\mu_1 = 0.449$, $\mu_2 = 0.950$, and $A = 1.108$. For Case 3, $\mu_1 = 0.442$, $\mu_2 = 0.966$, and $A = 1.172$.

[8]We considered several cases under $\delta^R > \delta^T$ and verified that our main results are nearly independent of these parameter values.

assumed value of ρ_1 differs for each case.[9]

Table 2: Calibrated Parameter Values: Heterogeneous Capital

c^A	c^R	ψ_A	ψ_R	b^A	b^R	α	μ_1	μ_2	A
0.205	0.211	0.644	1.169	0.587	0.349	0.350	0.580	0.958	0.484

4.2 Threshold Elasticity

We can now quantify the terms in (39) of Proposition 3 for the U.S. economy. Under the selected set of parameters, we obtain $s = 0.3656$, $\varepsilon^A = 0.0447$, and $1/(1-\rho_1) = 1.4$. This implies that the left-hand side of (39) is 3.91. Thus, the demand for routine labor increases in response to the robotics revolution if and only if the elasticity of substitution between routine labor and robots is less than 3.91. It is also easy to verify that the left-hand side of (26) in Proposition 2 is 3.83, which implies that robots increase the marginal product of routine labor if and only if the elasticity of substitution between routine labor and robots is less than 3.83.

We can also quantify the terms in Proposition 5, which is based on a richer model for the U.S. economy. Under the selected set of parameters, $s = 0.3656$, $\varepsilon^A = 0.0436$, and $1/(1-\rho_1) = 1.4$. This implies that the left-hand side of (43) is 2.52. Thus, for "technology optimism" to arise, the elasticity of substitution between routine labor and robot must be less than 2.52.

4.3 Robots and Labor Market Outcomes

We are interested in the long-run quantitative effects of a significant increase in the level of robot productivity on joblessness as well as wage inequality. As in Berg et al. (2018), we consider the scenario in which the level of productivity of robots doubles. In Appendix G, we present how our results depend on our calibration target regarding the flow value of nonwork. In Appendix H, we show how the results depend on our choice of matching function.

An important caveat concerning the model with homogeneous capital is that a unit increase in the productivity of robots does not translate into the same magnitude of increase

[9] The parameter values of μ_1, μ_2, and A presented in the table are those for Case 1. For Case 2, $\mu_1 = 0.579$, $\mu_2 = 0.969$, and $A = 0.486$. For Case 3, $\mu_1 = 0.579$, $\mu_2 = 0.974$, and $A = 0.487$.

in the productivity of the *aggregate* stock of capital as the share of robots is relatively small. Indeed, Berg et al. (2018) document that the income share of robot capital in the U.S. is 0.04 while that of traditional capital is 0.35.

Suppose (implicitly) that there are two types of capital, k^T and k^R. The aggregate stock of capital satisfies $k = k^T + k^R$ and we set $k^T/k^R = 0.35/0.04$. The effective capital input is $k^e = k^T + a^R k^R$, where a^R is the level of robot productivity. Suppose that a^R increases from 1 to 2. We then obtain

$$k_1^e = \frac{\frac{0.35}{0.04} + 2}{\frac{0.35}{0.04} + 1} k_0^e = 1.10 \times k_0^e.$$

From this calculation, the magnitude of CATP we consider for the model with homogeneous capital is 1.10.

Table 3 presents the steady-state values of some selected endogenous variables for the model with homogeneous capital (presented in Section 3.1) and with heterogeneous capital (presented in Section 3.6). The column labeled "Initial" shows the steady-state values under the calibrated parameters. We verify that both models replicate our calibration targets. The columns labeled "Basic" show the steady-state values of the homogeneous capital model under $a = 1.1$, whereas the columns labeled "Capital" show the steady-state values of the heterogeneous capital model under $a^R = 2$.

Table 3: CATP and Labor Market Outcomes

	Initial	Case 1: $\frac{1}{1-\rho_2} = 2.5$		Case 2: $\frac{1}{1-\rho_2} = 5$		Case 3: $\frac{1}{1-\rho_2} = 10$	
Model		Basic	Capital	Basic	Capital	Basic	Capital
Productivity		$a = 1.1$	$a^R = 2$	$a = 1.1$	$a^R = 2$	$a = 1.1$	$a^R = 2$
y	1.00	1.13	1.14	1.18	1.43	1.22	1.85
Q^A	0.36	0.41	0.41	0.43	0.49	0.44	0.55
Q^R	0.42	0.44	0.42	0.41	0.28	0.37	0.05
u^A/ϕ	2.6%	2.3%	2.3%	2.2%	2.0%	2.2%	1.7%
$u^R/(1-\phi)$	8.3%	7.8%	8.2%	8.5%	12.0%	9.2%	43.7%
u	5.7%	5.4%	5.6%	5.7%	7.5%	6.0%	24.8%
w^A	0.83	0.90	0.91	0.93	1.06	0.95	1.27
w^R	0.49	0.50	0.49	0.49	0.43	0.47	0.37
w^A/w^R	1.68	1.78	1.84	1.91	2.45	2.02	3.45
Labor share	0.61	0.58	0.57	0.55	0.47	0.53	0.37

First, consider Case 1 of the homogeneous capital model, in which the elasticity of substitution $1/(1-\rho_2)$ is 2.5. Results in Table 3 confirms Proposition 3. Because the elasticity of substitution is below the threshold of 3.91, the demand for labor will increase in both labor markets. Thus, CATP reduces the overall unemployment rate by increasing job creation in both labor markets. Similarly, the wage rates for both abstract and routine tasks increase. While this case is largely consistent with technology optimism, wage inequality expands and the labor share decreases.

Next, consider Case 2, in which the elasticity of substitution $1/(1-\rho_2)$ is 5, which is above the threshold level of 3.91. In this case, CATP increases only the demand for abstract-task intensive jobs and a larger proportion of routine tasks is performed by robots. Thus, the job-finding rate for routine-task intensive jobs decreases. There is little change in the overall unemployment rate because the increase in the number of unskilled job seekers is offset by a simultaneous decrease in the number of skilled job seekers. Albeit its small magnitude, the wage rate for routine jobs declines. In contrast, the wage rate for the skilled increases significantly and wage inequality expands.

Finally, consider Case 3, in which the elasticity of substitution $1/(1-\rho_2)$ is 10. In this case, the increase in job creation for abstract-task intensive jobs cannot compensate for the decline in the demand for routine-task intensive jobs. As a result, the overall unemployment rate increases. The wage rate for routine-task intensive occupations declines and wage inequality expands significantly.

Overall, our predictions based on the homogeneous capital model are not a disaster even under Case 3 as the magnitudes of job losses and wage reductions are modest in response to a large increase in the productivity of robots.

In contrast, our predictions based on the heterogeneous capital model are alarming. The reason is twofold. First, we now face a severer threshold elasticity for technology optimism, which is 2.52. Our optimistic scenario of Case 1 is now near the border. Second, the magnitudes of job losses are now significant. In particular, Case 3 is a disaster for those with routine-task intensive occupations. For type-R workers, the average duration of unemployment increases from 2.4 months to 20.6 months and the unemployment rate rises from 8.3% to 43.7%. Because the associated increase in the demand for abstract-task intensive occupations is moderate, the overall unemployment rate rises from 5.7% to 24.8%.

The large impact on the labor market in our heterogeneous capital model is partly explained by a surge in capital investment (Brynjolfsson and Hitt, 2003). In Case 2, the level of robot capital increases by a factor of about 8 when its productivity doubles, and traditional capital increases by 43%, whereas the labor input of type-A increases only by 0.7%. Similarly, in Case 3, the level of robot capital increases by a factor of about 17 and the level of traditional capital increases by 85%, whereas the labor input of type-A increases by 0.9%.

5 Unemployment Policy

This section conducts a policy analysis based on the model with heterogeneous capital. Our analysis is similar in spirit to Mortensen and Pissarides (1999) and Hornstein et al. (2007). We are interested in a scenario in which the level of unemployment insurance benefits is set higher than its current level.

In our benchmark model, the unemployment benefit replacement ratio $\bar{b}^i/w^i$ is 0.25. Given the set of model parameters summarized in Table 2, we consider the scenario in which the unemployment benefit replacement ratio increases by 20% to 0.30. This implies that the new flow value of unemployment satisfies $b^i/w^i = 0.76$.

Table 4: Institution and Labor Market Outcomes

	Initial	Case 1: $\frac{1}{1-\rho_2} = 2.5$		Case 2: $\frac{1}{1-\rho_2} = 5$		Case 3: $\frac{1}{1-\rho_2} = 10$	
a^R	1	1	2	1	2	1	2
$\bar{b}^i/w^i$	0.25	0.30	0.30	0.30	0.30	0.30	0.30
u^A/ϕ	2.6%	2.9%	2.4%	2.9%	2.0%	2.9%	1.8%
$u^R/(1-\phi)$	8.3%	9.2%	9.2%	9.2%	14.9%	9.3%	63.1%
u	5.7%	6.4%	6.1%	6.4%	9.1%	6.4%	35.5%
w^A	0.83	0.82	0.91	0.83	1.06	0.83	1.26
w^R	0.49	0.49	0.50	0.49	0.44	0.49	0.39
w^A/w^R	1.68	1.67	1.83	1.67	2.42	1.67	3.26
Labor share	0.61	0.61	0.57	0.61	0.47	0.61	0.35

Table 4 summarizes the results. The column labeled "Initial" shows the steady-state values under $b^i/w^i = 0.71$ and $a^R = 1$. All other columns show the values under $b^i/w^i = 0.76$. For each case, we present the steady-state values under $a^R = 1$ and those under $a^R = 2$.

First, consider the economy prior to CATP. The results are nearly identical across all cases. As is standard in the search-matching literature, a higher flow value of unemployment implies a higher wage rate and a higher unemployment rate. In all cases we consider, the increases in the wage rates are negligible and the wage premium declines only slightly. The unemployment rates for both types increase but the unskilled unemployment rate increases by more than the skilled unemployment rate.

The effects of CATP on the overall unemployment rate and especially joblessness for type-R workers are much more serious than those under a less generous unemployment policy. In Case 3, the unemployment rate for type-A workers declines from 2.9% to 1.8%. However, the unemployment rate for type-R workers increases from 9.3% to 63.1%. The magnitude amounts to 53.9 percentage points. Under a less generous unemployment policy (in Table 3), the magnitude is 35.4 percentage points. Under the more generous unemployment policy, the overall unemployment rate increases from 6.4% to 35.5%, while it increases from 5.7% to 24.8% under the less generous policy.

6 Computerization of Abstract Tasks

The previous sections have assumed that abstract tasks cannot be automated. This view, however, is controversial (Brynjolfsson and McAfee, 2014; Ford, 2015). The recent breakthrough in the field of machine learning opens the door to a new economy in which tasks that are considered as sufficiently complicated, such as driving, recognizing handwritings and verbal expressions, and translating one language into another, can be performed by computers.

This section considers the case in which skills that type-A workers acquired through education and training are subject to automation. Because our definition of skilled workers is those with skills that are hard to be automated, we consider the impact of a reduction in ϕ on the labor market outcomes and interpret it as reallocation of workers across the skill levels caused by computerization of abstract tasks.

Using a machine learning method, Frey and Osborne (2017) categorize 702 occupations currently exist in the U.S. into those facing the risk of automation and those that are resilient. They obtain the probability of computerization for each of 702 occupations and find that 47% of the occupations have computerization probabilities greater than 70%.

Given the set of model parameters summarized in Table 2, we change the proportion of the skilled by -47% to set $\phi = 0.2385$. When the level of capital productivity is at its original level ($a^R = 1$), computerization neutralizes part of human skills, which results in an output loss. When the level of robot productivity is 2, computerization does two things: reallocating 47% of skilled workers into unskilled and doubling robot productivity.

Table 5: Computerization of Abstract Tasks

	Initial	Case 1: $\frac{1}{1-\rho_2} = 2.5$		Case 2: $\frac{1}{1-\rho_2} = 5$		Case 3: $\frac{1}{1-\rho_2} = 10$	
a^R	1	1	2	1	2	1	2
ϕ	0.45	0.2385	0.2385	0.2385	0.2385	0.2385	0.2385
y	1.00	0.71	0.79	0.71	0.85	0.71	0.99
Q^A	0.36	0.47	0.50	0.47	0.52	0.47	0.55
Q^R	0.42	0.09	0.09	0.10	0.06	0.11	0.01
u^A/ϕ	2.6%	2.0%	1.9%	2.0%	1.8%	2.0%	1.7%
$u^R/(1-\phi)$	8.3%	30.0%	29.0%	27.9%	39.2%	26.2%	74.1%
u	5.7%	23.3%	22.5%	21.8%	30.3%	20.4%	56.8%
w^A	0.83	1.01	1.09	1.01	1.05	1.01	1.28
w^R	0.49	0.38	0.38	0.38	0.37	0.38	0.36
w^A/w^R	1.68	2.68	2.89	2.66	3.11	2.65	3.54
Labor share	0.61	0.62	0.58	0.63	0.52	0.64	0.38

Table 5 presents the results. The column labeled "Initial" shows the steady-state values under $\phi = 0.45$ and $a^R = 1$. All other columns show the values under $\phi = 0.2385$. For each case, we report the steady-state values under $a^R = 1$ and those under $a^R = 2$.

The results are striking. If automation of abstract tasks takes place when the productivity of robots doubles, then the joblessness for routine workers reaches 74.1% and the overall unemployment rate becomes 56.8% in Case 3. The overall unemployment rate becomes 30.3% even in Case 2.

If computerization is directed solely at neutralizing the skills that individuals acquired through education and training without any productivity gain, then the aggregate output significantly declines because those who were previously considered as skilled are now considered as unskilled in production. The output loss associated with this effect cannot be compensated for by the productivity gains by robots if the magnitude is factor of 2.

An important observation is asymmetry between the two labor markets. In the labor market for abstract-task intensive jobs, the impact of automation on the unemployment rate is small while the impact on the wage rate is significant. On the other hand, in the labor market for routine-task intensive jobs, the impact of automation on the wage rate is negligible while the unemployment rate deteriorates significantly.

7 Labor Force Participation

In many scenarios we consider, the robotics revolution seriously hurts the routine labor market. However, the hike in the unemployment rate may be (at least partially) masked by the "discouraged worker effect" as the declines in both job-finding rate and wage rate induce individuals to exit from the labor market (Cortes et al., 2017; Jaimovich et al., 2020), thereby creating a downward pressure on the unemployment rate. To assess the importance of this adjustment margin, this section extends the model with traditional capital presented in Section 3.6 to further allow for endogenous labor force participation.

There are mainly two avenues to modeling labor force participation in the literature. One way is to consider an exogenous type distribution for individuals and look for a cutoff individual who is indifferent between participation and nonparticipation (Pissarides, 2000; Garibaldi and Wasmer, 2005; Jaimovich et al., 2020). The other way, which we adopt, is to describe the labor force participation decision through the representative family's time allocation problem (Greenwood and Hercowitz, 1991; Tripier, 2003; Veracierto, 2008). The reservation wage rate plays a central role in both approaches, but we adopt the latter because our model has a representative decision maker. Further, as we shall see below, this approach can capture the income effect on labor supply.

Let n_t^i denote the measure of type-i individuals who engage in home activity. As in Veracierto (2008), we assume a linear home production technology so that n_t^i also represents the units of home goods available for the household. With labor force participation choice, the representative household's objective function is given by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \zeta_A \ln n_t^A + \zeta_R \ln n_t^R \right], \tag{44}$$

where $\zeta_A > 0$ and $\zeta_R > 0$. As in the basic model, the representative household pools the resources of all its members. The equilibrium conditions are derived in Appendix I.

In any steady state, the participation decision is summarized by the following:

$$\zeta_i \frac{1}{n^i} C^\sigma = b^i + \frac{Q^i \beta \left(w^i - b^i \right)}{1 - \beta \left(1 - \lambda^i - Q^i \right)} \tag{45}$$

for $i = A, R$. Note that the right-hand side of (45) is the reservation wage rate for each worker type, $(1-\beta)\, U^i$. The labor force participation rates satisfy $lpr^A = (\ell^A + u^A)/\phi = (\phi - n^A)/\phi$ and $lpr^R = (\ell^R + u^R)/(1-\phi) = (1-\phi-n^R)/(1-\phi)$. The aggregate labor market participation rate satisfies $lpr = \ell^A + u^A + \ell^R + u^R = 1 - n^A - n^R$. Similarly, the unemployment rates are $u^A/(\ell^A+u^A)$ and $u^R/(\ell^R+u^R)$, and the aggregate unemployment rate is $u=(u^A+u^R)/(\ell^A+u^A+\ell^R+u^R)$.

We now have two additional endogenous variables (n^A and n^R) and two additional parameters (ζ_A and ζ_R). To pin down these two parameter values, we need two additional calibration targets. As in Garibaldi and Wasmer (2005), we target the aggregate unemployment rate and the labor market participation rate. However, we update the target unemployment rate to be 5.7% (so as to be consistent with our benchmark model) and the target labor market participation rate to be 0.66 (Krusell et al., 2017). The calibrated parameter values are summarized in Table 6[10].

Table 6: Calibrated Parameter Values: Labor Force Participation

c^A	c^R	ψ_A	ψ_R	b^A	b^R	ζ_A	ζ_R	α	μ_1	μ_2	A
0.310	0.320	0.645	1.172	0.886	0.527	0.256	0.192	0.350	0.585	0.966	0.631

Table 7: Labor Force Participation

	Initial	Case 1: $\frac{1}{1-\rho_2}=2.5$		Case 2: $\frac{1}{1-\rho_2}=5$		Case 3: $\frac{1}{1-\rho_2}=10$	
a^R	1	2	2	2	2	2	2
ϕ	0.45	0.45	0.2385	0.45	0.2385	0.45	0.2385
y	1.00	1.11	0.83	1.35	0.90	1.73	1.06
C	0.72	0.76	0.58	0.83	0.59	0.92	0.59
Q^A	0.36	0.41	0.50	0.48	0.52	0.55	0.55
Q^R	0.42	0.43	0.07	0.34	0.05	0.13	0.01
$u^A/(\ell^A+u^A)$	2.6%	2.3%	1.9%	2.0%	1.8%	1.7%	1.7%
$u^R/(\ell^R+u^R)$	8.3%	8.0%	36.3%	10.1%	45.5%	22.7%	76.5%
u	5.7%	5.4%	28.9%	6.0%	35.9%	9.0%	59.5%
lpr^A	0.67	0.66	0.69	0.65	0.70	0.64	0.73
lpr^R	0.65	0.62	0.78	0.51	0.78	0.28	0.77
lpr	0.66	0.63	0.76	0.57	0.76	0.44	0.76
w^A	1.25	1.36	1.67	1.56	1.74	1.89	1.91
w^R	0.74	0.75	0.56	0.69	0.56	0.59	0.54
Labor share	0.61	0.57	0.58	0.46	0.52	0.35	0.38

[10]The parameter values of μ_1, μ_2, and A presented in the table are those for Case 1. For Case 2, $\mu_1=0.584$, $\mu_2=0.977$, and $A=0.634$. For Case 3, $\mu_1=0.583$, $\mu_2=0.982$, and $A=0.635$.

Table 7 presents the steady-state values of some selected variables. The column labeled "Initial" shows the steady-state values under $\phi = 0.45$ and $a^R = 1$. All other columns show the steady-state values under $a^R = 2$. For each case, we provide the results under $\phi = 0.45$ and $\phi = 0.2385$.

First, consider the benchmark case in which the robotics revolution increases the level of a^R without any influence on ϕ. In our most pessimistic scenario of Case 3, the long-run unemployment rate for the unskilled is 22.7%, which is nearly a half of that under the basic model. This reduction is driven by the decline in the labor force participation rate of the unskilled, which decreases significantly from 0.65 to 0.28. The skilled unemployment rate is 1.7% and this is the same as that under the basic model. The labor force participation rate for the skilled changes only slightly from 0.67 to 0.64. Consistent with Cortes et al. (2017) and Jaimovich et al. (2020), our results verify that labor force participation can be an important adjustment margin in response to the robotics revolution.

However, this is not the whole story. We find that the unskilled labor force participation rate may *increase* through the *income effect* if the robotics revolution decreases the level of consumption. Labor force participation decision is made so that (45) is satisfied for each type. The right-hand side of (45) is the reservation wage rate while the left-hand side is the marginal utility from home production in units of consumption goods. A decrease in consumption induces the household to supply more labor because the marginal value of home production declines. This is the "added worker effect" (Pissarides, 2000). Our results show the importance of the tension between the discouraged worker effect and the added worker effect.

8 Conclusion

This paper explored whether the robotics revolution would make human obsolete in the process of production. We first demonstrated that the right question is whether the new technology harms the labor market prospect for a certain group of workers, not the entire labor force because we proved that the robotics revolution is beneficial for all workers if workers are homogeneous. We then addressed whether the new technology would cause joblessness for unskilled workers using a multi-factor model with segmented labor markets and search-matching frictions. We found and quantified a testable condition under which the robotics revolution increases the unskilled unemployment. Namely, the threshold elasticity of substitution between routine labor and capital, above which the new technology harms the routine labor market in the long run, is 3.91. This threshold is further reduced to 2.52 if robots and traditional capital are modeled as distinct inputs.

Our results are obtained under two important assumptions, flexible wages and perfect competition in the product market. We conjecture that introduction of wage rigidity in our framework would strengthen our results because the impact of the robotics revolution on the unemployment rate in our model is quantitatively very strong even under flexible wage adjustment through Nash bargaining. On the other hand, the impact of the robotics revolution on the labor market outcomes under imperfect product market competition, especially with the rising market power of firms, is an open question. For this investigation, one requires a richer model of market structure.

Another important avenue for future work is to study household heterogeneity. For tractability, we maintained the assumption that the representative family pools all consumption goods to provide perfect consumption insurance among the family members. While convenient, this assumption prevents us from making any welfare comparison across individuals. Further, because all individuals share the same level of consumption, the income effect on labor market participation decision is the same across all individuals. Further investigations on (the distribution of) nonmarket payoffs such as home production and wealth are needed to make a better prediction on how peoples' participation decisions are affected by the robotics revolution.

Appendix

A Properties of Homogeneous Functions

Because F is homogeneous of degree 1, F_1 is homogeneous of degree 0.[11] This implies

$$F_{11}(x_1, x_2, x_3)x_1 + F_{12}(x_1, x_2, x_3)x_2 + F_{13}(x_1, x_2, x_3)x_3 = 0.$$

Similarly, we obtain

$$F_{21}(x_1, x_2, x_3)x_1 + F_{22}(x_1, x_2, x_3)x_2 + F_{23}(x_1, x_2, x_3)x_3 = 0,$$
$$F_{31}(x_1, x_2, x_3)x_1 + F_{32}(x_1, x_2, x_3)x_2 + F_{33}(x_1, x_2, x_3)x_3 = 0.$$

Decreasing marginal products ($F_{11} < 0, F_{22} < 0, F_{33} < 0$) imply

$$F_{12}(x_1, x_2, x_3)x_2 + F_{13}(x_1, x_2, x_3)x_3 = -F_{11}(x_1, x_2, x_3)x_1 > 0,$$
$$F_{21}(x_1, x_2, x_3)x_1 + F_{23}(x_1, x_2, x_3)x_3 = -F_{22}(x_1, x_2, x_3)x_2 > 0,$$
$$F_{31}(x_1, x_2, x_3)x_1 + F_{32}(x_1, x_2, x_3)x_2 = -F_{33}(x_1, x_2, x_3)x_3 > 0.$$

By symmetry of cross-partial derivatives, these expressions imply that one of the three cross partials ($F_{12} = F_{21}, F_{23} = F_{32}, F_{31} = F_{13}$) can be negative and that if this is the case, then the other two cross partials must be strictly positive.

B Proof of Proposition 2

(i) Substitute (22) into (21) to consider

$$F(x_t^A, x_t^R, a_t k_t) = A\{\mu_1 (x_t^A)^{\rho_1} + (1-\mu_1)[\mu_2 (x_t^R)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}]^{\frac{\rho_1}{\rho_2}}\}^{\frac{1}{\rho_1}}. \tag{46}$$

It is then straightforward to obtain

$$F_1(x_t^A, x_t^R, a_t k_t) = A\{\mu_1 (x_t^A)^{\rho_1} + (1-\mu_1)[\mu_2 (x_t^R)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}]^{\frac{\rho_1}{\rho_2}}\}^{\frac{1}{\rho_1}-1} \mu_1 (x_t^A)^{\rho_1 - 1}.$$

[11] Remember that if $f(x_1, x_2, x_3)$ is homogeneous of degree h, then it satisfies $f_1 x_1 + f_2 x_2 + f_3 x_3 = h f(x_1, x_2, x_3)$.

Thus,

$$\begin{aligned}
F_{13}(x_t^A, x_t^R, a_t k_t) &= A\{\mu_1(x_t^A)^{\rho_1} + (1-\mu_1)[\mu_2(x_t^R)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}]^{\frac{\rho_1}{\rho_2}}\}^{\frac{1}{\rho_1}-2} \\
&\times (1-\rho_1)\mu_1(x_t^A)^{\rho_1-1}(1-\mu_1)[\mu_2(x_t^R)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}]^{\frac{\rho_1}{\rho_2}-1} \\
&\times (1-\mu_2)(a_t k_t)^{\rho_2-1} \\
&\geq 0.
\end{aligned}$$

(ii) From (46), we obtain

$$\begin{aligned}
F_2(x_t^A, x_t^R, ak_t) = A\frac{1}{\rho_1}&\{\mu_1(x_t^A)^{\rho_1} + (1-\mu_1)[\mu_2(x_t^R)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}]^{\frac{\rho_1}{\rho_2}}\}^{\frac{1}{\rho_1}-1} \\
\times \frac{\rho_1}{\rho_2}&(1-\mu_1)[\mu_2(x_t^R)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}]^{\frac{\rho_1}{\rho_2}-1}\rho_2\mu_2(x_t^R)^{\rho_2-1}.
\end{aligned}$$

Thus, we obtain

$$\begin{aligned}
F_{23}(x_t^A, x_t^R, ak_t) = &\left[\frac{1-\rho_1}{\rho_2}\frac{(1-\mu_1)\left[\mu_2\left(x_t^R\right)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}\right]^{\frac{\rho_1}{\rho_2}}}{\mu_1\left(x_t^A\right)^{\rho_1} + (1-\mu_1)\left[\mu_2\left(x_t^R\right)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}\right]^{\frac{\rho_1}{\rho_2}}} + \frac{\rho_1-\rho_2}{\rho_2}\right] \\
&\times A\frac{1}{\rho_2}\left[\mu_2\left(x_t^R\right)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}\right]^{-1} \\
&\times \frac{1}{\rho_2}(1-\mu_1)\left[\mu_2\left(x_t^R\right)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}\right]^{\frac{\rho_1-\rho_2}{\rho_2}} \\
&\times \left\{\mu_1\left(x_t^A\right)^{\rho_1} + (1-\mu_1)\left[\mu_2\left(x_t^R\right)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}\right]^{\frac{\rho_1}{\rho_2}}\right\}^{\frac{1-\rho_1}{\rho_1}} \\
&\times \rho_2\mu_2\left(x_t^R\right)^{\rho_2-1}\rho_2(1-\mu_2)(a_t k_t)^{\rho_2-1}.
\end{aligned}$$

Thus, $F_{23}(x_t^A, x_t^R, ak_t) < 0$ if and only if

$$\frac{1-\rho_1}{\rho_2}\frac{(1-\mu_1)\left[\mu_2\left(x_t^R\right)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}\right]^{\frac{\rho_1}{\rho_2}}}{\mu_1\left(x_t^A\right)^{\rho_1} + (1-\mu_1)\left[\mu_2\left(x_t^R\right)^{\rho_2} + (1-\mu_2)(a_t k_t)^{\rho_2}\right]^{\frac{\rho_1}{\rho_2}}} + \frac{\rho_1-\rho_2}{\rho_2} < 0.$$

Rewrite the condition as

$$(1-\rho_1)\frac{(1-\mu_1)z_t^{\rho_1}}{\mu_1\left(x_t^A\right)^{\rho_1} + (1-\mu_1)z_t^{\rho_1}} + (\rho_1-\rho_2) < 0,$$

or

$$\frac{\rho_2-\rho_1}{1-\rho_1} > \frac{(1-\mu_1)z_t^{\rho_1}}{\mu_1\left(x_t^A\right)^{\rho_1}+(1-\mu_1)z_t^{\rho_1}} \in (0,1). \tag{47}$$

For this inequality to be satisfied, $\rho_2 > \rho_1$ is necessary, and $\rho_2 = 1$ is sufficient. Observe that the cost share of type-A input, which is given by

$$\frac{p_t^A x_t^A}{y_t} = \frac{\mu_1(x_t^A)^{\rho_1}}{\mu_1(x_t^A)^{\rho_1}+(1-\mu_1)z_t^{\rho_1}}.$$

Thus, (47) implies

$$\frac{\rho_2-\rho_1}{1-\rho_1} > \frac{(1-\mu_1)z_t^{\rho_1}}{\mu_1\left(x_t^A\right)^{\rho_1}+(1-\mu_1)z_t^{\rho_1}} = 1-\frac{\mu_1(x_t^A)^{\rho_1}}{\mu_1(x_t^A)^{\rho_1}+(1-\mu_1)z_t^{\rho_1}} = 1-\frac{p_t^A x_t^A}{y_t}.$$

Thus, $F_{23}(x_t^A, x_t^R, ak_t) < 0$ holds if and only if

$$\frac{\rho_2-\rho_1}{1-\rho_1} > 1-\frac{p_t^A x_t^A}{y_t} \Leftrightarrow \frac{p_t^A x_t^A}{y_t} > \frac{1-\rho_2}{1-\rho_1} \Leftrightarrow \frac{1}{1-\rho_2} > \frac{\frac{1}{1-\rho_1}}{\frac{p_t^A x_t^A}{y}}.$$

Note that this inequality is identical to condition (14) in Berg et al. (2018).

C Proof of Lemma 2

(i) Totally differentiate (31) to obtain

$$\frac{da}{a} = (1-\rho_1)\frac{\mu_1}{\mu_1+(1-\mu_1)Z^{\rho_1}}\frac{dZ}{Z} + (1-\rho_2)\frac{\mu_2}{\mu_2+(1-\mu_2)K^{\rho_2}}\frac{dK}{K}, \tag{48}$$

where all coefficients are positive because $\rho_i \leq 1$. If we impose $da = 0$ on (48), then we obtain $dZ/dK < 0$. This defines the shape of the CM locus.

(ii) If we impose $dK = 0$ on (48), then we obtain $dZ/da > 0$. This implies that for each level of K, an increase in a shifts the CM locus upward.

(iii) Solve (34) for p^i to obtain

$$p^i = \frac{(r-\delta+\lambda^i)c^i}{(1-\eta)[1+(\theta^i)^{\psi_i}]^{-1/\psi_i}} + \frac{\eta}{1-\eta}c^i\theta^i + b^i \equiv \Theta_i\left(\theta^i\right),$$

where $\Theta_i'\left(\theta^i\right) > 0$. (36)-(38) imply

$$Z = \frac{1-\phi-u^R}{\phi-u^A}\left[\mu_2+(1-\mu_2)K^{\rho_2}\right]^{\frac{1}{\rho_2}}. \tag{49}$$

Thus, the system of equations determining the LM locus is given by

$$\begin{aligned}
p^A &= A\left[\mu_1 + (1-\mu_1)Z^{\rho_1}\right]^{\frac{1}{\rho_1}-1}\mu_1, \\
p^R &= A\left[\mu_1 + (1-\mu_1)Z^{\rho_1}\right]^{\frac{1}{\rho_1}-1}(1-\mu_1)Z^{\rho_1-1}\left[\mu_2 + (1-\mu_2)K^{\rho_2}\right]^{\frac{1}{\rho_2}-1}\mu_2, \\
p^i &= \frac{(r-\delta+\lambda^i)c^i}{(1-\eta)[1+(\theta^i)^{\psi_i}]^{-1/\psi_i}} + \frac{\eta}{1-\eta}c^i\theta^i + b^i \equiv \Theta_i\left(\theta^i\right), \\
u^A &= \frac{\phi\lambda^A}{\lambda^A + \theta^A[1+(\theta^A)^{\psi_A}]^{-1/\psi_A}} \equiv M_A\left(\theta^A\right), \\
u^R &= \frac{(1-\phi)\,\lambda^R}{\lambda^R + \theta^R[1+(\theta^R)^{\psi_R}]^{-1/\psi_R}} \equiv M_R\left(\theta^R\right), \\
Z &= \frac{1-\phi-u^R}{\phi-u^A}\left[\mu_2 + (1-\mu_2)K^{\rho_2}\right]^{\frac{1}{\rho_2}}.
\end{aligned}$$

Evidently, this system is independent of a. Now, take total differentiation of these equations to obtain

$$\frac{dp^A}{p^A} = (1-\rho_1)\frac{(1-\mu_1)Z^{\rho_1}}{\mu_1+(1-\mu_1)Z^{\rho_1}}\frac{dZ}{Z}, \tag{50}$$

$$\frac{dp^R}{p^R} = (\rho_1-1)\frac{\mu_1}{\mu_1+(1-\mu_1)Z^{\rho_1}}\frac{dZ}{Z} + (1-\rho_2)\frac{(1-\mu_2)K^{\rho_2}}{\mu_2+(1-\mu_2)K^{\rho_2}}\frac{dK}{K}, \tag{51}$$

$$dp^i = \Theta_i'\left(\theta^i\right)d\theta^i, \tag{52}$$

$$du^i = M_i'\left(\theta^i\right)d\theta^i, \tag{53}$$

$$\frac{dZ}{Z} = \frac{1}{\phi-u^A}du^A + \frac{(1-\mu_2)K^{\rho_2}}{\mu_2+(1-\mu_2)K^{\rho_2}}\frac{dK}{K} - \frac{1}{1-\phi-u^R}du^R, \tag{54}$$

where

$$\begin{aligned}
\Theta_i'\left(\theta^i\right) &= \frac{(r-\delta+\lambda^i)c^i}{(1-\eta)}[1+(\theta^i)^{\psi_i}]^{1/\psi_i-1}(\theta^i)^{\psi_i-1} + \frac{\eta}{1-\eta}c^i > 0, \\
M_A'\left(\theta^A\right) &= \frac{-\phi\lambda^A[1+(\theta^A)^{\psi_A}]^{-1/\psi_A-1}}{\{\lambda^A+\theta^A[1+(\theta^A)^{\psi_A}]^{-1/\psi_A}\}^2} < 0, \\
M_R'\left(\theta^R\right) &= \frac{-(1-\phi)\,\lambda^R[1+(\theta^R)^{\psi_R}]^{-1/\psi_R-1}}{\{\lambda^R+\theta^R[1+(\theta^R)^{\psi_R}]^{-1/\psi_R}\}^2} < 0.
\end{aligned}$$

From these expressions, we obtain

$$\frac{dZ}{dK} = \frac{Z}{K}\Omega, \tag{55}$$

where

$$\Omega = \frac{\left[1 - \frac{(1-\rho_2)p^R}{1-\phi-u^R}\frac{M_R'\left(\theta^R\right)}{\Theta_R'\left(\theta^R\right)}\right]\frac{(1-\mu_2)K^{\rho_2}}{\mu_2+(1-\mu_2)K^{\rho_2}}}{1-\frac{(1-\rho_1)p^A}{\phi-u^A}\frac{M_A'\left(\theta^A\right)}{\Theta_A'\left(\theta^A\right)}\frac{(1-\mu_1)Z^{\rho_1}}{\mu_1+(1-\mu_1)Z^{\rho_1}} - \frac{(1-\rho_1)p^R}{1-\phi-u^R}\frac{M_R'(\theta^R)}{\Theta_R'(\theta^R)}\frac{\mu_1}{\mu_1+(1-\mu_1)Z^{\rho_1}}} > 0. \tag{56}$$

Thus, Z and K are positively related along the LM locus.

D Proof of Proposition 3

We totally differentiate the system of equations determining a steady state. From (55), we obtain

$$\frac{dZ}{Z} = \Omega\frac{dK}{K}, \tag{57}$$

where Ω is defined in (56). Substitute (57) into (48) and (51) to obtain

$$\frac{da}{a} = \left[\frac{(1-\rho_1)\,\mu_1}{\mu_1+(1-\mu_1)Z^{\rho_1}}\Omega + \frac{(1-\rho_2)\,\mu_2}{\mu_2+(1-\mu_2)K^{\rho_2}}\right]\frac{dK}{K}, \tag{58}$$

$$\frac{dp^R}{p^R} = \left[\frac{(\rho_1-1)\,\mu_1}{\mu_1+(1-\mu_1)Z^{\rho_1}}\Omega + \frac{(1-\rho_2)\,(1-\mu_2)K^{\rho_2}}{\mu_2+(1-\mu_2)K^{\rho_2}}\right]\frac{dK}{K}. \tag{59}$$

These expressions imply

$$\frac{dp^R}{da} = \frac{\frac{(\rho_1-1)\mu_1}{\mu_1+(1-\mu_1)Z^{\rho_1}}\Omega + \frac{(1-\rho_2)(1-\mu_2)K^{\rho_2}}{\mu_2+(1-\mu_2)K^{\rho_2}}}{\frac{(1-\rho_1)\mu_1}{\mu_1+(1-\mu_1)Z^{\rho_1}}\Omega + \frac{(1-\rho_2)\mu_2}{\mu_2+(1-\mu_2)K^{\rho_2}}}\frac{p^R}{a}.$$

Because the denominator is positive, we can show that $dp^R/da < 0$ holds if and only if

$$\frac{(1-\rho_2)\,(1-\mu_2)K^{\rho_2}}{\mu_2+(1-\mu_2)K^{\rho_2}} < \frac{(1-\rho_1)\,\mu_1}{\mu_1+(1-\mu_1)Z^{\rho_1}}\Omega,$$

which is equivalent to

$$\frac{1}{1-\rho_1} < \frac{1}{1-\rho_2}\frac{\mu_1}{\mu_1+(1-\mu_1)Z^{\rho_1}} + \frac{p^A}{\phi-u^A}\frac{M_A'\left(\theta^A\right)}{\Theta_A'\left(\theta^A\right)}\frac{(1-\mu_1)Z^{\rho_1}}{\mu_1+(1-\mu_1)Z^{\rho_1}},$$

or

$$\frac{1}{1-\rho_1} < \frac{1}{1-\rho_2}\frac{p^A x^A}{y} - \varepsilon^A\left(1-\frac{p^A x^A}{y}\right),$$

where

$$\begin{aligned}\varepsilon^A &= \frac{dx^A}{dp^A}\frac{p^A}{x^A} = -\frac{M_A'\left(\theta^A\right)}{\Theta_A'\left(\theta^A\right)}\frac{p^A}{\phi-u^A}\\ &= \frac{\phi\lambda^A[1+(\theta^A)^{\psi_A}]^{-1/\psi_A-1}\{\lambda^A+\theta^A[1+(\theta^A)^{\psi_A}]^{-1/\psi_A}\}^{-2}}{\frac{(r-\delta+\lambda^A)c^A}{1-\eta}[1+(\theta^A)^{\psi_A}]^{1/\psi_A-1}(\theta^A)^{\psi_A-1}+\frac{\eta}{1-\eta}c^A}\frac{p^A}{\phi-u^A} > 0.\end{aligned} \tag{60}$$

E Proof of Proposition 4

For both types of capital to exist, $r_t^T = r_t^R$ must be satisfied in any equilibrium. Other equilibrium conditions are obtained as in the previous sections. We summarize the equilibrium conditions as follows: $r_t^T = A[\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1}{\rho_1}-1}\mu_1$, $p_t = A[\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1}{\rho_1}-1}(1-\mu_1)Z_t^{\rho_1-1}[\mu_2 + (1-\mu_2)K_t^{\rho_2}]^{\frac{1}{\rho_2}-1}\mu_2$, $r_t^R = A[\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1}{\rho_1}-1}(1-\mu_1)Z_t^{\rho_1-1}[\mu_2 + (1-\mu_2)K_t^{\rho_2}]^{\frac{1}{\rho_2}-1}(1-\mu_2)K_t^{\rho_2-1}a_t^R$, $Z_t = z_t/k_t^T$ and $K_t = a_t^R k_t^R / x_t$, $r_t = r_t^T = r_t^R$, $x_t = \ell_t = 1 - u_t$, $y_t - cv_t = C_t + k_{t+1}^T + k_{t+1}^R - (1-\delta)(k_t^T + k_t^R)$, (4), (6), (7), and (8).

In any steady state, $r^T = r^R = r = \beta^{-1} - 1 + \delta$. Thus, we obtain

$$r = A\left[\mu_1 + (1-\mu_1)Z^{\rho_1}\right]^{\frac{1}{\rho_1}-1}\mu_1, \tag{61}$$

$$r = A\left[\mu_1 + (1-\mu_1)Z^{\rho_1}\right]^{\frac{1}{\rho_1}-1}(1-\mu_1)Z^{\rho_1-1}\left[\mu_2 + (1-\mu_2)K^{\rho_2}\right]^{\frac{1}{\rho_2}-1}(1-\mu_2)K^{\rho_2-1}a^R. \tag{62}$$

Note that (61) uniquely determines Z. Given this value, (62) uniquely determines K for each level of a^R. Given the value of Z and K as a function of a^R, the price of labor service,

$$p = A\left[\mu_1 + (1-\mu_1)Z^{\rho_1}\right]^{\frac{1}{\rho_1}-1}(1-\mu_1)Z^{\rho_1-1}\left[\mu_2 + (1-\mu_2)K^{\rho_2}\right]^{\frac{1}{\rho_2}-1}\mu_2, \tag{63}$$

is determined separately from the rest of the model (just as in the model of Section 2). Because Z is constant and independent of a^R in any steady state, (62) implies that an increase in a^R increases K. Thus, (63) implies $dp/da^R > 0$.

F Heterogeneous Capital: Equilibrium Conditions

The representative household's lifetime utility is given by (3) and the budget constraint is now $C_t + \tau_t + k_{t+1}^R + k_{t+1}^T - (1-\delta_R)k_t^R - (1-\delta_T)k_t^T = w_t^A \ell_t^A + w_t^R \ell_t^R + b^A u_t^A + b^R u_t^R + r_t^R k_t^R + r_t^T k_t^T + \pi_t$. The household chooses the sequences of C_t, k_{t+1}^T, and k_{t+1}^R to maximize (3) subject to the budget constraint. The first-order conditions imply $C_t^{-\sigma} = \Lambda_t$ and $\Lambda_t = \beta\Lambda_{t+1}(r_{t+1}^T + 1 - \delta_T) = \beta\Lambda_{t+1}(r_{t+1}^R + 1 - \delta_R)$, from which we obtain $r_t^T - \delta_T = r_t^R - \delta_R$ for all t. The transversality condition is $\lim_{t\to\infty}\Lambda_t\beta^t k_t^j = 0$, $(j = T, R)$ where Λ_t is the Lagrange multiplier for this problem. Each individual discounts future payoffs by $B_t = (r_{t+1}^T + 1 - \delta_T)^{-1} = (r_{t+1}^R + 1 - \delta_R)^{-1} = \beta(C_{t+1}/C_t)^{-\sigma}$, from which $r^T - \delta_T = r^R - \delta_R = \beta^{-1} - 1$ in any steady state. Thus, $\delta_T < \delta_R$ implies $r^T < r^R$.

The production function for the final good producer is now

$$y_t = A(k_t^T)^\alpha [\mu_1 (x_t^A)^{\rho_1} + (1-\mu_1) z_t^{\rho_1}]^{\frac{1-\alpha}{\rho_1}}, \tag{64}$$

$$z_t = [\mu_2 (x_t^R)^{\rho_2} + (1-\mu_2)(a_t^R k_t^R)^{\rho_2}]^{\frac{1}{\rho_2}}, \tag{65}$$

where $0 < \alpha < 1$ and $\rho_1 < \rho_2 < 1$. Profit maximization implies $p_t^A = (1-\alpha)A(K_t^T)^\alpha [\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1-\alpha}{\rho_1}-1}\mu_1$, $p_t^R = (1-\alpha)A(K_t^T)^\alpha[\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1-\alpha}{\rho_1}-1}(1-\mu_1)Z_t^{\rho_1-1}[\mu_2 + (1-\mu_2)(K_t^R)^{\rho_2}]^{\frac{1}{\rho_2}-1}\mu_2$, $r_t^R = (1-\alpha)A(K_t^T)^\alpha[\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1-\alpha}{\rho_1}-1}(1-\mu_1)Z_t^{\rho_1-1}[\mu_2 + (1-\mu_2)(K_t^R)^{\rho_2}]^{\frac{1}{\rho_2}-1}(1-\mu_2)(K_t^R)^{\rho_2-1}a_t^R$, and $r_t^T = \alpha A(K_t^T)^{\alpha-1}[\mu_1 + (1-\mu_1)Z_t^{\rho_1}]^{\frac{1-\alpha}{\rho_1}}$, where $Z_t = z_t/x_t^A$, $K_t^T = k_t^T/x_t^A$, and $K_t^R = a_t^R k_t^R / x_t^R$.

Because $r^T = \beta^{-1} - 1 + \delta_T$ in any steady state, $r^T = \alpha A (K^T)^{\alpha-1}[\mu_1 + (1-\mu_1)Z^{\rho_1}]^{\frac{1-\alpha}{\rho_1}}$ determines K^T as a function of Z. We then substitute it into the other expressions above to obtain the following:

$$\begin{aligned} r^R &= \tilde{A}[\mu_1 + (1-\mu_1)Z^{\rho_1}]^{\frac{1}{\rho_1}-1}(1-\mu_1)Z^{\rho_1-1} \\ &\quad \times [\mu_2 + (1-\mu_2)(K^R)^{\rho_2}]^{\frac{1}{\rho_2}-1}(1-\mu_2)(K^R)^{\rho_2-1}a^R, \end{aligned} \tag{66}$$

$$p^A = \tilde{A}[\mu_1 + (1-\mu_1)Z^{\rho_1}]^{\frac{1}{\rho_1}-1}\mu_1, \tag{67}$$

$$p^R = \tilde{A}[\mu_1 + (1-\mu_1)Z^{\rho_1}]^{\frac{1}{\rho_1}-1}(1-\mu_1)Z^{\rho_1-1}[\mu_2 + (1-\mu_2)(K^R)^{\rho_2}]^{\frac{1}{\rho_2}-1}\mu_2, \tag{68}$$

where

$$\tilde{A} = (1-\alpha)\, A \left(\frac{\alpha A}{r^T}\right)^{\frac{\alpha}{1-\alpha}} = (1-\alpha)\, A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\beta^{-1} - 1 + \delta_T}\right)^{\frac{\alpha}{1-\alpha}}. \tag{69}$$

A steady-state equilibrium under heterogeneous capital is determined by a set $\{Z, K^T, K^R, p^A, p^R, \theta^A, \theta^R, u^A, u^R, z, k\}$ that satisfies (66)-(68), (34)-(38), and $k^T = x^A K^T = (\phi - u^A)K^T$.

G Flow Value of Nonwork

Our calibration strategy in Section 4 follows that of Hall and Milgrom (2008) so that the flow value of nonwork satisfies $b^i = 0.71 w^i$. This value is the sum of an estimate of the unemployment benefit replacement rate, which is 0.25, and other components including the saved disutility of work measured in units of consumption. However, it is widely recognized in the business cycle literature that unemployment tends to be more responsive to a change in the demand for labor when the target flow value of nonwork, b^i in our model, is closer to the flow value of work w^i (Hagedorn and Manovskii, 2008). It is therefore useful to conduct a sensitivity analysis.

Suppose now that the flow value of nonwork b^i is calibrated to satisfy $b^i = 0.4w^i$. This corresponds to Shimer (2005), who assumes that the unemployment benefit replacement rate is 0.40 and that job seekers do not receive any utility value such as the saved disutility of work.

Table 8: Flow Value of Nonwork

	Case 1: $\frac{1}{1-\rho_2} = 2.5$		Case 2: $\frac{1}{1-\rho_2} = 5$		Case 3: $\frac{1}{1-\rho_2} = 10$	
b^i/w^i	0.71	0.40	0.71	0.40	0.71	0.40
$-\frac{\Delta u^A}{\Delta p^A}\frac{p^A}{u^A}$	1.25	0.73	0.88	0.60	0.62	0.47
$-\frac{\Delta u^R}{\Delta p^R}\frac{p^R}{u^R}$	2.10	1.05	3.70	1.29	17.00	2.00
$\frac{\Delta w^A}{\Delta p^A}\frac{p^A}{w^A}$	0.99	1.00	0.99	1.00	0.99	1.00
$\frac{\Delta w^R}{\Delta p^R}\frac{p^R}{w^R}$	0.99	1.02	0.99	1.02	0.99	1.02

The results, summarized in Table 8, are in line with the business cycle literature. When the (target) flow value of nonwork satisfies $b^i = 0.4w^i$ rather than $b^i = 0.71w^i$, the unemployment elasticity in response to a change in the demand for labor is lower. Interestingly, the wage elasticity is about 1.0 for all models and for all cases.

H Cobb-Douglas Matching Function

Our specification of the matching function, which we adopt from den Haan et al. (2000) and Hagedorn and Manovskii (2008), is motivated by the restriction that the job-finding probabilities Q^i (as well as the vacancy-filling probabilities q^i) must be restricted to be in between 0 and 1 even under a large persistent shock such as the robotics revolution. An important drawback of this choice is that this specification is not widely used in the literature.

Because the most widely accepted specification in the literature is a Cobb-Douglas form, it is useful to study this case as a robustness analysis. In particular, we are interested in whether the large magnitudes of the effects of the robotics revolution on the labor market variables depend on our specification of the matching function. With Cobb-Douglas, we can no longer guarantee $0 < Q^i < 1$ and $0 < q^i < 1$, but we can interpret the results as those for a continuous-time model because the steady-state equations are essentially identical to those for a continuous-time counterpart of our model.

Suppose now that the number of new employees in market i ($i = A, R$) is determined by $m^i(u_t^i)^{\xi}(v_t^i)^{1-\xi}$. We assume that each market has a distinct matching constant $m^i > 0$ and that both markets share the same matching elasticity, $0 < \xi < 1$. Each type-i vacancy is matched to a type-i job-seeker with probability $q_t^i = m^i(u_t^i)^{\xi}(v_t^i)^{1-\xi}/v_t^i = m^i(\theta_t^i)^{-\xi}$, where $\theta_t^i = v_t^i/u_t^i$ is labor market tightness in market i. Similarly, the probability that a type-i job-seeker is matched with a type-i vacancy is given by $Q_t^i = m^i(u_t^i)^{\xi}(v_t^i)^{1-\xi}/u_t^i = m^i(\theta_t^i)^{1-\xi} = \theta_t^i q_t^i$. The calibrated parameters for Case 2 is given in Table 9.

Table 9: Calibrated Parameter Values: Cobb-Douglas Matching Function

c^A	c^R	m_A	m_R	b^A	b^R	α	μ_1	μ_2	A
0.205	0.211	0.340	0.534	0.587	0.349	0.350	0.579	0.969	0.486

Table 10 shows that the two different matching functions generate nearly identical results, except for the unemployment rate for the unskilled.

Table 10: Comparison of Matching Functions

	Initial	Case 1: $\frac{1}{1-\rho_2} = 2.5$		Case 2: $\frac{1}{1-\rho_2} = 5$		Case 3: $\frac{1}{1-\rho_2} = 10$	
	$a^R = 1$	CD	HRW	CD	HRW	CD	HRW
y	1.00	1.14	1.14	1.43	1.43	1.87	1.85
u^A/ϕ	2.6%	2.3%	2.3%	1.9%	2.0%	1.5%	1.7%
$u^R/(1-\phi)$	8.3%	8.2%	8.2%	10.7%	12.0%	25.2%	43.7%
u	5.7%	5.6%	5.6%	6.7%	7.5%	14.6%	24.8%
w^A	0.83	0.91	0.91	1.06	1.06	1.28	1.27
w^R	0.49	0.49	0.49	0.43	0.43	0.36	0.37
w^A/w^R	1.68	1.84	1.84	2.46	2.45	3.55	3.45
Labor share	0.61	0.57	0.57	0.48	0.47	0.38	0.37

I Labor Market Participation: Equilibrium Conditions

The household chooses a set of sequences $\{C_t, k_{t+1}^R, k_{t+1}^T, \ell_{t+1}^A, \ell_{t+1}^R, u_t^A, u_t^R\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \zeta_A \ln n_t^A + \zeta_R \ln n_t^R \right]$$

subject to $C_t + \tau_t + k^R_{t+1} + k^T_{t+1} - (1-\delta_R)k^R_t - (1-\delta_T)k^T_t = w^A_t \ell^A_t + w^R_t \ell^R_t + \bar{b}^A u^A_t + \bar{b}^R u^R_t + r^R_t k^R_t + r^T_t k^T_t + \pi_t$, $\phi = \ell^A_t + u^A_t + n^A_t$, $1-\phi = \ell^R_t + u^R_t + n^R_t$, $\ell^A_{t+1} = \left(1-\lambda^A\right)\ell^A_t + Q^A_t u^A_t$, and $\ell^R_{t+1} = \left(1-\lambda^R\right)\ell^R_t + Q^R_t u^R_t$. The Lagrangian for this problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \zeta_A \ln n^A_t + \zeta_R \ln n^R_t \right.$$
$$+ \Lambda_t \left[\begin{array}{c} w^A_t \ell^A_t + w^R_t \ell^R_t + b^A\left(\phi - \ell^A_t - n^A_t\right) + b^R\left(1-\phi-\ell^R_t - n^R_t\right) + r^R_t k^R_t + r^T_t k^T_t + \pi_t \\ -C_t - \tau_t - k^R_{t+1} - k^T_{t+1} + (1-\delta_R)k^R_t + (1-\delta_T)k^T_t \end{array} \right]$$
$$+ \Lambda^A_t \left[\left(1-\lambda^A\right)\ell^A_t + Q^A_t\left(\phi - \ell^A_t - n^A_t\right) - \ell^A_{t+1}\right]$$
$$\left. + \Lambda^R_t \left[\left(1-\lambda^R\right)\ell^R_t + Q^R_t\left(1-\phi - \ell^R_t - n^R_t\right) - \ell^R_{t+1}\right] \right\}.$$

The first-order conditions are:

$$\begin{aligned}
C_t^{-\sigma} - \Lambda_t &= 0, \\
\zeta_A \frac{1}{n^A_t} - \Lambda_t b^A - \Lambda^A_t Q^A_t &= 0, \\
\zeta_R \frac{1}{n^R_t} - \Lambda_t b^R - \Lambda^R_t Q^R_t &= 0, \\
-\Lambda^A_t + \beta\Lambda^A_{t+1}\left[1-\lambda^A - Q^A_{t+1}\right] + \beta\Lambda_{t+1}\left[w^A_{t+1} - b^A\right] &= 0, \\
-\Lambda^R_t + \beta\Lambda^R_{t+1}\left[1-\lambda^R - Q^R_{t+1}\right] + \beta\Lambda_{t+1}\left[w^R_{t+1} - b^R\right] &= 0, \\
-\Lambda_t + \beta\Lambda_{t+1}\left[r^R_t + (1-\delta_R)\right] &= 0, \\
-\Lambda_t + \beta\Lambda_{t+1}\left[r^T_t + (1-\delta_T)\right] &= 0.
\end{aligned}$$

The transversality condition is $\lim_{t\to\infty} \Lambda_t \beta^t k^R_t = \lim_{t\to\infty} \Lambda_t \beta^t k^T_t = 0$.

The first-order conditions with respect to C_t, k^T_{t+1}, and k^R_{t+1} imply $C_t^{-\sigma} = \Lambda_t$ and $\Lambda_t = \beta\Lambda_{t+1}(r^T_{t+1} + 1 - \delta_T) = \beta\Lambda_{t+1}(r^R_{t+1} + 1 - \delta_R)$, from which we obtain $r^T_{t+1} - \delta_T = r^R_{t+1} - \delta_R$.

In any steady state,

$$\begin{aligned}
\zeta_A \frac{1}{n^A} &= \Lambda b^A + \Lambda^A Q^A \\
\zeta_R \frac{1}{n^R} &= \Lambda b^R + \Lambda^R Q^R, \\
\beta\Lambda \frac{w^A - b^A}{1-\beta\left(1-\lambda^A - Q^A\right)} &= \Lambda^A, \\
\beta\Lambda \frac{w^R - b^R}{1-\beta\left(1-\lambda^R - Q^R\right)} &= \Lambda^R,
\end{aligned}$$

from which we obtain

$$\zeta_A \frac{1}{n^A} C^\sigma = b^A + \frac{Q^A \beta \left(w^A - b^A\right)}{1 - \beta \left(1 - \lambda^A - Q^A\right)}, \tag{70}$$

$$\zeta_R \frac{1}{n^R} C^\sigma = b^R + \frac{Q^R \beta \left(w^R - b^R\right)}{1 - \beta \left(1 - \lambda^R - Q^R\right)}. \tag{71}$$

Accordingly, the steady-state conditions (36)-(38) must be replaced with $z = [\mu_2(1 - \phi - u^R - n^R)^{\rho_2} + (1 - \mu_2)(ak)^{\rho_2}]^{1/\rho_2}$, $z = (\phi - u^A - n^A)Z$, $ak = (1 - \phi - u^R - n^R)K$, respectively. Other steady-state conditions are identical to those presented in Appendix F.

References

[1] Acemoglu, Daron, and David Autor. "Skills, Tasks and Technologies: Implications for Employment and Earnings." in, *Handbook of Labor Economics.* Vol. 4, edited by David Card and Orley Ashenfelter, Elsevier, 2011, 1043-1171.

[2] Acemoglu, Daron, and Pascual Restrepo. "Modeling Automation." *American Economic Review* 108 (2018a) 48–53.

[3] Acemoglu, Daron, and Pascual Restrepo. "The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment." *American Economic Review* 108 (2018b) 1488–1542.

[4] Aghion, Philippe, and Peter Howitt. "Growth and Unemployment." *Review of Economic Studies* 61 (1994) 477–494.

[5] Aghion, Philippe, Benjamin F. Jones, and Charles I. Jones. "Artificial Intelligence and Economic Growth." in, *The Economics of Artificial Intelligence: An Agenda*, edited by Ajay Agrawal, Joshua Gans, and Avi Goldfarb, The University of Chicago Press, 2019.

[6] Andolfatto, David. "Business Cycles and Labor-Market Search." *American Economic Review* 86 (1996) 112–132.

[7] Autor, David H., and David Dorn. "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market." *American Economic Review* 103 (2013) 1553–1597.

[8] Autor, David H., Frank Levy, and Richard J. Murnane. "The Skill Content of Recent Technological Change: An Empirical Exploration." *Quarterly Journal of Economics* 118 (2003) 1279–1333.

[9] Baldwin, Richard. *The Globotics Upheaval: Globalization, Robotics, and the Future of Work.* Oxford University Press, 2019.

[10] Berg, Andrew, Edward F. Buffie, and Luis-Felipe Zanna. "Should We Fear the Robot Revolution? (The Correct Answer Is Yes)." *Journal of Monetary Economics* 97 (2018) 117–148.

[11] Brynjolfsson, Erik, and Lorin M. Hitt. "Computing Productivity: Firm-Level Evidence." *Review of Economics and Statistics* 85 (2003) 793–808.

[12] Brynjolfsson, Erik, and Andrew McAfee. *The Second Machine Age: Work, Progress, and Prosperity in a Time of Brilliant Technologies*. WW Norton & Company, 2014.

[13] Caselli, Francesco, and Alan Manning. "Robot Arithmetic: New Technology and Wages." *American Economic Review: Insights* 1 (2019) 1–12.

[14] Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. "Unemployment and Business Cycles." *Econometrica* 84 (2016) 1523–1569.

[15] Ciccone, Antonio, and Giovanni Peri. "Long-Run Substitutability between More and Less Educated Workers: Evidence from U.S. States, 1950–1990." *Review of Economics and Statistics* 87 (2005) 652–663.

[16] Coles, Melvyn G., and Eric Smith. "Cross-Section Estimation of the Matching Function: Evidence from England and Wales." *Economica* (1996) 589–597.

[17] Cortes, Guido Matias, Nir Jaimovich, and Henry E. Siu. "Disappearing Routine Jobs: Who, How, and Why?" *Journal of Monetary Economics* 91 (2017) 69–87.

[18] den Haan, Wouter J., Garey Ramey, and Joel Watson. "Job Destruction and Propagation of Shocks." *American Economic Review* 90 (2000) 482–498.

[19] Diamond, Peter A. "Aggregate Demand Management in Search Equilibrium." *Journal of Political Economy* 90 (1982) 881–894.

[20] Fallick, Bruce, and Charles A. Fleischman. "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows." Federal Reserve Board, *Finance and Economics Discussion Series* 2004-34, 2004.

[21] Ford, Martin. *Rise of the Robots: Technology and the Threat of a Jobless Future*. Basic Books, Yew York, 2015.

[22] Frey, Carl Benedikt, and Michael A. Osborne. "The Future of Employment: How Susceptible Are Jobs to Computerisation?" *Technological Forecasting and Social Change* 114 (2017) 254–280.

[23] Garibaldi, Pietro, and Etienne Wasmer. "Equilibrium Search Unemployment, Endogenous Participation, and Labor Market Flows." *Journal of the European Economic Association* 3 (2005) 851–882.

[24] Goldin, Claudia Dale, and Lawrence F. Katz. *The Race between Education and Technology*. Harvard University Press, 2009.

[25] Goos, Maarten, and Alan Manning. "Lousy and Lovely Jobs: The Rising Polarization of Work in Britain." *Review of Economics and Statistics* 89 (2007) 118–133.

[26] Graetz, Georg, and Guy Michaels. "Robots at Work." *Review of Economics and Statistics* 100 (2018) 753–768.

[27] Greenwood, Jeremy, and Zvi Hercowitz. "The Allocation of Capital and Time over the Business Cycle." *Journal of Political Economy* 99 (1991) 1188–1214.

[28] Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell. "Long-Run Implications of Investment-Specific Technological Change." *American Economic Review* (1997) 342–362.

[29] Hagedorn, Marcus, and Iourii Manovskii. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review* 98 (2008) 1692–1706.

[30] Hagedorn, Marcus, Iourii Manovskii, and Sergiy Stetsenko. "Taxation and Unemployment in Models with Heterogeneous Workers." *Review of Economic Dynamics* 19 (2016) 161–189.

[31] Hall, Robert E., and Paul R. Milgrom. "The Limited Influence of Unemployment on the Wage Bargain." *American Economic Review* 98 (2008) 1653–74.

[32] Harari, Yuval Noah. *Homo Deus: A Brief History of Tomorrow*. Random House, 2016.

[33] Hornstein, Andreas, Per Krusell, and Giovanni L. Violante. "Technology-Policy Interaction in Frictional Labour-Markets." *Review of Economic Studies* 74 (2007) 1089–1124.

[34] Jaimovich, Nir, Itay Saporta-Eksten, Henry E. Siu, and Yaniv Yedid-Leviet. "The Macroeconomics of Automation: Data, Theory, and Policy Analysis," NBER Working Paper 27122, National Bureau of Economic Research, 2020. http://www.nber.org/papers/w27122.

[35] Katz, Lawrence F., and Kevin M. Murphy. "Changes in Relative Wages, 1963–1987: Supply and Demand Factors." *Quarterly Journal of Economics* 107 (1992) 35–78.

[36] Krause, Michael U., and Thomas A. Lubik. "The (Ir) relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions." *Journal of Monetary Economics* 54 (2007) 706–727.

[37] Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, and Ayşegül Şahin. "Gross Worker Flows over the Business Cycle." *American Economic Review* 107 (2017) 3447–3476.

[38] Krusell, Per, Lee E. Ohanian, José-Víctor Ríos-Rull, and Giovanni Violante. "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis." *Econometrica* 68 (2000) 1029–1053.

[39] Merz, Monika. "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics* 36 (1995) 269–300.

[40] Mortensen, Dale T., and Eva Nagypal. "More on Unemployment and Vacancy Fluctuations." *Review of Economic Dynamics* 10 (2007) 327-347.

[41] Mortensen, Dale T., and Christopher A. Pissarides. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies* 61 (1994) 397–415.

[42] Mortensen, Dale T., and Christopher A. Pissarides. "Technological Progress, Job Creation, and Job Destruction." *Review of Economic Dynamics* 1 (1998) 733–753.

[43] Mortensen, Dale T., and Christopher A. Pissarides. "Unemployment Responses to 'Skill-Biased' Technology Shocks: The Role of Labour Market Policy." *Economic Journal* 109 (1999) 242–265.

[44] Petrongolo, Barbara, and Christopher A. Pissarides. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature* 39 (2001) 390–431.

[45] Pissarides, Christopher A. *Equilibrium Unemployment Theory*. MIT press, 2000.

[46] Pissarides, Christopher A. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica* 77 (2009) 1339–1369.

[47] Pissarides, Christopher A., and Giovanna Vallanti. "The Impact of TFP Growth on Steady-State Unemployment." *International Economic Review* 48 (2007) 607–640.

[48] Shimer, Robert. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95 (2005) 25–49.

[49] Trigari, Antonella. "Equilibrium Unemployment, Job Flows, and Inflation Dynamics." *Journal of Money, Credit and Banking* 41 (2009) 1–33.

[50] Tripier, Fabien. "Can the Labor Market Search Model Explain the Fluctuations of Allocations of Time?" *Economic Modelling* 21 (2004) 131–146.

[51] Veracierto, Marcelo. "On the Cyclical Behavior of Employment, Unemployment and Labor Force Participation." *Journal of Monetary Economics* 55 (2008) 1143–1157.

[52] vom Lehn, Christian. "Labor Market Polarization, the Decline of Routine Work, and Technological Change: A Quantitative Analysis." *Journal of Monetary Economics* 110 (2020) 62–80.