KOCHI UNIVERSITY OF TECHNOLOGY

Social Design Engineering Series SDES-2016-5

# Globalisation and inequality: A dynamic general equilibrium model of unequal exchange

Roberto Veneziani
*Queen Mary University of London*

Naoki Yoshihara
*Department of Economics, University of Massachusetts Amherst*
*Institute of Economic Research, Hitotsubashi University*
*School of Economics and Management, Kochi University of Technology*

15th April, 2016

School of Economics and Management
Research Center for Social Design Engineering
Kochi University of Technology

# Globalisation and Inequality: A Dynamic General Equilibrium Model of Unequal Exchange*

Roberto Veneziani† and Naoki Yoshihara‡

April 14, 2016

**Abstract**

A dynamic general equilibrium model that generalises Roemer's [23] economy with a global capital market is analysed. An axiomatic analysis of the concept of unequal exchange (UE) between countries is developed at general dynamic equilibria. The class of UE definitions that satisfy three fundamental properties - including a correspondence between wealth, class and UE exploitation status - is completely characterised. It is shown that this class is nonempty and a definition of UE exploitation between countries is proposed, which is theoretically robust and firmly anchored to empirically observable data. The full class and UE exploitation structure of the international economy is derived in equilibrium.

**JEL classification**: D63; F02; B51.

**Keywords**: Exploitation, classes, unequal exchange, international economy.

---

*Paper prepared for the special issue in honour of John Roemer. We thank Jon Cogliano, Amitava Dutt, Gary Dymski, Peter H. Matthews, Rajiv Sethi, Gil Skillman, Peter Skott, and audiences in London, New York, Chicago, Amherst, and Sendai, for useful comments and suggestions. The usual disclaimer applies.

†School of Economics and Finance, Queen Mary University of London, Mile End Road, London E1 4NS, UK. E-mail: r.veneziani@qmul.ac.uk

‡(Corresponding) Department of Economics, University of Massachusetts Amherst, 200 Hicks Way, Amherst, MA 01003, USA; The Institute of Economic Research, Hitotsubashi University, Kunitachi, Tokyo 186-0004 Japan; and School of Management, Kochi University of Technology, Kochi 782-8502, Japan. E-mail: n_yoshihara_1967@yahoo.co.jp

# 1 Introduction

The last four decades have witnessed the increasing integration of different national economies and the widespread adoption of 'neoliberal' policies. This phenomenon, often labelled 'globalisation', has far-reaching economic, social, and political implications, and has stimulated a vast debate. Globalisation has significant effects within each economy, but special attention has been paid to its repercussions on the relations between developed and less developed countries. For although many poor countries have benefited from trade and have experienced spectacular growth, which has lifted a big part of the world population above the poverty line, globalisation has also been characterised by the economic stagnation of backward areas, large inequalities in income and standard of living among countries, and asymmetries in bargaining power in the international arena.

Unsurprisingly, different schools of thought have emphasised different aspects of the global economy. Radical authors and the so-called *dependence school* have historically emphasised the inequalities that characterise international relations as a product of exploitative relations between rich and poor nations.[1] In his classic work, for example, Arghiri Emmanuel [8] has argued that the core-periphery structure of international relations generates an *unequal exchange* (UE) between rich and poor nations. According to Emmanuel, in a world economy characterised by institutionalised wage differentials between developed and less developed nations, the international trade of commodities and capital mobility across borders cause a transfer of surplus labour from poor nations with lower capital-labour ratios to wealthy nations with higher capital-labour ratios, which results in the impoverishment of the former to the advantage of the latter.

In contrast, the conventional view emphasises the positive effects of globalisation and various authors have questioned the empirical and theoretical relevance of UE theory. Empirically, for example, Ben-David ([3], p.653) has argued that evidence suggests that "movement toward free trade may actually have just the opposite effect [than predicted by UE theory], leading to a *reduction* in income disparity among countries".[2] Theoretically, UE theory is criticised because it is inconsistent with the principle of comparative

[1] The literature is too vast for a comprehensive list of references. We refer the reader to the excellent reviews by Bacha [2] and Griffin and Gurley [12].

[2] Yet, recent empirical studies reach exactly the opposite conclusion. See, for example, Slaughter [27].

advantage, according to which profit equalisation and capital flows from rich to poor countries have growth-inducing and inequality-reducing effects.[3] Interestingly, however, both the 'dependence school' and mainstream authors suggest that any increase in international inequalities should be attributed to some major imperfections in global markets, such as institutionalised wage differentials, increasing returns to scale, wealth-dependent borrowing constraints in international financial markets, and so on.[4]

In a seminal contribution, Roemer [23] has provided a rigorous, consistent definition of UE, and an elegant analysis of UE between countries, in a static model with revenue-maximising countries and a Leontief technology. He has shown that even assuming perfectly competitive global commodity and credit markets, the international economy is characterised by the emergence of classes in the credit market and UE exploitative relations. In equilibrium, the world economy is characterised both by mutual gains from trade and by asymmetric international relations because the economic development of less developed countries is crucially dependent on capital exports from developed countries, and surplus is transferred *from* the former *to* the latter via international capital markets. "Unequal exchange does not preempt mutual gains from trade" (Roemer [23], p.35).[5] Thus, Roemer [23] helps to clarify the scope both of the standard, neoclassical approach to free trade and of the UE theory of international relations. Crucially, deviations from competitive assumptions and market imperfections are unessential for his results: unequal and asymmetric relations between countries "can be entirely explained by differential capital-labor ratios across countries" (Roemer [23], p.34).

As insightful as Roemer's [22, 23] contributions are, it is still an open question whether UE theory provides a general framework to analyse international relations. It is not clear, for example, whether Roemer's key conclusions hold under more general assumptions concerning preferences and technology, and outside of static, one-period models (or, at least, outside of what may be interpreted as stationary equilibria of dynamic models). In later contributions, Roemer himself has raised doubts on the generality of

[3] See, for example, the debate between Paul Samuelson and Arghiri Emmanuel in *The Journal of International Economics* in 1978.

[4] The literatue is vast here, too. See, among the many others, the classic contributions by Krugman [13] and Matsuyama [16], and the comprehensive discussion in Raj [21].

[5] This insight is compatible with the classical Marxian theory of exploitation, as Marx ([15], chapter 20, (e)) notes that "a richer country exploits a poorer one, even when the latter benefits from the exchange."

UE theory,[6] and on the possibility of identifying a rigorous definition of UE exploitation that captures our fundamental normative intuitions.

In this paper, we set up a dynamic general equilibrium model of a global economy, which generalises Roemer's [23] economies, in order to analyse the robustness of UE theory and how unequal international relations may arise in a competitive setting. We extend Roemer's [22, 23] analysis in two main directions. Formally, the model incorporates more general assumptions on preferences - by assuming that countries welfare depends both on consumption *and* on leisure - and on technology - by allowing for convex production sets. Furthermore, we analyse international relations in a general dynamic context by explicitly considering intertemporal decisions *and* by focusing on general equilibria, without restricting attention to stationary states.

Methodologically, instead of analysing international relations based on a specific definition of UE exploitation, we start from first principles and adopt a novel axiomatic approach to UE theory.[7] We formalise three fundamental intuitions of UE theory. The first one, called *Labour Exploitation* (**LE**), is a domain axiom that captures some basic properties defining the core of UE theory that all admissible definitions should (and all of the main definitions do) satisfy. Intuitively, according to **LE**, exploitative international relations are characterised by systematic differences between the labour performed by agents in a country $\nu$ and the amount of labour 'contained' in some reference commodity bundles that capture consumption possibilities of citizens in $\nu$.[8] The other two properties, called the *Wealth-Exploitation Correspondence Principle* (**WECP**) and the *Class-Exploitation Correspondence Principle* (**CECP**) capture axiomatically two fundamental intuitions of UE theory originally introduced by Roemer [22, 23], who proved them to hold in certain economies, under specific UE definitions. The **WECP** states that the exploitation status of countries in the international arena should be determined by their level of development (proxied by the value of their productive endowments). According to the **CECP**, a correspondence should exist between a country's status in the international capital market and its

[6] For example, Roemer [24]. See Veneziani [29] for a thorough discussion.

[7] We have applied the axiomatic method to exploitation theory in other contributions (see, for example, [32, 34, 31]). In this paper, however, we extend the analysis to dynamic economies *and* consider a different set of axioms.

[8] A rigorous statement of **LE** - and of the other axioms - is in section 4. This characterisation is conceptually related to the classic theories of unequal exchange (Emmanuel [8]) and underdevelopment (Amin [1], Frank [9]).

exploitation status: in equilibrium, nations that can only optimise by lending capital on the credit market should emerge as UE exploiters, whereas nations that can only optimise by borrowing capital should be UE exploited.

We derive a complete characterisation of the class of definitions of UE exploitation that satisfy all three axioms in general dynamic equilibria of the international economy. Then, we prove that this class is nonempty: a definition recently proposed by Yoshihara and Veneziani [32, 34, 31] based on the 'New Interpretation' (Duménil [5, 6]; Foley [10, 11]; Duménil et al [7]) satisfies all three properties. Contrary to the received view, a rigorous and logically consistent definition of unequal, exploitative exchange exists, which is firmly anchored to empirically observable data and generalises the key insights of UE theory to general, dynamic international economies.

To be sure, this does not fully answer the question of the normative relevance of UE and the wrongfulness of exploitative international relations. Yet the rigorous, axiomatic characterisation of a nonempty class of definitions that preserve some key insights of UE theory is a crucial first step in order to address that question. In the Appendix we briefly elaborate on this issue and explore some of the normative implications of UE theory in a general dynamic setting.

# 2 The Model

The economy consists of a set $\mathcal{N} = \{1, ..., N\}$ of countries, with generic element $\nu$, in which a sequence of nonoverlapping generations exist, each living for $T$ periods,[9] and indexed by the date of birth $kT$, $k = 0, 1, 2, ...$ In every period $t$, countries consume $n$ produced commodities, and leisure.

Technology is freely available to all countries: in every period $t$, capitalists in each country can operate any activity in the production set $P \subseteq \mathbb{R}^{2n+1}$, with elements of the form $\alpha = (-\alpha_l, -\underline{\alpha}, \overline{\alpha})$, where $\alpha_l \in \mathbb{R}_+$ is the direct labour input; $\underline{\alpha} \in \mathbb{R}^n_+$ are the inputs of the $n$ goods; and $\overline{\alpha} \in \mathbb{R}^n_+$ are the outputs of the $n$ goods. The net output vector arising from $\alpha$ is denoted as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. Let $\mathbf{0}$ be the null vector. The set $P$ is assumed to be a closed convex cone containing the origin in $\mathbb{R}^{2n+1}$, and to satisfy the following standard properties.[10]

[9] Unless otherwise specified, our results hold both if $T$ is finite and if it is infinite. In the latter case, only one infinitely-lived generation exists.

[10] Vector inequalities: for all $x, y \in \mathbb{R}^p$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, p)$; $x \geq y$

**Assumption 1 (A1).** For all $\alpha \in P$, if $\overline{\alpha} \geq \mathbf{0}$ then $\alpha_l > 0$ and $\underline{\alpha} \geq \mathbf{0}$.
**Assumption 2 (A2).** For all $c \in \mathbb{R}^n_+$, there exists $\alpha \in P$ such that $\widehat{\alpha} \geqq c$.
**Assumption 3 (A3).** For all $\alpha \in P$, and for all $(-\underline{\alpha}', \overline{\alpha}') \in \mathbb{R}^n_- \times \mathbb{R}^n_+$, if $(-\underline{\alpha}', \overline{\alpha}') \leqq (-\underline{\alpha}, \overline{\alpha})$ then $(-\alpha_l, -\underline{\alpha}', \overline{\alpha}') \in P$.

A1 implies that some labour and some capital are indispensable to produce any output; A2 states that any non-negative commodity bundle is producible as net output; A3 is a standard free disposal condition. The set of productively efficient activities is $\partial P = \{\alpha \in P : \nexists \alpha' \in P \text{ such that } \alpha' > \alpha\}$.

Commodities and capital can freely migrate across borders, while labour is immobile. In every $t$, $(p_t, r_t)$ is the $1 \times (n+1)$ international price vector, where $p_t$ denotes the prices of the $n$ commodities and $r_t$ is the interest rate that prevails in competitive capital markets. In order to focus on international inequalities, agents are assumed to be identical within each country; thus, the superscript $\nu$ denotes both a country and its representative agent.

Following Roemer [22, 23], we explicitly model the time structure of individual exchange and production decisions, and incorporate the fact that production takes time. To be specific, each production period $t$ is divided into two stages: the capital market and the market for productive assets operate at the beginning of $t$, where goods are exchanged at the prices $p_{t-1}$ ruling at the end of $t-1$ and beginning of $t$. Thus, at the beginning of every period $t$, $\omega^\nu_t$ is the vector of productive assets owned by $\nu$, - where $\omega^\nu_{kT}$ denotes the endowments inherited when born in $kT$, - and the market value of $\nu$'s endowments, $\nu$'s wealth, is $W^\nu_t = p_{t-1}\omega^\nu_t$.

At the beginning of every $t$, each $\nu \in \mathcal{N}$ can borrow an amount $p_{t-1}\underline{\beta}^\nu_t$ on the international credit market to purchase $\underline{\beta}^\nu_t$ in order to operate production activity $\beta^\nu_t = \left(-\beta^\nu_{lt}, -\underline{\beta}^\nu_t, \overline{\beta}^\nu_t\right) \in P$. Otherwise, it can use its wealth $W^\nu_t$ either to purchase capital goods $\underline{\alpha}^\nu_t$ to operate activity $\alpha^\nu_t = (-\alpha^\nu_{lt}, -\underline{\alpha}^\nu_t, \overline{\alpha}^\nu_t) \in P$; or to buy commodities $\delta^\nu_t \in \mathbb{R}^n_+$ to be stored and sold at the end of the period; or to lend capital $z^\nu_t \in \mathbb{R}_+$ abroad.

Because production takes time, output is exchanged on the final goods market at the end of $t$, at end-of-period prices $p_t$. For each country $\nu \in \mathcal{N}$, proceedings from production are given by $p_t\left(\overline{\alpha}^\nu_t + \overline{\beta}^\nu_t\right)$ and the return to lending $z^\nu_t$ is $(1+r_t)\, z^\nu_t$, thus gross national income at the end of $t$ is $p_t\left(\overline{\alpha}^\nu_t + \overline{\beta}^\nu_t\right) + (1+r_t)\, z^\nu_t$ from which the rental cost of the borrowed capital

if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, p)$.

$(1+r_t)\, p_{t-1}\underline{\beta}_t^\nu$ must be paid. The rest of $\nu$'s income can be used to purchase consumption goods $c_t^\nu \in \mathbb{R}_+^n$ and to finance accumulation $\omega_{t+1}^\nu \in \mathbb{R}_+^n$ .

Given production decisions $(\alpha_t^\nu, \beta_t^\nu)$, in every period $t$, the total amount of labour performed by agents in $\nu \in \mathcal{N}$ is given by $\Lambda_t^\nu = \alpha_{lt}^\nu + \beta_{lt}^\nu$ and it cannot exceed $\nu$'s endowment, $L^\nu$. For simplicity, and without loss of generality, all countries are assumed to be endowed with the same amount of labour $L^\nu = L > 0$. Therefore for each $\nu \in \mathcal{N}$, leisure enjoyed at $t$ is $l_t^\nu = L - \Lambda_t^\nu$, and we assume that country $\nu$'s welfare at $t$ can be represented by a function $u : \mathbb{R}_+^n \times [0, L] \rightarrow \mathbb{R}_+$: $u(c_t, l_t)$ can be interpreted either as a standard neoclassical utility function or as an objectivist index of primary goods, or capabilities. The latter interpretation is more in line with exploitation theory, but the two interpretations are formally equivalent.[11]

In order to characterise the structure of international relations and the dynamic pattern of exploitation and classes, it is necessary to impose some structure on the function $u$. For the sake of simplicity, we assume that $u(c_t, l_t) = \phi(L - \Lambda_t) + v(c_t)$, where $v : \mathbb{R}_+^n \rightarrow \mathbb{R}$ and $\phi : [0, L] \rightarrow \mathbb{R}$ are strictly increasing and twice differentiable. Furthermore, in order to avoid a number of unnecessary technicalities, we shall assume that $v$ is strictly quasi-concave and homogeneous of degree one, while $\phi$ is strictly concave with $\lim_{\Lambda \rightarrow 0} \phi'(L - \Lambda) = 0$ and $\lim_{\Lambda \rightarrow L} \phi'(L - \Lambda) = \infty$.

These assumptions significantly generalise the canonical models of exploitation theory by Roemer [22, 23], and are standard in international economics - and specifically, in the literature on Heckscher-Ohlin models (see, for example, Chen [4]). The assumptions on $\phi$ rule out rather implausible equilibria with countries performing zero labour, or enjoying no leisure at all. Further, as forcefully argued by Silvestre [26], it is theoretically appropriate to assume $v$ to be linearly homogeneous if an objectivist view is adopted and $u$ is interpreted as an objectivist welfare index.[12] It is worth stressing, however, that the restrictions on $u$ are imposed mostly for technical convenience, and the main results of this paper can be derived under more general assumptions, albeit at the cost of a significant increase in technicalities.[13]

[11] For a discussion of subjective and objective principles, see Roemer and Veneziani [25] and, in the context of exploitation theory, Yoshihara and Veneziani [35].

[12] Even if $u$ is interpreted as a subjective welfare index, the assumptions seem reasonable, given that $u$ represents the welfare of a nation, which depends on the country's consumption expenditure (and national income), and working hours.

[13] For example, it is possible to allow for heterogeneous preferences over consumption goods with $u^\nu(c_t^\nu, l_t^\nu) = \phi(L - \Lambda_t^\nu) + v^\nu(c_t^\nu)$; a weakly concave $\phi$; $v$ being homogeneous of

Let $c^\nu = \{c_t^\nu\}_{t=kT}^{(k+1)T-1}$ be $\nu$'s lifetime consumption plan; and likewise for $\alpha^\nu, \beta^\nu, z^\nu, \delta^\nu$, and $\Lambda^\nu$, and let $\omega^\nu = \left\{\omega_{t+1}^\nu\right\}_{t=kT}^{(k+1)T-1}$ be $\nu$'s lifetime accumulation plan. Let $(\mathbf{p}, \mathbf{r}) = \{(p_t, r_t)\}_{t=kT}^{(k+1)T-1}$ be the path of international price vectors during the lifetime of a generation. Let $\xi^\nu = (\alpha^\nu, \beta^\nu, z^\nu, \delta^\nu, c^\nu, \omega^\nu)$ denote a generic intertemporal plan for $\nu$. Let $0 < \rho \leqq 1$ be the time preference factor. Each $\nu$ is assumed to choose $\xi^\nu$ to maximise welfare subject to the constraints that in every $t$, (1) gross national income is sufficient for consumption and accumulation; (2) wealth is sufficient for production and lending; (3) production activities are technologically feasible. Finally, (4) reproducibility requires resources not to be depleted: generation $k$ is constrained to bequeath at least as much wealth as they inherited. Formally, each $\nu$ solves programme $MP^\nu$.[14]

$$MP^\nu\colon\; V(\omega_{kT}^\nu) = \max_{\xi^\nu} \sum_{t=kT}^{(k+1)T-1} \rho^t u\left(c_t^\nu, l_t^\nu\right),$$

subject to: for every $t = kT, \ldots ,(k+1)\,T-1$,

$$\begin{aligned}
p_t\overline{\alpha}_t^\nu + \left[p_t\overline{\beta}_t^\nu - (1+r_t)\,p_{t-1}\underline{\beta}_t^\nu\right] + (1+r_t)\,z_t^\nu + p_t\delta_t^\nu &= p_t c_t^\nu + p_t\omega_{t+1}^\nu, &(1)\\
p_{t-1}\left(\underline{\alpha}_t^\nu + \delta_t^\nu\right) + z_t^\nu &= p_{t-1}\omega_t^\nu, &(2)\\
\alpha_t^\nu, \beta_t^\nu &\in P,\ \Lambda_t^\nu \leqq L, &(3)\\
p_{(k+1)T-1}\omega_{(k+1)T}^\nu &\geqq p_{(k+1)T-1}\omega_{kT}^\nu. &(4)
\end{aligned}$$

$MP^\nu$ is a suitable way of modelling country $\nu$'s decision problem, given the representative-agent assumption, and it generalises Roemer's [22, 23] static models in which countries maximise national income.

In order to capture the role of financial markets in exploitative international relations, only short-term credit contracts are considered as in Roemer [22, 23]: within each period, countries can operate on the international capital market to finance their production plans, but contracts do not extend over time and credit plays a limited role in fostering accumulation. Consumption, debt, and savings must be financed out of current revenue. Due to the possibility of saving, and noting that net savings are allowed to be

degree $k < 1$; and so on.

[14] The first two constraints are written as equalities without loss of generality, given the monotonicity of $u$.

negative, however, Roemer's [22, 23] static models are generalised by allowing for intertemporal trade-offs *within* a country, consistently with a dynamic setting in which agents live for more than one period.

For all $\nu \in \mathcal{N}$, let $\mathcal{O}^{\nu}(\mathbf{p}, \mathbf{r})$ be the set of vectors $\xi^{\nu}$ that solve $MP^{\nu}$ at prices $(\mathbf{p}, \mathbf{r})$. Let $\Omega_{kT} = \left(\omega_{kT}^{1}, \omega_{kT}^{2}, ..., \omega_{kT}^{N}\right)$. Let $E(P, \mathcal{N}, u, \rho, \Omega_{kT})$, or as a shorthand notation $E(\Omega_{kT})$, denote the international economy with technology $P$, countries $\mathcal{N}$, welfare function $u$ with discount factor $\rho$, and productive endowments $\Omega_{kT}$. Let $c_t = \sum_{\nu \in \mathcal{N}} c_t^{\nu}$; and likewise for all other variables. For the sake of simplicity, let "for all $t$" stand for "for all $t = kT, \ldots, (k+1)T-1$". Following Roemer [22, 23], the equilibrium concept can now be defined.[15]

**Definition 1:** A *reproducible solution* (RS) for $E(\Omega_{kT})$ is a price vector $(\mathbf{p}, \mathbf{r})$ and an associated profile of actions $(\xi^{\nu})_{\nu \in \mathcal{N}}$ such that:

(i) $\xi^{\nu} \in \mathcal{O}^{\nu}(\mathbf{p}, \mathbf{r})$ for all $\nu \in \mathcal{N}$;
(ii) $\overline{\alpha}_t + \overline{\beta}_t + \delta_t \geqq c_t + \omega_{t+1}$ for all $t$;
(iii) $\underline{\alpha}_t + \underline{\beta}_t + \delta_t \leqq \omega_t$ for all $t$;
(iv) $p_{t-1}\underline{\beta}_t = z_t$ for all $t$;
(v) $\omega_{(k+1)T} \geqq \omega_{kT}$.

In other words, at a RS, (i) every country optimises. Conditions (ii) and (iii) are standard excess demand conditions in the markets for final goods and capital goods respectively: in each market, aggregate demand should not exceed aggregate supply in any period. Condition (iii) also ensures that in each period, aggregate production is feasible given the stock of capital goods. Condition (iv) requires that the international credit market clears in every period. Finally, reproducibility - condition (v) - requires that every generation leaves to the following at least as many resources as they inherited. This is a standard condition in Ramsey-type growth models with a finite horizon (see, for example, Morishima's [18] classic model) and it is quite natural given that countries - rather than individuals - are the focus of analysis. For, although each generation dies, the country itself exists across successive generations, and so its capital stock should not be depleted.

In the rest of this section, we derive some preliminary results that describe the characteristics of the equilibria of the international economy. In what follows, even if it is not explicitly stated, we shall focus only on *non-trivial* RS's in which *some* production takes place in every period.

[15] The existence of a reproducible solution is proved in the Addendum.

First of all, the strict monotonicity of $v$ implies that at any RS, it must be $p_t > \mathbf{0}$ for all $t$. Moreover, at a non-trivial RS, it must be $\max_{\alpha\in P\setminus\{\mathbf{0}\}} p_t\overline{\alpha} - (1+r_t)\, p_{t-1}\underline{\alpha} \geqq 0$ at all $t$. To see this, note that if $\max_{\alpha\in P\setminus\{\mathbf{0}\}} p_t\overline{\alpha} - (1+r_t)\, p_{t-1}\underline{\alpha} < 0$ at some $t$, then at the solution to $MP^\nu$ it must be $\alpha_t^\nu = \beta_t^\nu = \mathbf{0}$ for all $\nu \in \mathcal{N}$, contradicting the assumption that the RS is non-trivial. This implies that at a non-trivial RS $\max_{\alpha\in P\setminus\{\mathbf{0}\}} \frac{p_t\overline{\alpha}}{p_{t-1}\underline{\alpha}} \geqq (1+r_t)$, all $t$, which in turn implies that $\max_{\alpha\in P\setminus\{\mathbf{0}\}} \frac{p_t\overline{\alpha}}{p_{t-1}\underline{\alpha}} \geqq \max_i \frac{p_{it}}{p_{it-1}}$, all $t$. For if there were some commodity $i$ such that $\frac{p_{it}}{p_{it-1}} > \max_{\alpha\in P\setminus\{\mathbf{0}\}} \frac{p_t\overline{\alpha}}{p_{t-1}\underline{\alpha}} \geqq (1+r_t)$ at some $t$, then at the solution to $MP^\nu$ it would be $\alpha_t^\nu = \mathbf{0}$ and $z_t^\nu = 0$ for all $\nu \in \mathcal{N}$. By Definition 1(iv), this implies that $\beta_t^\nu = \mathbf{0}$ for all $\nu \in \mathcal{N}$, contradicting the assumption that the RS is non-trivial.

Given $(\mathbf{p}, \mathbf{r})$, at any $t$, let $w_t^{\max} = \max_{\alpha\in P} \frac{p_t\overline{\alpha}-(1+r_t)p_{t-1}\underline{\alpha}}{\alpha_l}$: by the assumptions on $P$, $w_t^{\max}$ is well-defined. Hence let $P_t^w(\mathbf{p},\mathbf{r}) = \left\{\alpha \in P \mid w_t^{\max} = \frac{p_t\overline{\alpha}-(1+r_t)p_{t-1}\underline{\alpha}}{\alpha_l}\right\}$. Proposition 1 proves that only processes with the highest return to labour are activated and, as is well-known in international economics, even without an international labour market, wages are equalised in all countries at all $t$.

**Proposition 1:** *Let $\left((\mathbf{p},\mathbf{r}),(\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be a RS for $E(\Omega_{kT})$. Then, $w_t^{\max} > 0$ for all $t$, and $\alpha_t^\nu, \beta_t^\nu \in P_t^w(\mathbf{p},\mathbf{r})$ for all $\nu$ and all $t$.*

**Proof:** 1. Suppose, contrary to the statement, that $w_t^{\max} \leqq 0$ for some $t$. Then, by A1, at the solution to $MP^\nu$, it must be $\alpha_t^\nu = \beta_t^\nu = \mathbf{0}$ for all $\nu$, which contradicts the nontriviality of the RS.

2. The second part of the statement follows immediately from $MP^\nu$. ■

Lemma 1 proves a useful property of the set of solutions of $MP^\nu$.

**Lemma 1:** *Let $(\mathbf{p},\mathbf{r})$ be a price vector such that $w_t^{\max} > 0$ for all $t$. For all $\nu \in \mathcal{N}$, if $(\alpha^\nu, \beta^\nu, z^\nu, \delta^\nu, c^\nu, \omega^\nu)$ solves $MP^\nu$, then $(\alpha'^\nu, \beta'^\nu, z'^\nu, \delta^\nu, c^\nu, \omega^\nu)$ also solves $MP^\nu$ whenever $\alpha_t'^\nu + \beta_t'^\nu = \alpha_t^\nu + \beta_t^\nu$, $z_t'^\nu + p_{t-1}\underline{\alpha}_t'^\nu = z_t^\nu + p_{t-1}\underline{\alpha}_t^\nu$, $\alpha_t'^\nu, \beta_t'^\nu \in P$, and $z_t'^\nu \geqq 0$, all $t$.*

**Proof:** By construction, it is immediate to check that the constraints of $MP^\nu$ are all satisfied. Furthermore, $\beta_{lt}'^\nu + \alpha_{lt}'^\nu = \beta_{lt}^\nu + \alpha_{lt}^\nu$, at all $t$ and $c'^\nu = c^\nu$. Hence $(\alpha'^\nu, \beta'^\nu, z'^\nu, \delta^\nu, c^\nu, \omega^\nu)$ yields the same welfare. ■

By Lemma 1, and the convexity of $P$, we can consider solutions of $MP^\nu$ with $\alpha^\nu = \mathbf{0}$, without loss of generality.

At the solution to $MP^{\nu}$, $p_{(k+1)T-1}\omega^{\nu}_{(k+1)T} = p_{(k+1)T-1}\omega^{\nu}_{kT}$ for all $\nu$ and all $k$. Therefore, at a RS, $\omega_{(k+1)T} = \omega_{kT}$ since $p_{(k+1)T-1} > \mathbf{0}$. Moreover, $\omega^{\nu}_{(k+1)T} = \omega^{\nu}_{kT}$ is feasible and optimal for any $\nu \in \mathcal{N}$. Therefore, without loss of generality, we can focus on solutions with $\omega^{\nu}_{(k+1)T} = \omega^{\nu}_{kT}$. Hence, if $\left((\mathbf{p},\mathbf{r}), (\xi^{\nu})_{\nu\in\mathcal{N}}\right)$ is a RS for $E(\Omega_{kT})$, then it is also a RS for $E(\Omega_{(k+1)T})$, and in what follows generation $k = 0$ can be considered without loss of generality.

A subset of equilibria that are of particular interest are those where agents optimise at an interior solution. Thus:

**Definition 2:** An *interior RS* (IRS) for $E(\Omega_{kT})$ is a RS such that for all $\nu$, $\xi^{\nu} \in \mathcal{O}^{\nu}(\mathbf{p},\mathbf{r})$ is an interior solution to $MP^{\nu}$ with $c^{\nu}_t > \mathbf{0}$ for all $t$.

The next result proves a necessary condition for an IRS.

**Lemma 2:** *Let* $\left((\mathbf{p},\mathbf{r}), (\xi^{\nu})_{\nu\in\mathcal{N}}\right)$ *be an IRS for* $E(\Omega_0)$. *Then for all* $\nu \in \mathcal{N}$, $\Lambda^{\nu} = \Lambda^{*} = \{\Lambda^{*}_t\}_{t=kT}^{(k+1)T-1}$ *where* $\frac{\phi'(L-\Lambda^{*}_t)}{w^{\max}_t} = \rho(1+r_{t+1})\frac{\phi'\left(L-\Lambda^{*}_{t+1}\right)}{w^{\max}_{t+1}}$, *for all* $t$.

**Proof:** 1. By Proposition 1, for all $\nu \in \mathcal{N}$, at the solution to $MP^{\nu}$, it must be $(1+r_t)\, p_{t-1}\omega^{\nu}_t + w^{\max}_t \Lambda^{\nu}_t = p_t c_t + p_t \omega^{\nu}_{t+1}$, all $t$. Then it is immediate to prove that, at an interior solution $\xi^{\nu} \in \mathcal{O}^{\nu}(\mathbf{p},\mathbf{r})$ to $MP^{\nu}$, for all $t$ and all $\nu \in \mathcal{N}$, it must be $\frac{v'_i(c^{*\nu}_t)}{v'_j(c^{*\nu}_t)} = \frac{p_{it}}{p_{jt}}$ and $\phi'\left(L - \Lambda^{*\nu}_t\right) = w^{\max}_t \frac{v'_i(c^{*\nu}_t)}{p_{it}}$ for all $i, j$. By the linear homogeneity of $v(.)$, this implies that at an interior solution to $MP^{\nu}$, at all $t$, it must be $\frac{c^{*\nu}_{it}}{c^{*\nu}_{jt}} = \frac{c^{*\mu}_{it}}{c^{*\mu}_{jt}}$ for all $\nu, \mu \in \mathcal{N}$, and therefore $v'_i\left(c^{*\nu}_t\right) = v'_i\left(c^{*\mu}_t\right)$ and $\phi'\left(L - \Lambda^{*\nu}_t\right) = \phi'\left(L - \Lambda^{*\mu}_t\right)$ for all $\nu, \mu \in \mathcal{N}$. The first part of the statement then follows from the strict concavity of $\phi$.

2. At any $t$, let $c^{*}_t \in \mathbb{R}^n_+$ be such that $\frac{v'_i(c^{*}_t)}{v'_j(c^{*}_t)} = \frac{p_{it}}{p_{jt}}$, for all $i, j$. Then by step 1, at an IRS, it must be $c^{*\nu}_t = k^{\nu}_t c^{*}_t$ where $k^{\nu}_t = \frac{(1+r_t)p_{t-1}\omega^{\nu}_t + w^{\max}_t\Lambda^{*}_t - p_t\omega^{\nu}_{t+1}}{p_t c^{*}_t} > 0$, at all $t$ and for all $\nu \in \mathcal{N}$. Take any two adjacent periods $t$,$t+1$, and consider $\nu \in \mathcal{N}$ such that $p_t\omega^{\nu}_{t+1} > 0$. Consider a small one-period perturbation of $\omega^{\nu}$ such that $\mathrm{d}k^{\nu}_t = -\frac{1}{p_t c^{*}_t} p_t \mathrm{d}\omega^{\nu}_{t+1}$ and $\mathrm{d}k^{\nu}_{t+1} = \frac{(1+r_{t+1})}{p_{t+1}c^{*}_{t+1}} p_t \mathrm{d}\omega^{\nu}_{t+1}$. By the linear homogeneity of $v$, the resulting change in welfare is $v(c^{*}_t)\mathrm{d}k^{\nu}_t + \rho v(c^{*}_{t+1})\mathrm{d}k^{\nu}_{t+1} = -v(c^{*}_t)\frac{1}{p_t c^{*}_t} p_t \mathrm{d}\omega^{\nu}_{t+1} + v(c^{*}_{t+1})\rho\frac{(1+r_{t+1})}{p_{t+1}c^{*}_{t+1}} p_t \mathrm{d}\omega^{\nu}_{t+1}$.

3. By step 1, at an IRS, at all $t$, $\phi'\left(L - \Lambda^{*}_t\right) = w^{\max}_t \frac{v'_i(c^{*\nu}_t)}{p_{it}}$ for all $i$ and all $\nu \in \mathcal{N}$. By the linear homogeneity of $v$, this implies that $\phi'\left(L - \Lambda^{*}_t\right) = w^{\max}_t \frac{v(c^{*\nu}_t)}{p_t c^{*\nu}_t} = w^{\max}_t \frac{v(c^{*}_t)}{p_t c^{*}_t}$. Using the latter expression, the change in welfare can

be written equivalently as $-\frac{\phi'(L-\Lambda_t^*)}{w_t^{\max}}p_t \mathrm{d}\omega_{t+1}^{\nu} + \rho\left(1+r_{t+1}\right)\frac{\phi'\left(L-\Lambda_{t+1}^*\right)}{w_{t+1}^{\max}}p_t \mathrm{d}\omega_{t+1}^{\nu}$. Therefore a necessary condition for $\xi^{\nu} \in \mathcal{O}^{\nu}(\mathbf{p}, \mathbf{r})$ to be an interior solution to $MP^{\nu}$ is that $\left[-\frac{\phi'(L-\Lambda_t^*)}{w_t^{\max}} + \rho(1+r_{t+1})\frac{\phi'\left(L-\Lambda_{t+1}^*\right)}{w_{t+1}^{\max}}\right] p_t \mathrm{d}\omega_{t+1}^{\nu} \leqq 0$ for all $\mathrm{d}\omega_{t+1}^{\nu}$, which holds only if the expression in brackets is equal to zero. ■

The condition in Lemma 2 is the Euler equation deriving from $MP^{\nu}$. Observe that if $\phi$ assumed to be concave but not necessarily strictly so, each country $\nu$ may have a continuum of optimal values of $\Lambda^{\nu}$, but it would still be optimal for all countries to choose the common intertemporal profile of labour supply identified in Lemma 2.

# 3 The International Class Structure

As a first step in the analysis of the structure of international relations, we derive the international class structure, where "classes of countries can be defined with reference to the use of the credit market" (Roemer [23], p.54). Let $(a_1, a_2, a_3)$ be a vector where $a_i \in \{+, \mathbf{0}\}$, $i = 1, 3$, $a_2 \in \{+, 0\}$, and "+" means a non-zero vector in the appropriate place. Let $A^{\nu} = \sum_{t=0}^{T-1} \alpha_t^{\nu}$, $B^{\nu} = \sum_{t=0}^{T-1} \beta_t^{\nu}$, and $Z^{\nu} = \sum_{t=0}^{T-1} z_t^{\nu}$. Because agents live for more than one period, there are two extensions of Roemer's definition of classes.

**Definition 3:** Let $\left((\mathbf{p}, \mathbf{r}), (\xi^{\nu})_{\nu\in\mathcal{N}}\right)$ be a RS for $E(\Omega_0)$. Country $\nu$ is said to be a *member of $WP_t$ class* $(a_1, a_2, a_3)$ *in* $t$, if there is a $\xi'^{\nu} \in \mathcal{O}^{\nu}(\mathbf{p}, \mathbf{r})$ with $\omega'^{\nu} = \omega^{\nu}$ such that $(\alpha_t'^{\nu}, z_t'^{\nu}, \beta_t'^{\nu})$ has the form $(a_1, a_2, a_3)$ in $t$. Similarly, $\nu$ is said to be a *member of $WL$ class* $(a_1, a_2, a_3)$, if there is a $\xi'^{\nu} \in \mathcal{O}^{\nu}(\mathbf{p}, \mathbf{r})$ such that $(A'^{\nu}, Z'^{\nu}, B'^{\nu})$ has the form $(a_1, a_2, a_3)$.

There are eight conceivable classes $(a_1, a_2, a_3)$ for each definition, but only four of them are theoretically relevant, as forcefully argued by Roemer [22]. At a RS $\left((\mathbf{p}, \mathbf{r}), (\xi^{\nu})_{\nu\in\mathcal{N}}\right)$ for $E(\Omega_0)$, let $\Gamma^{\nu} = \{(A'^{\nu}, Z'^{\nu}, B'^{\nu}) \mid \xi'^{\nu} \in \mathcal{O}^{\nu}(\mathbf{p}, \mathbf{r})\}$, and at all $t$, $\Gamma_t^{\nu} = \{(\alpha_t'^{\nu}, z_t'^{\nu}, \beta_t'^{\nu}) \mid \xi'^{\nu} \in \mathcal{O}^{\nu}(\mathbf{p}, \mathbf{r})$ with $\omega'^{\nu} = \omega^{\nu}\}$. As a shorthand, we shall say that $\Gamma^{\nu}$ has a solution of the form $(a_1, a_2, a_3) \setminus (a_1', a_2', a_3')$ to mean that $\Gamma^{\nu}$ contains a vector $(A'^{\nu}, Z'^{\nu}, B'^{\nu})$ of the form $(a_1, a_2, a_3)$ but not one of the form $(a_1', a_2', a_3')$; and likewise for $\Gamma_t^{\nu}$. Then, according to the

$WL$ definition:

$$\begin{aligned} C^1 &= \{\nu \in \mathcal{N} \mid \Gamma^\nu \text{ has a solution of the form } (+,+,\mathbf{0}) \setminus (+,0,\mathbf{0})\}, \\ C^2 &= \{\nu \in \mathcal{N} \mid \Gamma^\nu \text{ has a solution of the form } (+,0,\mathbf{0})\}, \\ C^3 &= \{\nu \in \mathcal{N} \mid \Gamma^\nu \text{ has a solution of the form } (+,0,+) \setminus (+,0,\mathbf{0})\}, \\ C^4 &= \{\nu \in \mathcal{N} \mid \Gamma^\nu \text{ has a solution of the form } (\mathbf{0},0,+)\}. \end{aligned}$$

$WP_t$ classes $C_t^1 - C_t^4$ are similarly specified, replacing $\Gamma^\nu$ with $\Gamma_t^\nu$. Countries in $C^1$ (*resp.* $C_t^1$) are net lenders of capital in the global market; countries in $C^2$ (*resp.* $C_t^2$) can optimise without using the capital market; countries in $C^3$ (*resp.* $C_t^3$) must borrow foreign capital to optimise; countries in $C^4$ (*resp.* $C_t^4$) must borrow all of their operating capital. This definition of classes based on credit relations conveys the intuition that a country's position in the capital market affects its international status.

Below, it is proved that in equilibrium the set of countries $\mathcal{N}$ can indeed be partitioned into these four $WP_t$ classes at all $t$. First, Lemma 3 proves that $WP_t$ classes $(+,+,+)$ and $(\mathbf{0},+,+)$ can be ignored.

**Lemma 3**: *Let $\left((\mathbf{p},\mathbf{r}),(\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be an IRS for $E(\Omega_0)$. Let $\xi^\nu \in \mathcal{O}^\nu(\mathbf{p},\mathbf{r})$ be such that $\nu$ is a member of $WP_t$ class $(+,+,+)$ or $(\mathbf{0},+,+)$ in $t$. Then:*

$$\begin{aligned} &\textit{if } z_t'^\nu > p_{t-1}\underline{\beta}_t'^\nu \textit{ for all } (\alpha_t'^\nu, z_t'^\nu, \beta_t'^\nu) \in \Gamma_t^\nu, \textit{ then } \nu \in C_t^1; \\ &\textit{if } z_t'^\nu = p_{t-1}\underline{\beta}_t'^\nu \textit{ for some } (\alpha_t'^\nu, z_t'^\nu, \beta_t'^\nu) \in \Gamma_t^\nu, \textit{ then } \nu \in C_t^2; \\ &\textit{if } z_t'^\nu < p_{t-1}\underline{\beta}_t'^\nu \textit{ for all } (\alpha_t'^\nu, z_t'^\nu, \beta_t'^\nu) \in \Gamma_t^\nu, \textit{ then } \nu \in C_t^3. \end{aligned}$$

**Proof:** 1. By the convexity of $MP^\nu$, it follows that if $z_t'^\nu < p_{t-1}\underline{\beta}_t'^\nu$ for some $(\alpha_t'^\nu, z_t'^\nu, \beta_t'^\nu) \in \Gamma_t^\nu$ and $z_t''^\nu > p_{t-1}\underline{\beta}_t''^\nu$ for some other $(\alpha_t''^\nu, z_t''^\nu, \beta_t''^\nu) \in \Gamma_t^\nu$, then there exists $(\alpha_t'''^\nu, z_t'''^\nu, \beta_t'''^\nu) \in \Gamma_t^\nu$ such that $z_t'''^\nu = p_{t-1}\underline{\beta}_t'''^\nu$. Therefore, the three cases in the statement are mutually exclusive and they decompose the set of agents with $W_t^\nu > 0$ into disjoint sets.

2. Suppose $z_t'^\nu > p_{t-1}\underline{\beta}_t'^\nu$ for all $(\alpha_t'^\nu, z_t'^\nu, \beta_t'^\nu) \in \Gamma_t^\nu$. Construct $(\alpha_t'^\nu, z_t'^\nu, \beta_t'^\nu)$ such that $\alpha_t'^\nu = \alpha_t^\nu + \beta_t^\nu$, $z_t'^\nu = z_t^\nu - p_{t-1}\underline{\beta}_t^\nu > 0$, and $\beta_t'^\nu = \mathbf{0}$. By the convexity of $P$, $\alpha_t'^\nu \in P$, $\alpha_{lt}'^\nu = \alpha_{lt}^\nu + \beta_{lt}^\nu$ and $z_t'^\nu + p_{t-1}\underline{\alpha}_t'^\nu = z_t^\nu + p_{t-1}\underline{\alpha}_t^\nu$. Then by Lemma 1, $(\alpha_t'^\nu, z_t'^\nu, \beta_t'^\nu) \in \Gamma_t^\nu$. Thus, it remains to show that there is no solution in $\Gamma_t^\nu$ of the form $(+,0,\mathbf{0})$. Suppose, by way of contradiction, that $\Gamma_t^\nu$ contains a solution of the form $(+,0,\mathbf{0})$ at $t$. Construct $(\alpha_t'^\nu, z_t'^\nu, \beta_t'^\nu)$ such that $\alpha_t'^\nu = \mathbf{0}$, $z_t'^\nu = p_{t-1}\underline{\alpha}_t^\nu > 0$, and $\beta_t'^\nu = \alpha_t^\nu + \beta_t^\nu$. Clearly, $\beta_t'^\nu \in P$, $\beta_{lt}'^\nu = \alpha_{lt}^\nu + \beta_{lt}^\nu$

and $z_t^{\prime\nu} + p_{t-1}\underline{\alpha}_t^{\prime\nu} = z_t^{\nu} + p_{t-1}\underline{\alpha}_t^{\nu}$. Then by Lemma 1, $(\alpha_t^{\prime\nu}, z_t^{\prime\nu}, \beta_t^{\prime\nu}) \in \Gamma_t^{\nu}$, contradicting the assumption that $z_t^{\prime\nu} > p_{t-1}\underline{\beta}_t^{\prime\nu}$ for all $(\alpha_t^{\prime\nu}, z_t^{\prime\nu}, \beta_t^{\prime\nu}) \in \Gamma_t^{\nu}$.

3. The other two cases are proved similarly. ■

Lemma 3 substantiates the claim that a country's class status is determined by its position in the capital market: net lenders form the 'upper' strata of the global economy, whereas net borrowers occupy 'lower' positions. It is therefore natural to investigate whether class status (and so a country's position in the credit market) is determined by its wealth.

Consider an IRS. At all $t$, let $\alpha_t^{\min}$ be defined as follows: $\alpha_t^{\min} \in P_t^w(\mathbf{p}, \mathbf{r})$, $\frac{p_{t-1}\underline{\alpha}_t^{\min}}{\alpha_{lt}^{\min}} = \min_{\alpha \in P_t^w(\mathbf{p},\mathbf{r})} \left[\frac{p_{t-1}\underline{\alpha}}{\alpha_l}\right]$, and $\alpha_{lt}^{\min} = \Lambda_t^*$, where $\Lambda_t^* > 0$ is the optimal amount of labour that every $\nu$ optimally spends at $t$, by Lemma 2. Similarly, let $\alpha_t^{\max}$ be defined as follows: $\alpha_t^{\max} \in P_t^w(\mathbf{p}, \mathbf{r})$, $\frac{p_{t-1}\underline{\alpha}_t^{\max}}{\alpha_{lt}^{\max}} = \max_{\alpha \in P_t^w(\mathbf{p},\mathbf{r})} \left[\frac{p_{t-1}\underline{\alpha}}{\alpha_l}\right]$, and $\alpha_{lt}^{\max} = \Lambda_t^*$. Note that $p_{t-1}\underline{\alpha}_t^{\min} \leqq p_{t-1}\underline{\alpha}_t^{\max}$ and that $\alpha_t^{\min}$, $\alpha_t^{\max}$ are well-defined. Theorem 1 generalises one of the main results of Roemer's theory of classes: at an IRS, $WP$ classes are pairwise disjoint and exhaustive, and $WP$ class status depends on a country's wealth.

**Theorem 1** (The Dependence School Theorem): *Let* $\left((\mathbf{p}, \mathbf{r}), (\xi^{\nu})_{\nu \in \mathcal{N}}\right)$ *be an IRS for* $E(\Omega_0)$ *such that* $1 + r_t > \max_i \frac{p_{it}}{p_{it-1}}$ *for all* $t$. *Then at all* $t$:

(i) $\nu \in C_t^1 \Leftrightarrow p_{t-1}\underline{\alpha}_t^{\max} < W_t^{\nu}$;
(ii) $\nu \in C_t^2 \Leftrightarrow p_{t-1}\underline{\alpha}_t^{\min} \leqq W_t^{\nu} \leqq p_{t-1}\underline{\alpha}_t^{\max}$;
(iii) $\nu \in C_t^3 \Leftrightarrow 0 < W_t^{\nu} < p_{t-1}\underline{\alpha}_t^{\min}$;
(iv) $\nu \in C_t^4 \Leftrightarrow W_t^{\nu} = 0$.

**Proof:** 1. By Lemma 2, at an IRS, $\Lambda_t^{\nu} = \Lambda_t^* = \alpha_{lt}^{\nu} + \beta_{lt}^{\nu} > 0$, all $\nu \in \mathcal{N}$.
2. By step 1, it immediately follows that at any $t$, $\nu \in C_t^4$ if and only if $W_t^{\nu} = 0$.
3. Consider part (ii). Suppose $p_{t-1}\underline{\alpha}_t^{\min} \leqq W_t^{\nu} \leqq p_{t-1}\underline{\alpha}_t^{\max}$. We show that $\Gamma_t^{\nu}$ has a solution of the form $(+, 0, \mathbf{0})$. By step 1, and noting that $1 + r_t > 0$, at an IRS it must be $w_t^{\max}\Lambda_t^* + (1 + r_t) W_t^{\nu} = p_t c_t^{*\nu} + p_t \omega_{t+1}^{\nu}$, and any $(\alpha_t^{\prime\nu}, z_t^{\prime\nu}, \beta_t^{\prime\nu})$ with $\alpha_t^{\prime\nu}, \beta_t^{\prime\nu} \in P_t^w(\mathbf{p}, \mathbf{r})$, $\alpha_{lt}^{\prime\nu} + \beta_{lt}^{\prime\nu} = \Lambda_t^*$, $z_t^{\prime\nu} \geqq 0$, and $p_{t-1}\underline{\alpha}_t^{\prime\nu} + z_t^{\prime\nu} = W_t^{\nu}$ is part of an optimal solution. But then, since $p_{t-1}\underline{\alpha}_t^{\min} \leqq W_t^{\nu} \leqq p_{t-1}\underline{\alpha}_t^{\max}$, by the convexity of $P$, it follows that there exists some $\alpha_t^{\prime\prime\nu} \in P_t^w(\mathbf{p}, \mathbf{r})$, such that $w_t^{\max}\alpha_{lt}^{\prime\prime\nu} + (1 + r_t) p_{t-1}\underline{\alpha}_t^{\prime\prime\nu} = p_t c_t^{*\nu} + p_t \omega_{t+1}^{\nu}$, with $\alpha_{lt}^{\prime\prime\nu} = \Lambda_t^*$ and $p_{t-1}\underline{\alpha}_t^{\prime\prime\nu} = W_t^{\nu}$. Conversely, suppose that $\nu \in C_t^2$, so that $\Gamma_t^{\nu}$ has a solution of the form $(+, 0, \mathbf{0})$. This implies that there exists $\alpha_t \in P_t^w(\mathbf{p}, \mathbf{r})$ such that

$(1+r_t)\, p_{t-1}\underline{\alpha}_t + w_t^{\max}\alpha_{lt} = p_t c_t^{*\nu} + p_t \omega_{t+1}^{\nu}$, with $p_{t-1}\underline{\alpha}_t = W_t^{\nu}$ and $\alpha_{lt} = \Lambda_t^*$, which implies $p_{t-1}\underline{\alpha}_t^{\min} \leqq W_t^{\nu} \leqq p_{t-1}\underline{\alpha}_t^{\max}$.
4. Parts (i) and (iii) are proved similarly. ■

In order to interpret Theorem 1, note that country $\nu$'s wealth, $W_t^{\nu} = p_{t-1}\omega_t^{\nu}$, can be seen as the main proxy for its level of development. A higher $W_t^{\nu}$ is associated to advanced countries, less developed countries are characterised by a lower $W_t^{\nu}$, and "non-capitalist strata and countries" (Luxemburg [14], p.352) have $W_t^{\nu} = 0$. Theorem 1 implies that in equilibrium less developed countries are net borrowers, whereas developed countries are net lenders: a nation's wealth (and development) level determines its class status. Given that $E(\Omega_0)$ is an international economy with perfectly competitive markets for commodities and capital, in equilibrium all countries enjoy benefits from trade, as international capital flows allow poor countries to improve their lot. Yet, the IRS is also characterised by a four-class structure which reflects the wealth hierarchy and an asymmetric relationship between countries. For the economic development of the countries in $C_t^3 \cup C_t^4$ is crucially dependent on the existence of the rich countries in $C_t^1$ who can export their capital to the poor, whereas the rich in $C_t^1$ could realise a certain economic development with full employment by themselves alone. In this sense, Theorem 1 captures some of the key insights of the 'dependence school' discussed in the Introduction, and therefore it is named accordingly.

# 4 Exploitative International Relations

Exploitation in international relations is conceived of as the *unequal exchange of labour* between countries, which is defined following Roemer [22, 23]: exploitative international relations are characterised by systematic differences between the labour 'contributed', in some relevant sense, by agents in country $\nu$ and the labour 'received', in some relevant sense, by them via their national income. As intuitive as this definition may seem at first sight, in general economies, the notions of labour contributed and labour received are not obvious. Indeed, the very existence of a general, consistent definition which preserves the key insights of UE theory has been put into doubt. In this section, we develop an axiomatic analysis of UE exploitation theory and characterise a class of definitions that satisfy three important properties. In the next section, we prove that the class is nonempty.

## 4.1 A domain axiom

In order to define UE exploitation, it is necessary to identify both the labour 'contributed' and the labour 'received' by $\nu$. In economies with homogeneous labour, the former amount coincides with the labour performed by workers in $\nu$, $\Lambda_t^\nu$.[16] Outside of simple, static two-class Leontief economies with subsistence wages, instead, many different definitions of the labour 'received' by $\nu$ can be, and have in fact been proposed, which incorporate different normative and positive views.[17] In recent work, Yoshihara and Veneziani [34, 32, 30, 31] have proposed an axiom that identifies the domain of admissible UE definitions: it imposes some weak restrictions on the notion of labour received and all of the main approaches satisfy it in static economies. In this subsection, we generalise their analysis and define a domain axiom for all admissible definitions of UE exploitation in general dynamic equilibria.

In all of the main UE approaches, the amount of labour 'received' by agent $\nu$ is determined by some reference bundles that are, or can be, consumed by $\nu$. The focus of analysis is on the income that agents do, or can devote to consumption net of the replacement of any wealth they possess. When focusing on static models, or on the steady state of dynamic economies, this implies focusing on bundles that are, or can be purchased with $\nu$'s actual *net* income. In the general equilibria considered here, the choice of the relevant budget set is not unambiguous.

Let $\left((\mathbf{p},\mathbf{r}),(\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be a RS for $E(\Omega_0)$. The gross income of each $\nu\in\mathcal{N}$ at $t$ is given by $(1+r_t)\,p_{t-1}\omega_t^\nu + w_t^{\max}\Lambda_t^\nu$. In order to identify $\nu$'s 'net' income at $t$ in this context, the fund for replenishing $\nu$'s wealth $p_{t-1}\omega_t^\nu$ in the next period should be deducted after adjusting for the difference in prices between $t-1$ and $t$. To do so, we define the *inflation index at* $t$, $R_t \equiv \frac{p_t\omega_t}{p_{t-1}\omega_t}$, taking $\omega_t$ as the numéraire bundle. Given this index, $\nu$'s wealth $p_{t-1}\omega_t^\nu$ at $t-1$ is evaluated as being equivalent to $R_t p_{t-1}\omega_t^\nu$ at $t$. Then, $\nu$'s 'net' income at $t$ can be defined as $(1+r_t)\,p_{t-1}\omega_t^\nu + w_t^{\max}\Lambda_t^\nu - R_t p_{t-1}\omega_t^\nu$, and it identifies the normatively relevant set of commodity bundles 'received' by $\nu$:

$$B_t\left((\mathbf{p},\mathbf{r})\,;p_{t-1}\omega_t^\nu,\Lambda_t^\nu\right) \equiv \left\{c\in\mathbb{R}_+^n \mid p_t c = (1+r_t)\,p_{t-1}\omega_t^\nu + w_t^{\max}\Lambda_t^\nu - R_t p_{t-1}\omega_t^\nu\right\}.$$

In other words, the set of commodity bundles 'received' by $\nu$ at $t$ is de-

[16] For a generalisation to economies with heterogeneous inputs and skills, see Veneziani and Yoshihara [30, 31].

[17] See, for example, Morishima [19] and Roemer [22]. See Yoshihara [32, 33] and Veneziani and Yoshihara [31] for a thorough discussion.

fined counterfactually by considering the net income that *could* be devoted to consumption if $\nu$ decided only to replace its wealth, i.e. to carry forward the real asset value of $W_t^\nu$ to the next period.[18] The reason for this choice is threefold. First, every country $\nu$ is interested in wealth $W_t^\nu$, $W_{t+1}^\nu$, rather than in the specific vector of capital endowments, $\omega_t^\nu$, $\omega_{t+1}^\nu$. Second, from a normative perspective, for a given gross income, in every $t$ exploitation status should not depend on saving and investment decisions, or on the specific vector of productive endowments purchased. As Roemer ([23], p.53) forcefully argued, the appropriate notion of UE exploitation should be preference-independent. Third, it is immediate to show that the focus on bundles in $B_t\left((\mathbf{p},\mathbf{r});p_{t-1}\omega_t^\nu,\Lambda_t^\nu\right)$ is a generalisation of the standard approach and it reduces to the latter at a RS with stationary prices and capital.

Let $\mathcal{E}$ denote the set of all economies $E(P,\mathcal{N},u,\rho,\Omega_0)$ that satisfy our basic assumptions on technology, agents, preferences and endowments. For all $c\in\mathbb{R}_+^n$, let $\psi(c)\equiv\{\alpha\in P\mid\widehat{\alpha}\geqq c\}$ be the set of production activities that can produce $c$ as net output. Given any definition of exploitation, let $\mathcal{N}_t^{ter}\subseteq\mathcal{N}$ and $\mathcal{N}_t^{ted}\subseteq\mathcal{N}$ denote, respectively, the *set of exploiters at* $t$, or $WP_t$ exploiters, and the *set of exploited agents at* $t$, or $WP_t$ exploited agents, at a given allocation, where $\mathcal{N}_t^{ter}\cap\mathcal{N}_t^{ted}=\varnothing$. A domain axiom can now be formally introduced that captures the basic intuitions of UE theory.

**Labour Exploitation (LE):** *Consider any economy* $E(\Omega_0)\in\mathcal{E}$. *Let* $\left((\mathbf{p},\mathbf{r}),(\xi^\nu)_{\nu\in\mathcal{N}}\right)$ *be a RS for* $E(\Omega_0)$. *Given any definition of UE exploitation, in each period* $t$ *two subsets* $\mathcal{N}_t^{ter}\subseteq\mathcal{N}$ *and* $\mathcal{N}_t^{ted}\subseteq\mathcal{N}$, $\mathcal{N}_t^{ter}\cap\mathcal{N}_t^{ted}=\varnothing$, *constitute* the set of $WP_t$ *exploiters and* the set of $WP_t$ *exploited* agents *if and only if for any* $\nu\in\mathcal{N}$, *there exist* $\overline{c}_t^\nu,\underline{c}_t^\nu\in B_t\left((\mathbf{p},\mathbf{r});p_{t-1}\omega_t^\nu,\Lambda_t^\nu\right)$ *such that there exist* $\alpha^{\overline{c}_t^\nu}\in\psi\left(\overline{c}_t^\nu\right)\cap\partial P$ *with* $\widehat{\alpha}^{\overline{c}_t^\nu}\ngeq\overline{c}_t^\nu$ *and* $\alpha^{\underline{c}_t^\nu}\in\psi\left(\underline{c}_t^\nu\right)\cap\partial P$ *with* $\widehat{\alpha}^{\underline{c}_t^\nu}\ngeq\underline{c}_t^\nu$ *such that* $\alpha_l^{\underline{c}_t^\nu}\geqq\alpha_l^{\overline{c}_t^\nu}$, *and the following condition holds,*

$$\begin{aligned}\nu &\in \mathcal{N}_t^{ter}\Leftrightarrow\Lambda_t^\nu<\alpha_l^{\overline{c}_t^\nu};\\ \nu &\in \mathcal{N}_t^{ted}\Leftrightarrow\Lambda_t^\nu>\alpha_l^{\underline{c}_t^\nu}.\end{aligned}$$

Axiom **LE** requires UE exploitation status to be determined based on the labour contributed by countries (the labour performed by their citizens) and on the labour received by them, where the latter is determined in relation both to purchasing power, and to productive conditions.

[18] The set $B_t\left((\mathbf{p},\mathbf{r});p_{t-1}\omega_t^\nu,\Lambda_t^\nu\right)$ does not necessarily contain $\nu$'s real consumption bundle at $t$, as $p_t\omega_{t+1}^\nu$ may be different from $R_tp_{t-1}\omega_t^\nu$, in equilibrium.

To be specific, by **LE** under any admissible definition, in equilibrium the sets $\mathcal{N}_t^{ter}$ and $\mathcal{N}_t^{ted}$ are characterised in each period $t$ by identifying two (possibly identical) reference commodity bundles $\overline{c}_t^\nu, \underline{c}_t^\nu \in \mathbb{R}_+^n$ for each $\nu \in \mathcal{N}$. The reference bundles must be affordable for any $\nu$ who simply replaces its wealth at period $t$, i.e. $\overline{c}_t^\nu, \underline{c}_t^\nu \in B_t\left((\mathbf{p},\mathbf{r})\,; p_{t-1}\omega_t^\nu, \Lambda_t^\nu\right)$, and must be technically producible as net output of efficient production activities, i.e. $\alpha^{\overline{c}_t^\nu} \in \psi\left(\overline{c}_t^\nu\right) \cap \partial P$ with $\widehat{\alpha}^{\overline{c}_t^\nu} \not> \overline{c}_t^\nu$ and $\alpha^{\underline{c}_t^\nu} \in \psi\left(\underline{c}_t^\nu\right) \cap \partial P$ with $\widehat{\alpha}^{\underline{c}_t^\nu} \not> \underline{c}_t^\nu$. The labour contained in $\overline{c}_t^\nu, \underline{c}_t^\nu$ is equal to the amount of labour required to produce them as net output, respectively, $\alpha_l^{\overline{c}_t^\nu}$, $\alpha_l^{\underline{c}_t^\nu}$. Given $\alpha_l^{\underline{c}_t^\nu} \geqq \alpha_l^{\overline{c}_t^\nu}$, the (possibly degenerate) interval $\left[\alpha_l^{\overline{c}_t^\nu}, \alpha_l^{\underline{c}_t^\nu}\right]$ is regarded as the labour that $\nu$ can receive via its 'net income' at $t$ and it determines $\nu$'s UE exploitation status at $t$, once compared with the labour contributed by $\nu$, $\Lambda_t^\nu$. In equilibrium, at any $t$, $\nu$ is a $WP_t$ *exploiter* ($\nu \in \mathcal{N}_t^{ter}$) if and only if $\nu$ works less than the *minimum* amount of labour $\alpha_l^{\overline{c}_t^\nu}$ that $\nu$ can receive via its 'net income' ($\Lambda_t^\nu < \alpha_l^{\overline{c}_t^\nu}$); whereas $\nu$ is $WP_t$ *exploited* ($\nu \in \mathcal{N}_t^{ted}$) if and only if $\nu$ works more than the *maximum* amount of labour $\alpha_l^{\underline{c}_t^\nu}$ that $\nu$ can receive via its 'net income' ($\Lambda_t^\nu > \alpha_l^{\underline{c}_t^\nu}$).

Axiom **LE** is a rather weak condition that captures some fundamental insights of UE exploitation theory shared by all of the main approaches in the literature, in general convex dynamic economies. It provides a minimal necessary condition that identifies the domain of admissible UE definitions, but it cannot discriminate among alternative definitions *within* the admissible domain, which can be large indeed. For this purpose, some additional properties must be imposed. To this task we turn next.

## 4.2 Class, Wealth and Exploitation

A fundamental insight of UE theory is the existence of a strict relation between development - or wealth, - exploitation status, and class position in the global economy. The existence of such relation is often proved as a *result* in a given economic environment, under certain conditions. Yet its central relevance in UE theory is such that "its epistemological status in our understanding is as a postulate. We seek a model which will make our postulated belief true" (Roemer [22], p.152). In this subsection, we state this intuition axiomatically and formalise two properties that incorporate, on the one hand, the relation between wealth and exploitation and, on the other hand, the relation between class and exploitation status. Then, we provide a characterisation of the class of definitions of UE exploitation (within the admissible domain identified by **LE**) that satisfy both properties, in the dynamic international economies considered here.

The first property captures the intuition that richer countries are UE exploiters while less developed countries suffer from UE exploitation:

**Wealth-Exploitation Correspondence Principle (WECP):** *Let* $\left((\mathbf{p},\mathbf{r}),(\xi^{\nu})_{\nu\in\mathcal{N}}\right)$ *be an IRS for* $E(P,\mathcal{N},u,\rho,\Omega_0)$ *such that* $1+r_t-R_t>0$ *for all* $t$*. For each* $t$*, there exist* $\overline{W}_t,\underline{W}_t>0$ *with* $\overline{W}_t\geqq\underline{W}_t$ *such that for any* $E(P,\mathcal{N},u,\rho,\Omega_0')\in\mathcal{E}$ *with* $\sum_{\nu\in\mathcal{N}}\omega_0'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_0^{\nu}$ *and any IRS* $\left((\mathbf{p},\mathbf{r}),(\xi'^{\nu})_{\nu\in\mathcal{N}}\right)$ *for* $E(P,\mathcal{N},u,\rho,\Omega_0')$ *with* $\sum_{\nu\in\mathcal{N}}\omega_t'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_t^{\nu}$*, all* $t$:

$$W_t'^{\nu}>\overline{W}_t\Leftrightarrow\nu\in\mathcal{N}_t^{ter};$$
$$W_t'^{\nu}<\underline{W}_t\Leftrightarrow\nu\in\mathcal{N}_t^{ted}.$$

**WECP** states that, in equilibrium, in any given period there should be two (possibly equal) threshold wealth levels, $\overline{W}_t,\underline{W}_t$, such that the set of UE exploiters (resp., UE exploited) corresponds to the set of countries with wealth higher than $\overline{W}_t$ (resp., lower than $\underline{W}_t$). The threshold levels may depend on equilibrium prices and aggregate endowments, but not on the equilibrium wealth distribution.

The next Lemma characterises the set of definitions that satisfy **WECP**.

**Lemma 4** (**WECP**): *Let* $\left((\mathbf{p},\mathbf{r}),(\xi^{\nu})_{\nu\in\mathcal{N}}\right)$ *be an IRS for* $E(P,\mathcal{N},u,\rho,\Omega_0)$ *such that* $1+r_t-R_t>0$ *for all* $t$*. Given any definition of UE exploitation satisfying* **LE***, the following statements are equivalent*:
(i) **WECP** *holds;*
(ii) *at all* $t$*, there exist* $\overline{W}_t,\underline{W}_t>0$ *with* $\overline{W}_t\geqq\underline{W}_t$ *such that for any* $E(P,\mathcal{N},u,\rho,\Omega_0')\in\mathcal{E}$ *with* $\sum_{\nu\in\mathcal{N}}\omega_0'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_0^{\nu}$ *and any IRS* $\left((\mathbf{p},\mathbf{r}),(\xi'^{\nu})_{\nu\in\mathcal{N}}\right)$ *for* $E(P,\mathcal{N},u,\rho,\Omega_0')$ *with* $\sum_{\nu\in\mathcal{N}}\omega_t'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_t^{\nu}$*, all* $t$*, and for each* $\nu\in\mathcal{N}$*,*

$$W_t'^{\nu}>\overline{W}_t\Leftrightarrow W_t'^{\nu}>\frac{p_t\overline{c}_t'^{\nu}-w_t^{\max}\alpha_l^{\overline{c}_t'^{\nu}}}{1+r_t-R_t};$$
$$W_t'^{\nu}<\underline{W}_t\Leftrightarrow W_t'^{\nu}<\frac{p_t\underline{c}_t'^{\nu}-w_t^{\max}\alpha_l^{\underline{c}_t'^{\nu}}}{1+r_t-R_t}.$$

**Proof:** 1. Consider any economy $E(P, \mathcal{N}, u, \rho, \Omega_0') \in \mathcal{E}$ and any IRS $\left((\mathbf{p}, \mathbf{r}), (\xi'^{\nu})_{\nu \in \mathcal{N}}\right)$ for $E(P, \mathcal{N}, u, \rho, \Omega_0')$. Given a definition of exploitation satisfying **LE**, at any $t$, for each $\nu \in \mathcal{N}$, there exist $\overline{c}_t'^{\nu}, \underline{c}_t'^{\nu} \in B_t\left((\mathbf{p}, \mathbf{r}); W_t'^{\nu}, \Lambda_t'^{\nu}\right)$ such that there exist $\alpha^{\overline{c}_t'^{\nu}} \in \psi\left(\overline{c}_t'^{\nu}\right) \cap \partial P$ with $\widehat{\alpha}^{\overline{c}_t'^{\nu}} \ngtr \overline{c}_t'^{\nu}$ and $\alpha^{\underline{c}_t'^{\nu}} \in \psi\left(\underline{c}_t'^{\nu}\right) \cap \partial P$ with $\widehat{\alpha}^{\underline{c}_t'^{\nu}} \ngtr \underline{c}_t'^{\nu}$ such that $\alpha_l^{\underline{c}_t'^{\nu}} \geqq \alpha_l^{\overline{c}_t'^{\nu}}$ and $\nu \in \mathcal{N}_t^{ter} \Leftrightarrow \Lambda_t'^{\nu} < \alpha_l^{\overline{c}_t'^{\nu}}$, and $\nu \in \mathcal{N}_t^{ted} \Leftrightarrow \Lambda_t'^{\nu} > \alpha_l^{\underline{c}_t'^{\nu}}$.

2. In order to prove the result, it is sufficient to show that for any economy $E(P, \mathcal{N}, u, \rho, \Omega_0') \in \mathcal{E}$ and any IRS $\left((\mathbf{p}, \mathbf{r}), (\xi'^{\nu})_{\nu \in \mathcal{N}}\right)$ for $E(P, \mathcal{N}, u, \rho, \Omega_0')$, the following conditions hold at any $t$ and for each $\nu \in \mathcal{N}$,

$$\nu \in \mathcal{N}_t^{ter} \Leftrightarrow W_t'^{\nu} > \frac{p_t \overline{c}_t'^{\nu} - w_t^{\max} \alpha_l^{\overline{c}_t'^{\nu}}}{1 + r_t - R_t}; \tag{5}$$

$$\nu \in \mathcal{N}_t^{ted} \Leftrightarrow W_t'^{\nu} < \frac{p_t \underline{c}_t'^{\nu} - w_t^{\max} \alpha_l^{\underline{c}_t'^{\nu}}}{1 + r_t - R_t}. \tag{6}$$

Consider (5). By **LE**, $\nu \in \mathcal{N}_t^{ter} \Leftrightarrow \Lambda_t'^{\nu} < \alpha_l^{\overline{c}_t'^{\nu}}$. Moreover, $\overline{c}_t'^{\nu} \in B_t\left((\mathbf{p}, \mathbf{r}); W_t'^{\nu}, \Lambda_t'^{\nu}\right)$ implies $p_t \overline{c}_t'^{\nu} = (1 + r_t - R_t) W_t'^{\nu} + w_t^{\max} \Lambda_t'^{\nu}$. Therefore $\Lambda_t'^{\nu} < \alpha_l^{\overline{c}_t'^{\nu}} \Leftrightarrow \alpha_l^{\overline{c}_t'^{\nu}} > \frac{p_t \overline{c}_t'^{\nu} - (1 + r_t - R_t) W_t'^{\nu}}{w_t^{\max}}$ and the desired inequality follows by rearranging the latter expression and noting that $1 + r_t - R_t > 0$.

A similar argument proves that (6) also holds. ■

Theorem 1 and Lemma 4 provide two different partitions of the set of countries, according to their UE exploitation or class status, based on wealth inequalities. Depending on the UE definition that one adopts, the two partitions may or may not coincide. Yet, as noted above, an important intuition of UE theory is the existence of a robust relation between class and UE exploitation status. Based on Roemer [22], we formulate this intuition explicitly in the following axiom:

**Class-Exploitation Correspondence Principle (CECP):** *Let* $\left((\mathbf{p}, \mathbf{r}), (\xi^{\nu})_{\nu \in \mathcal{N}}\right)$ *be an IRS for* $E(\Omega_0)$ *such that* $1 + r_t - R_t > 0$ *for all* $t$*. Then, at all* $t$

$$\begin{aligned} \nu &\in C_t^1 \Rightarrow \nu \in \mathcal{N}_t^{ter}; \\ \nu &\in C_t^3 \cup C_t^4 \Rightarrow \nu \in \mathcal{N}_t^{ted}. \end{aligned}$$

**CECP** formalises the relation between a country's $WP$ class status (and its position in the global capital market) and its UE exploitation status: in equilibrium, countries in the upper classes in the credit market should emerge as UE exploiters, while those in the lower classes should be UE exploited.

Theorem 2 provides necessary and sufficient conditions for a UE definition in the admissible domain to satisfy both **WECP** and **CECP**:

**Theorem 2** (**CECP**): *Let* $\left((\mathbf{p},\mathbf{r}),(\xi^{\nu})_{\nu\in\mathcal{N}}\right)$ *be an IRS for* $E(P,\mathcal{N},u,\rho,\Omega_0)$ *such that* $1+r_t>\max_i \frac{p_{it}}{p_{it-1}}$ *for all* $t$. *Given any definition of UE exploitation satisfying* **LE**, *the following statements are equivalent*:
(i) **WECP** *and* **CECP** *hold;*
(ii) *at all* $t$, *there exist* $\overline{W}_t,\underline{W}_t>0$ *with* $p_{t-1}\underline{\alpha}_t^{\min}\leqq\underline{W}_t\leqq\overline{W}_t\leqq p_{t-1}\underline{\alpha}_t^{\max}$ *such that for any* $E(P,\mathcal{N},u,\rho,\Omega_0')\in\mathcal{E}$ *with* $\sum_{\nu\in\mathcal{N}}\omega_0'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_0^{\nu}$ *and any IRS* $\left((\mathbf{p},\mathbf{r}),(\xi'^{\nu})_{\nu\in\mathcal{N}}\right)$ *for* $E(P,\mathcal{N},u,\rho,\Omega_0')$ *with* $\sum_{\nu\in\mathcal{N}}\omega_t'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_t^{\nu}$, *all* $t$, *and for each* $\nu\in\mathcal{N}$,

$$W_t'^{\nu}>\overline{W}_t\Leftrightarrow W_t'^{\nu}>\frac{p_t\overline{c}_t'^{\nu}-w_t^{\max}\alpha_l^{\overline{c}_t'^{\nu}}}{1+r_t-R_t};$$

$$W_t'^{\nu}<\underline{W}_t\Leftrightarrow W_t'^{\nu}<\frac{p_t\underline{c}_t'^{\nu}-w_t^{\max}\alpha_l^{\underline{c}_t'^{\nu}}}{1+r_t-R_t}.$$

**Proof:** **1.** Consider any economy $E(P,\mathcal{N},u,\rho,\Omega_0')\in\mathcal{E}$ and any IRS $\left((\mathbf{p},\mathbf{r}),(\xi'^{\nu})_{\nu\in\mathcal{N}}\right)$ for $E(P,\mathcal{N},u,\rho,\Omega_0')$. Note that $1+r_t>\max_i \frac{p_{it}}{p_{it-1}}$ for all $t$ implies $1+r_t-R_t>0$ for all $t$. Given a definition of exploitation satisfying **LE**, at any $t$, for each $\nu\in\mathcal{N}$, there exist $\overline{c}_t'^{\nu},\underline{c}_t'^{\nu}\in B_t\left((\mathbf{p},\mathbf{r});W_t'^{\nu},\Lambda_t'^{\nu}\right)$ such that there exist $\alpha^{\overline{c}_t'^{\nu}}\in\psi\left(\overline{c}_t'^{\nu}\right)\cap\partial P$ with $\widehat{\alpha}^{\overline{c}_t'^{\nu}}\not>\overline{c}_t'^{\nu}$ and $\alpha^{\underline{c}_t'^{\nu}}\in\psi\left(\underline{c}_t'^{\nu}\right)\cap\partial P$ with $\widehat{\alpha}^{\underline{c}_t'^{\nu}}\not>\underline{c}_t'^{\nu}$ satisfying: $\alpha_l^{\underline{c}_t'^{\nu}}\geqq\alpha_l^{\overline{c}_t'^{\nu}}$, and $\nu\in\mathcal{N}_t^{ter}\Leftrightarrow\Lambda_t'^{\nu}<\alpha_l^{\overline{c}_t'^{\nu}}$ and $\nu\in\mathcal{N}_t^{ted}\Leftrightarrow\Lambda_t'^{\nu}>\alpha_l^{\underline{c}_t'^{\nu}}$.

**2.** ((ii)⇒(i)) Suppose that (ii) holds. Then by Lemma 4 and Theorem 1 it immediately follows that **WECP** and **CECP** hold.

**3.** ((i)⇒(ii)) Let **WECP** and **CECP** hold. By Lemma 4, it is sufficient to show that at all $t$, $\overline{W}_t\leqq p_{t-1}\underline{\alpha}_t^{\max}$ and $p_{t-1}\underline{\alpha}_t^{\min}\leqq\underline{W}_t$.

Take any period $t$ and suppose, by way of contradiction, that $\overline{W}_t>p_{t-1}\underline{\alpha}_t^{\max}$. We consider two cases.

Case 1: suppose that $p_{t-1}\underline{\alpha}_t^{\max}\geqq p_{t-1}\omega_t'^{\nu}$, all $\nu\in\mathcal{N}$, for any $E(P,\mathcal{N},u,\rho,\Omega_0')\in\mathcal{E}$ with $\sum_{\nu\in\mathcal{N}}\omega_0'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_0^{\nu}$ and any IRS $\left((\mathbf{p},\mathbf{r}),(\xi'^{\nu})_{\nu\in\mathcal{N}}\right)$ for $E(P,\mathcal{N},u,\rho,\Omega_0')$ with $\sum_{\nu\in\mathcal{N}}\omega_t'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_t^{\nu}$, all $t$. By Theorem 1, $C_t^1=\varnothing$. Moreover, because $\overline{W}_t>p_{t-1}\underline{\alpha}_t^{\max}$, by **WECP** $\mathcal{N}_t^{ter}=\varnothing$. But then, noting that the same holds for any $\overline{W}_t'\geqq p_{t-1}\underline{\alpha}_t^{\max}$ and that **WECP** does not require wealth thresholds to be unique, it is possible to set $\overline{W}_t=p_{t-1}\underline{\alpha}_t^{\max}$

Case 2: suppose that there exists an economy $E(P,\mathcal{N},u,\rho,\Omega_0')\in\mathcal{E}$ with $\sum_{\nu\in\mathcal{N}}\omega_0'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_0^{\nu}$ and an IRS $\left((\mathbf{p},\mathbf{r}),(\xi'^{\nu})_{\nu\in\mathcal{N}}\right)$ for $E(P,\mathcal{N},u,\rho,\Omega_0')$ with $\sum_{\nu\in\mathcal{N}}\omega_t'^{\nu}=\sum_{\nu\in\mathcal{N}}\omega_t^{\nu}$, all $t$, such that $p_{t-1}\underline{\alpha}_t^{\max}<p_{t-1}\omega_t'^{\nu}$, for some

$\nu \in \mathcal{N}$. If $p_{t-1}\underline{\alpha}_t^{\max} < p_{t-1}\omega_t''^{\nu} \leqq \overline{W}_t$, then the desired contradiction follows from Theorem 1, **CECP**, and **WECP**. So, suppose that $p_{t-1}\underline{\alpha}_t^{\max} < \overline{W}_t < p_{t-1}\omega_t''^{\nu}$. Then by Lemma 2 it is immediate to show that there exists another economy $E(P, \mathcal{N}, u, \rho, \Omega_0'') \in \mathcal{E}$ with $\sum_{\nu \in \mathcal{N}} \omega_0''^{\nu} = \sum_{\nu \in \mathcal{N}} \omega_0^{\nu}$ and an IRS $\left((\mathbf{p}, \mathbf{r}), (\xi''^{\nu})_{\nu \in \mathcal{N}}\right)$ for $E(P, \mathcal{N}, u, \rho, \Omega_0'')$ with $\sum_{\nu \in \mathcal{N}} \omega_t''^{\nu} = \sum_{\nu \in \mathcal{N}} \omega_t^{\nu}$, all $t$, such that $p_{t-1}\underline{\alpha}_t^{\max} < p_{t-1}\omega_t''^{\nu} \leqq \overline{W}_t$ for some $\nu \in \mathcal{N}$, which yields the desired contradiction.

A similar argument can be used to prove that $p_{t-1}\underline{\alpha}_t^{\min} \leqq \underline{W}_t$. ■

The above results fully characterise exploitative international relations in the intertemporal model. Theorem 1 and Lemma 3 identify the structure of the global capital market, in which developed countries emerge as net lenders and less developed countries as net borrowers. Lemma 4 derives necessary and sufficient conditions for exploitative international relations to map wealth inequalities, such that a country's UE exploitation status is related to its level of economic development as proxied by the value of its productive endowment. Finally, Theorem 2 provides necessary and sufficient conditions for international credit relations, and class positions in the global credit market, to map wealth inequalities and exploitation status.

Two points should be emphasised which highlight the generality of the results. First, the above characterisations are derived without adopting *any* specific definition of UE exploitation: they hold for *any* definition within the admissible domain identified by **LE**. Thus, the relation between wealth, class and exploitation is proved to hold for an entire (and potentially large) class of UE definitions. Second, unlike in the rest of the literature, the results hold in full blown intertemporal economies, under rather general assumptions concerning preferences and technology, *and* without restricting the analysis to steady state equilibria.

# 5 A Definition of UE Exploitation

Section 4 provides a complete characterisation of the class of UE definitions that satisfy **WECP** and **CECP**, within the admissible domain identified by **LE**, in general international dynamic economies. This immediately raises the question whether there actually exist any definitions that satisfy the condition in Theorem 2. This is not an idle question. Yoshihara [32] has shown that in static economies with revenue-maximising agents, some of the received definitions - including Morishima's [19] and Roemer's [22] - satisfy **LE** but not **CECP**. Roemer [22] himself has raised doubts about the robustness of the relation between wealth, exploitation, and class in general economies. In this section, we show that the class of definitions identified by Theorem 2 is indeed nonempty: the definition recently proposed by Yoshihara and Veneziani [34, 32, 31] satisfies **LE** and preserves **WECP** and **CECP**, in the dynamic international economies considered in this paper.

Let $\left((\mathbf{p},\mathbf{r}),(\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be a RS for $E(\Omega_0)$ and let $\alpha_t^{\mathbf{p},\mathbf{r}}+\beta_t^{\mathbf{p},\mathbf{r}}$ denote the aggregate equilibrium production activity at $t$. For any $c\in\mathbb{R}_+^n$, such that $p_t c \leqq p_t\left(\widehat{\alpha}_t^{\mathbf{p},\mathbf{r}}+\widehat{\beta}_t^{\mathbf{p},\mathbf{r}}\right)$, the *labour content of* $c$ is equal to $\tau_t^c\left(\alpha_{lt}^{\mathbf{p},\mathbf{r}}+\beta_{lt}^{\mathbf{p},\mathbf{r}}\right)$, where $\tau_t^c\in[0,1]$ is such that $\tau_t^c p_t\left(\widehat{\alpha}_t^{\mathbf{p},\mathbf{r}}+\widehat{\beta}_t^{\mathbf{p},\mathbf{r}}\right)=p_t c$.[19] Thus, the labour content of aggregate net output, $\widehat{\alpha}_t^{\mathbf{p},\mathbf{r}}+\widehat{\beta}_t^{\mathbf{p},\mathbf{r}}$, is equal to total social labour, $\alpha_{lt}^{\mathbf{p},\mathbf{r}}+\beta_{lt}^{\mathbf{p},\mathbf{r}}$, and the labour contained in any bundle $c$ (whose value does not exceed global income) is equal to the fraction $\tau^c$ of social labour, $\tau^c\left(\alpha_{lt}^{\mathbf{p},\mathbf{r}}+\beta_{lt}^{\mathbf{p},\mathbf{r}}\right)$, which is necessary to produce a fraction of aggregate net output, $\tau^c\left(\widehat{\alpha}_t^{\mathbf{p},\mathbf{r}}+\widehat{\beta}_t^{\mathbf{p},\mathbf{r}}\right)$, that has the same value as $c$. We denote this amount as $l.v.\left(c;(\mathbf{p},\mathbf{r}),\alpha_t^{\mathbf{p},\mathbf{r}}+\beta_t^{\mathbf{p},\mathbf{r}}\right)$: it is the labour value of $c$ at $t$, at a RS with prices $(\mathbf{p},\mathbf{r})$ and aggregate production, $\alpha_t^{\mathbf{p},\mathbf{r}}+\beta_t^{\mathbf{p},\mathbf{r}}$. Then:

**Definition 4:** Consider any economy $E(\Omega_0)\in\mathcal{E}$. Let $\left((\mathbf{p},\mathbf{r}),(\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be a RS for $E(\Omega_0)$. At all $t$, country $\nu\in\mathcal{N}$, which supplies $\Lambda_t^\nu$, is *$WP_t$-exploited* if and only if $\Lambda_t^\nu > l.v.\left(\widetilde{c}_t^\nu;(\mathbf{p},\mathbf{r}),\alpha_t^{\mathbf{p},\mathbf{r}}+\beta_t^{\mathbf{p},\mathbf{r}}\right)$ for any $\widetilde{c}_t^\nu\in B_t\left((\mathbf{p},\mathbf{r});p_{t-1}\omega_t^\nu,\Lambda_t^\nu\right)$, and a *$WP_t$-exploiter* if and only if $\Lambda_t^\nu < l.v.\left(\widetilde{c}_t^\nu;(\mathbf{p},\mathbf{r}),\alpha_t^{\mathbf{p},\mathbf{r}}+\beta_t^{\mathbf{p},\mathbf{r}}\right)$ for any $\widetilde{c}_t^\nu\in B_t\left((\mathbf{p},\mathbf{r});p_{t-1}\omega_t^\nu,\Lambda_t^\nu\right)$.

Definition 4 is conceptually related to the 'New Interpretation' (Duménil [5, 6]; Foley [10, 11]; Duménil et al [7]). In fact, $\tau^{\widetilde{c}_t^\nu}$ is $\nu$'s reference share of world's income, and so $\tau^{\widetilde{c}_t^\nu}\left(\alpha_{lt}^{\mathbf{p},\mathbf{r}}+\beta_{lt}^{\mathbf{p},\mathbf{r}}\right)$ is the share of total social labour that $\nu$ receives by earning national income $p\widetilde{c}_t^\nu$. Then, as in the New Interpretation, country $\nu$ is UE exploited if and only if the amount of social labour it receives is less than the amount of labour expended by its workers, $\Lambda_t^\nu$.

Definition 4 has several attractive features. First, it does not rely on the labour theory of value and it is more general than the standard approach, in that it is not restricted to arguably special economies with Leontief or von

[19] If $p_t\left(\widehat{\alpha}_t^{\mathbf{p},\mathbf{r}}+\widehat{\beta}_t^{\mathbf{p},\mathbf{r}}\right)=0$, we set $\tau_t^c=0$ by definition.

Neumann technologies. Second, unlike in the standard approach, exploitation is not a merely technological phenomenon and social relations play a central role in determining exploitation status. For, in Definition 4 the definition of UE exploitation requires knowledge of equilibrium prices and of the social reproduction point, and it is related to the production and distribution of global income and social labour. Third, UE exploitation is identified as a feature of the competitive allocation of social labour rather than as the result of productive inefficiencies, or labour market imperfections.

Fourth, Definition 4 transparently captures the fundamental intuitions of UE theory. For it identifies exploitation status by comparing the labour contributed by each country $\nu$ and the share of aggregate social labour received by $\nu$ via its national income. Moreover, Yoshihara and Veneziani [34] have shown that in a rich domain of (static) convex economies, Definition 4 is the *only* UE definition that satisfies a small set of formally weak and theoretically desirable properties.[20]

The next result proves that if Definition 4 is adopted then both **WECP** and **CECP** hold.

**Theorem 3**: *Let $\left((\mathbf{p},\mathbf{r}),(\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be an IRS for $E(\Omega_0)$ such that $1+r_t > \max_i \frac{p_{it}}{p_{it-1}}$ for all $t$. Then, under Definition 4,* **WECP** *and* **CECP** *hold.*

**Proof: 1.** First, we show that Definition 4 satisfies **LE** at an IRS. Since $\left((\mathbf{p},\mathbf{r}),(\xi^\nu)_{\nu\in\mathcal{N}}\right)$ is an IRS for $E(\Omega_0)$, it follows that $(\alpha_t^{\mathbf{p},\mathbf{r}}+\beta_t^{\mathbf{p},\mathbf{r}}) \in P_t^w(\mathbf{p},\mathbf{r})$ and $\underline{\alpha}_t^{\mathbf{p},\mathbf{r}}+\underline{\beta}_t^{\mathbf{p},\mathbf{r}}=\omega_t$, at all $t$. Further, by Lemma 2, $\Lambda_t^\nu=\Lambda_t^*$ for all $t$ and all $\nu\in\mathcal{N}$. At all $t$, let $(\theta_t^\nu)_{\nu\in\mathcal{N}}\in[0,1]^N$ be such that $\sum_{\nu\in\mathcal{N}}\theta_t^\nu=1$ and $p_{t-1}\omega_t^\nu=\theta_t^\nu p_{t-1}\omega_t$ for each $\nu\in\mathcal{N}$. Then, at all $t$,

$$\begin{aligned}
& (1+r_t)\,p_{t-1}\omega_t^\nu + w_t^{\max}\Lambda_t^* - R_t p_{t-1}\omega_t^\nu \\
= \;& (1+r_t-R_t)\,\theta_t^\nu p_{t-1}\omega_t + w_t^{\max}\Lambda_t^* \\
= \;& \theta_t^\nu\left[p_t\left(\widehat{\alpha}_t^{\mathbf{p},\mathbf{r}}+\widehat{\beta}_t^{\mathbf{p},\mathbf{r}}\right) - w_t^{\max}\left(\alpha_{lt}^{\mathbf{p},\mathbf{r}}+\beta_{lt}^{\mathbf{p},\mathbf{r}}\right)\right] + w_t^{\max}\Lambda_t^*.
\end{aligned}$$

Then, because $1+r_t-R_t>0$ and $w_t^{\max}>0$, all $t$, in each period $t$, there exists $(\tau_t^\nu)_{\nu\in\mathcal{N}}\in(0,1)^N$ such that $\sum_{\nu\in\mathcal{N}}\tau_t^\nu=1$ and

$$\tau_t^\nu p_t\left(\widehat{\alpha}_t^{\mathbf{p},\mathbf{r}}+\widehat{\beta}_t^{\mathbf{p},\mathbf{r}}\right)=\theta_t^\nu\left[p_t\left(\widehat{\alpha}_t^{\mathbf{p},\mathbf{r}}+\widehat{\beta}_t^{\mathbf{p},\mathbf{r}}\right)-w_t^{\max}\left(\alpha_{lt}^{\mathbf{p},\mathbf{r}}+\beta_{lt}^{\mathbf{p},\mathbf{r}}\right)\right]+w_t^{\max}\Lambda_t^*.$$

[20] See also Yoshihara [32] for an axiomatic analysis of Definition 4 in the context of accumulating economies.

According to Definition 4, at all $t$, $\nu \in \mathcal{N}_t^{ter}$ if and only if $\Lambda_t^* < \tau_t^\nu \left(\alpha_{lt}^{\mathbf{p},\mathbf{r}} + \beta_{lt}^{\mathbf{p},\mathbf{r}}\right)$; and $\nu \in \mathcal{N}_t^{ted}$ if and only if $\Lambda_t^* > \tau_t^\nu \left(\alpha_{lt}^{\mathbf{p},\mathbf{r}} + \beta_{lt}^{\mathbf{p},\mathbf{r}}\right)$. By taking $\overline{c}_t^\nu = \underline{c}_t^\nu = \tau_t^\nu \left(\widehat{\alpha}_t^{\mathbf{p},\mathbf{r}} + \widehat{\beta}_t^{\mathbf{p},\mathbf{r}}\right)$ and $\alpha^{\overline{c}_t^\nu} = \alpha^{\underline{c}_t^\nu} = \tau_t^\nu \left(\alpha_t^{\mathbf{p},\mathbf{r}} + \beta_t^{\mathbf{p},\mathbf{r}}\right)$, for all $\nu \in \mathcal{N}$, we can see that Definition 4 satisfies **LE**.

2. By step 1, it suffices to show that under Definition 4, statement (ii) of Theorem 2 holds. Let $\left((\mathbf{p},\mathbf{r}), (\xi^\nu)_{\nu\in\mathcal{N}}\right)$ be an IRS for $E(P,\mathcal{N},u,\rho,\Omega_0)$ such that $1 + r_t > \max_i \frac{p_{it}}{p_{it-1}}$ for all $t$. We show that at all $t$, $\overline{W}_t = \underline{W}_t = W_t^* \equiv \frac{1}{N} p_{t-1}\omega_t > 0$ satisfies all conditions in statement (ii).

First of all, note that for any $E(P,\mathcal{N},u,\rho,\Omega_0') \in \mathcal{E}$ with $\sum_{\nu\in\mathcal{N}} \omega_0'^\nu = \sum_{\nu\in\mathcal{N}} \omega_0^\nu$ and any IRS $\left((\mathbf{p},\mathbf{r}), (\xi'^\nu)_{\nu\in\mathcal{N}}\right)$ for $E(P,\mathcal{N},u,\rho,\Omega_0')$ with $\sum_{\nu\in\mathcal{N}} \omega_t'^\nu = \sum_{\nu\in\mathcal{N}} \omega_t^\nu$, all $t$, $W_t^*$ is well defined, unique and invariant.

Further, for any $E(P,\mathcal{N},u,\rho,\Omega_0') \in \mathcal{E}$ with $\sum_{\nu\in\mathcal{N}} \omega_0'^\nu = \sum_{\nu\in\mathcal{N}} \omega_0^\nu$ and any IRS $\left((\mathbf{p},\mathbf{r}), (\xi'^\nu)_{\nu\in\mathcal{N}}\right)$ for $E(P,\mathcal{N},u,\rho,\Omega_0')$ with $\sum_{\nu\in\mathcal{N}} \omega_t'^\nu = \sum_{\nu\in\mathcal{N}} \omega_t^\nu$, all $t$, for each $\nu \in \mathcal{N}$, we have

$$\begin{aligned} p_t \widehat{\alpha}^{\overline{c}_t'^\nu} - w_t^{\max} \alpha_l^{\overline{c}_t'^\nu} &= \tau_t^\nu \left[ p_t \left(\widehat{\alpha}_t'^{\mathbf{p},\mathbf{r}} + \widehat{\beta}_t'^{\mathbf{p},\mathbf{r}}\right) - w_t^{\max} \left(\alpha_{lt}'^{\mathbf{p},\mathbf{r}} + \beta_{lt}'^{\mathbf{p},\mathbf{r}}\right)\right] \\ &= \tau_t^\nu \left[ (1+r_t)\, p_{t-1} \left(\underline{\alpha}_t'^{\mathbf{p},\mathbf{r}} + \underline{\beta}_t'^{\mathbf{p},\mathbf{r}}\right) - p_t \left(\underline{\alpha}_t'^{\mathbf{p},\mathbf{r}} + \underline{\beta}_t'^{\mathbf{p},\mathbf{r}}\right)\right] \\ &= \tau_t^\nu \left(1 + r_t - R_t\right) p_{t-1}\omega_t. \end{aligned}$$

where the first equality follows from step 1, the second equality follows from the fact that $\left(\alpha_t'^{\mathbf{p},\mathbf{r}} + \beta_t'^{\mathbf{p},\mathbf{r}}\right) \in P_t^w(\mathbf{p},\mathbf{r})$ at a RS, and the last equality follows from the definition of $R_t$ noting that at a RS $\underline{\alpha}_t'^{\mathbf{p},\mathbf{r}} + \underline{\beta}_t'^{\mathbf{p},\mathbf{r}} = \omega_t$. Then, since $\widehat{\alpha}^{\overline{c}_t'^\nu} = \overline{c}_t'^\nu = \widehat{\alpha}^{\underline{c}_t'^\nu} = \underline{c}_t'^\nu$ and $\alpha^{\overline{c}_t'^\nu} = \alpha^{\underline{c}_t'^\nu}$ by step 1, it immediately follows that for any $E(P,\mathcal{N},u,\rho,\Omega_0') \in \mathcal{E}$ with $\sum_{\nu\in\mathcal{N}} \omega_0'^\nu = \sum_{\nu\in\mathcal{N}} \omega_0^\nu$ and any IRS $\left((\mathbf{p},\mathbf{r}), (\xi'^\nu)_{\nu\in\mathcal{N}}\right)$ for $E(P,\mathcal{N},u,\rho,\Omega_0')$ with $\sum_{\nu\in\mathcal{N}} \omega_t'^\nu = \sum_{\nu\in\mathcal{N}} \omega_t^\nu$, all $t$, for each $\nu \in \mathcal{N}$,

$$W_t'^\nu > W_t^* \Leftrightarrow W_t'^\nu > \frac{p_t \overline{c}_t'^\nu - w_t^{\max} \alpha_l^{\overline{c}_t'^\nu}}{1 + r_t - R_t};$$
$$W_t'^\nu < W_t^* \Leftrightarrow W_t'^\nu < \frac{p_t \underline{c}_t'^\nu - w_t^{\max} \alpha_l^{\underline{c}_t'^\nu}}{1 + r_t - R_t}.$$

Finally, since $W_t^* = \frac{1}{N} p_{t-1}\omega_t = p_{t-1}\left(\frac{\underline{\alpha}_t'^{\mathbf{p},\mathbf{r}} + \underline{\beta}_t'^{\mathbf{p},\mathbf{r}}}{N}\right)$, $\left(\alpha_t'^{\mathbf{p},\mathbf{r}} + \beta_t'^{\mathbf{p},\mathbf{r}}\right) \in P_t^w(\mathbf{p},\mathbf{r})$ implies that $p_{t-1}\underline{\alpha}_t^{\min} \leqq W_t^* \leqq p_{t-1}\underline{\alpha}_t^{\max}$.

In summary, statement (ii) of Theorem 2 holds under Definition 4 and therefore **WECP** and **CECP** hold under Definition 4. ■

Theorem 3 implies that the set of definitions identified in Lemma 4 and Theorem 2 is nonempty. If Definition 4 is adopted, then both Roemer's ([22], pp.78ff) *Class-Exploitation Correspondence Principle* and the *Wealth-Exploitation Correspondence Principle* can be generalised to the dynamic equilibrium paths of international economies with general convex technologies and welfare functions, without restricting attention to steady states.

If equilibria with stationary prices and zero net savings are considered, however, a more detailed picture of exploitative international relations can be derived and the standard insights of UE exploitation can be generalised further. First of all, in a dynamic context, UE exploitation status can be defined focusing either on exploitative relations within a given period - $WP_t$ exploitation - or on the whole life of a generation - $WL$ exploitation. We next provide the $WL$ extension of Definition 4. Let $\Delta^\nu = \sum_{t=0}^{T-1} \left(\Lambda_t^\nu - l.v.\left(\widetilde{c}_t^\nu; (\mathbf{p}, \mathbf{r}), \alpha_t^{\mathbf{p},\mathbf{r}} + \beta_t^{\mathbf{p},\mathbf{r}}\right)\right)$.

**Definition 5:** Let $\left((\mathbf{p}, \mathbf{r}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be a RS. Agents in country $\nu$ are *UE exploited during their whole life*, or *$WL$ exploited* if and only if $\Delta^\nu > 0$; they are *$WL$ exploiters* if and only if $\Delta^\nu < 0$.

Definitions 4 and 5 incorporate different normative concerns. The $WL$ definition reflects the intuition that, from a country's viewpoint, to be UE exploited in every period is certainly worse than being exploited only in some periods. An analysis based on the $WP$ definition captures the idea that the existence of UE exploitation is morally relevant *per se*, and a global economy where countries switch their exploitation status over time is not necessarily just. This distinction, however, is not relevant in stationary equilibria where the two criteria provide exactly the same information on the nature of exploitative international relations.[21]

At all $t$, let $W_t^* = \frac{1}{N} p_{t-1} \omega_t$. Given Definitions 4 and 5, the next results follow immediately from Theorems 1-3 at a stationary equilibrium:

**Corollary 1**: *Let $\left((\mathbf{p}, \mathbf{r}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be an IRS for $E(\Omega_0)$ such that at all $t$, $\omega_t^\nu = \omega_0^\nu$ for all $\nu \in \mathcal{N}$, $p_{t-1} = p_t$, and $r_t > 0$. Under Definitions 4 and 5, for all $\nu \in \mathcal{N}$:*
(i) *$\nu \in \mathcal{N}_t^{ted}$ for all $t$, and $\Delta^\nu > 0$ if and only if $W_0^\nu < W_0^*$;*
(ii) *$\nu \in \mathcal{N}_t^{ter}$ for all $t$, and $\Delta^\nu < 0$ if and only if $W_0^\nu > W_0^*$.*

**Corollary 2**: *Let $\left((\mathbf{p}, \mathbf{r}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be an IRS for $E(\Omega_0)$ such that at all*

[21] For a thorough discussion of $WP$ and $WL$ views, see Veneziani [28, 29].

*$t$, $\omega_t^\nu = \omega_0^\nu$ for all $\nu \in \mathcal{N}$, and $p_{t-1} = p_t$. Under Definitions 4 and 5, the following three statements are equivalent*:
(i) $r_t > 0$ *for all $t$;*
(ii) *if $\nu \in C_0^1$, then $\nu \in C^1$ and $\nu \in \mathcal{N}_t^{ter}$ for all $t$, and $\Delta^\nu < 0$;*
(iii) *if $\nu \in C_0^3 \cup C_0^4$, then $\nu \in C^3 \cup C^4$ and $\nu \in \mathcal{N}_t^{ted}$ for all $t$, and $\Delta^\nu > 0$.*

Corollaries 1 and 2 generalise Roemer's [22, 23] analysis of exploitative international relations to the intertemporal model. At a steady state, both class and UE exploitation status depend on a country's *initial* wealth: the level of development of a country at $t = 0$ determines its location in the international class and UE exploitation structure in every subsequent period and over the entire lifetime of a generation. Moreover, the *Class-Exploitation Correspondence Principle* holds both in each period and over the lifetime of a generation, as there exists a $WP$ and $WL$ correspondence between a country's position in the capital market and its exploitation status.

In summary, if one adopts Definitions 4-5, Theorems 1 and 3, and Corollaries 1-2 allow us to identify the structure of dependent and exploitative international relations emerging between developed and less developed countries as the equilibrium outcome of a perfectly competitive international economy, both in each period and (provided one focuses on stationary states) over the whole life of a generation. Mutual benefits from free international trade of commodities and capital coexist with an international stratification of countries in the credit market and with unequal flows of revenue and labour.

# 6 Conclusion

This paper analyses the phenomenon of unequal exchange between countries. A dynamic general equilibrium model is set up, which generalises Roemer's [22, 23] economy with a global capital market. First, the international class structure is completely characterised: a country's class status in the global capital market is determined in a general dynamic equilibrium as a function of its level of development (proxied by the value of the country's productive assets). Developed countries emerge as net lenders in the credit market, whereas less developed countries must borrow in order to optimise. Then, the structure of unequal exchange between countries is analysed axiomatically. The class of definitions that preserve three fundamental properties of UE exploitation theory - including the existence of a correspondence between wealth, class and exploitation status, - in general dynamic equilibria is completely characterised. This class is shown to be nonempty: there exists a UE definition that satisfies a basic domain axiom and both the *Wealth-Exploitation Correspondence Principle* and the *Class-Exploitation Correspondence Principle*. This definition is conceptually related to the so-called 'New Interpretation' (Duménil [5, 6]; Foley [10, 11]; Duménil et al [7]). It is logically consistent and general, and is firmly anchored to empirically observed data.

Based on this definition, unequal international relations are fully characterised and Roemer's [22, 23] results generalised. In equilibrium, countries are partitioned based on their UE exploitation status and on their position in the capital market: advanced countries are net lenders and exploiters, less developed countries are net borrowers and suffer from UE exploitation. Mutual gains from trade and UE exploitation coexist in competitive markets. The exploitative nature of international relations is the product of capital flows, which transfer surplus from less developed to more developed countries.[22]

This provides a normative benchmark to evaluate international relations under globalisation. For, inequalities in wealth and development among countries are at least partly due to past "robbery and plunder" - especially during the colonial period - which makes them, and the unequal exchanges and exploitative relations resulting from them hardly justifiable. To be sure, the radical change in ownership relations in the world economy necessary to eliminate UE exploitation may be considered politically infeasible. This does not make the concept of UE exploitation any less relevant. For it is essential to establish a robust normative benchmark against which to evaluate international relations, and even if it is not possible to eliminate UE exploitation in one stroke, there may be a number of measures to *reduce* it via international transfers and redistribution. An interesting question from this perspective concerns the development of a measure of the *degree or intensity of UE exploitation* of each country, and an index of aggregate UE exploitation in the international economy that goes beyond the rather coarse

[22] Empirical studies on the role of international capital markets and capital flows across countries reach mixed conclusions. There is, however, some evidence to suggest that our analysis does capture some relevant aspects of globalisation. As Nolan and Zhang ([20], p.101) have noted, "Between 1980 and 2008, the globalization decades, companies from the advanced capitalist core increased their outward stock of FDI from $503 billion to $13,623 billion. Developing-country firms also increased their outward stock of FDI, but by 2008 their total amounted to less than a fifth of the core's."

classification into UE exploiting and UE exploited nations. We leave this issue for further research.

# A Inequalities and the persistence of UE

We have shown that there exists a logically consistent and theoretically robust definition of UE exploitation (indeed, an entire *class* of definitions) that allows us to analyse the class and exploitation structure of the global economy in a rather general dynamic setting. In this appendix, we exploit the dynamic nature of our model in order to provide some preliminary insights on two related issues, namely the normative relevance of unequal exchange between countries, and the role of power, force and coercion in international exploitative relations.

It is often argued that in the global economy, international institutions and the use of force play a qualitatively different role with respect to the past, and thus traditional UE theories are inadequate. Even granting this (by no means uncontroversial) claim to be true, the model suggests that the notion of UE exploitation does provide relevant insights on international relations. UE exploitation emerges as the equilibrium feature of a global economy due to inequalities in development and wealth, and the functioning of global markets for commodities and capital, all of which are relevant features of the contemporary global economy. Exploitative international relations take the form of an international transfer of surplus mediated by the capital market. Yet, from a normative viewpoint, the fact that exploitative international relations derive from voluntary market interactions hardly makes them justifiable; UE can be condemned even if competitive conditions prevail and all countries gain from trade. Actually, although an exploitation-based approach has been adopted so far, the unfairness of international relations can also be analysed by focusing on international 'welfare' inequalities.

At all $t$, let $\Lambda_t^*$ be defined as in Lemma 2: recall that at any $t$, $\Lambda_t^*$ is only a function of $(p_{t-1}, p_t, r_t)$. The next Theorem characterises an important set of solutions to $MP^\nu$.

**Theorem A1**: *Let* $(\mathbf{p}, \mathbf{r})$ *be such that* $p_t > \mathbf{0}$, $1 + r_t \geqq \max_i \frac{p_{it}}{pit-1}$, $w_t^{\max} > 0$, *and* $\frac{\phi'(L-\Lambda_t^*)}{w_t^{\max}} = \rho(1 + r_{t+1})\frac{\phi'\left(L-\Lambda_{t+1}^*\right)}{w_{t+1}^{\max}}$ *for all* $t$*, where* $\Lambda_t^*$ *is specified as in Lemma 2. Then* $\Lambda_t^\nu = \Lambda_t^*$ *with* $0 < \Lambda_t^* < L$ *and* $\omega_{t+1}^\nu = \omega_t^\nu$ *for all* $t$ *are* *optimal for all* $\nu$. *If, in addition,* $p_t = p$ *and* $r_t = \frac{1-\rho}{\rho}$ *for all* $t$, *then* $\Lambda_t^* = \Lambda^*$, *all* $t$, *and* $V(\omega_0^\nu) = \left(1-\rho^T\right)\left[\frac{\phi(L-\Lambda^*)+\phi'(L-\Lambda^*)\Lambda^*}{1-\rho} + \frac{p\omega_0^\nu\phi'(L-\Lambda^*)}{w^{\max}\rho}\right]$.

**Proof:** 1. Let $\mathcal{W} \subseteq \mathbb{R}_+^n$ be the state space with generic element $\omega$. The feasibility correspondence $\Psi : \mathcal{W} \to \mathcal{W}$ is the set of feasible states at $t+1$ given the state at $t$: $\Psi(\omega_t^\nu) = \left\{\omega_{t+1}^\nu \in \mathcal{W} \mid p_t\omega_{t+1}^\nu \leqq w_t^{\max}L + (1+r_t)p_{t-1}\omega_t^\nu\right\}$. The set of feasible sequences is

$$\mathcal{F}(\omega_0^\nu) = \left\{\omega^\nu \mid \omega_{t+1}^\nu \in \Psi(\omega_t^\nu) \text{ for all } t,\, p_{T-1}\omega_T^\nu \geqq p_{T-1}\omega_0^\nu, \text{ and } \omega_0^\nu \text{ given}\right\}.$$

Let $\Phi = \left\{(\omega_t^\nu, \omega_{t+1}^\nu) \in \mathcal{W}\times\mathcal{W} \mid \omega_{t+1}^\nu \in \Psi(\omega_t^\nu)\right\}$ be the graph of $\Psi$. We can use a two-stage approach to simplify the intertemporal problem and by Lemma 2, write the one-period return function $F : \Phi \to \mathbb{R}$ at $t$ as

$$F(\omega_t^\nu, \omega_{t+1}^\nu) = \left[\phi\left(L-\Lambda_t^*\right) + \phi'\left(L-\Lambda_t^*\right)\Lambda_t^*\right] - \frac{\left[p_t\omega_{t+1}^\nu - (1+r_t)p_{t-1}\omega_t^\nu\right]\phi'\left(L-\Lambda_t^*\right)}{w_t^{\max}}.$$

Program $MP^\nu$ can then be written as

$$\begin{aligned} & V(\omega_0^\nu) \\ = & \max_{\omega^\nu\in\mathcal{F}(\omega_0^\nu)} \sum_{t=0}^{T-1}\rho^t\left\{\left[\phi\left(L-\Lambda_t^*\right) + \phi'\left(L-\Lambda_t^*\right)\Lambda_t^*\right] - \frac{\left[p_t\omega_{t+1}^\nu - (1+r_t)p_{t-1}\omega_t^\nu\right]\phi'\left(L-\Lambda_t^*\right)}{w_t^{\max}}\right\}. \end{aligned}$$

Clearly, $\Psi(\omega_t^\nu) \neq \varnothing$ for all $\omega_t^\nu \in \mathcal{W}$. Moreover, noting that $0 < \Lambda_t^* < L$, $0 < \phi'\left(L-\Lambda_t^*\right) < \infty$, so that $F$ is bounded, and $MP^\nu$ is well defined.

2. If $\frac{\phi'(L-\Lambda_t^*)}{w_t^{\max}} = \rho(1+r_{t+1})\frac{\phi'\left(L-\Lambda_{t+1}^*\right)}{w_{t+1}^{\max}}$ for all $t$, then $MP^\nu$ reduces to

$$\begin{aligned} V(\omega_0^\nu) &= \max_{\omega^\nu\in\mathcal{F}(\omega_0^\nu)} \sum_{t=0}^{T-1}\rho^t\left[\phi\left(L-\Lambda_t^*\right) + \phi'\left(L-\Lambda_t^*\right)\Lambda_t^*\right] \\ &\quad + \left[\frac{(1+r_0)p_{-1}\omega_0^\nu\phi'\left(L-\Lambda_0^*\right)}{w_0^{\max}} - \rho^{T-1}\frac{p_{T-1}\omega_T^\nu\phi'\left(L-\Lambda_{T-1}^*\right)}{w_{T-1}^{\max}}\right], \end{aligned}$$

and thus any $\omega^\nu \in \mathcal{F}(\omega_0^\nu)$ such that $p_{T-1}\omega_T^\nu = p_{T-1}\omega_0^\nu$ is optimal, including $\omega_t^\nu = \omega_0^\nu$ for all $t$.

**3.** If $p_t = p$ and $r_t = \frac{1-\rho}{\rho}$, all $t$, then $w_t^{\max} = w^{\max} = \max_{\alpha \in P} \frac{p\overline{\alpha} - \rho^{-1} p\underline{\alpha}}{\alpha_l}$, all $t$. Therefore, since $\frac{\phi'(L-\Lambda_t^*)}{w^{\max}} = \frac{\phi'\left(L-\Lambda_{t+1}^*\right)}{w^{\max}}$ for all $t$, $\Lambda_t^* = \Lambda^*$ holds for all $t$, and the rest of the statement follows from step 2. ■

Theorem A1 provides some additional insights on the normative implications of the model. Consider stationary equilibria in which Roemer's results are fully generalised, in that the intertemporal economy is precisely the $T$-fold iteration of the static model. Under Definitions 4-5, Corollaries 1 and 2 imply that wealth inequalities yield UE exploitation. By Theorem A1, for any two countries $\nu$, $\mu$, if $W_0^\nu > W_0^\mu$ then $V(\omega_0^\nu) > V(\omega_0^\mu)$, and thus wealth inequalities yield welfare inequalities, too. But then according to both the exploitation criterion and the 'welfare' inequality view, the model identifies the counterfactual for exploitative international relations: exploitative international relations should be evaluated against a benchmark economy in which morally arbitrary differences in initial endowments are eliminated.

According to Roemer ([23], p.57), "it is not immediately clear that the argument [for the socialization of capital within a country] applies as well to the socialization of capital among nation-states". This claim is not entirely convincing. As argued by Roemer himself, at least part of the international inequalities in capital endowments derive from acts of "robbery and plunder" at the expense of less developed countries during so-called primitive accumulation by developed countries, and thus they *are* morally arbitrary. Even if the socialisation of capital at the world level may seem utopian, the model can still provide the normative foundations for capital transfers to less developed countries as a requirement of justice, rather than charity.

Thus, our analysis confirms one of the core insights of Roemer's theory [22, 23]: inequalities in wealth and development are instrumental in yielding stratification in the global capital market, and exploitative international relations. It is not clear, however, that "the unequal exchange phenomenon can be driven *entirely* by different [productive endowments]" (Roemer [23], p.57, italics in the original). This is true in Roemer's one period, static economies but not necessarily in the intertemporal context: Veneziani [28, 29] has shown that, in dynamic subsistence economies, if agents are allowed to (but do not necessarily) save, wealth inequalities are necessary for the emergence of UE exploitation but they are not sufficient for its persistence. In the rest of the appendix, we provide further support to this conclusion.

Consider first an IRS with $p_t = p$, $r_t = r$, and $\omega_t = \omega_0$, all $t$, or *sta-*

*tionary RS* (henceforth, SRS). SRS's are theoretically relevant and can be considered as a normative and positive benchmark, because they allow us to fully generalise the standard insights of UE theory, as shown in section 5 above. By Lemma 2, however, it immediately follows that at a SRS, it must be $r_t = r^* = \frac{1-\rho}{\rho}$, all $t$, and therefore the existence of UE exploitation and classes crucially depends on a strictly positive rate of time preference. If $\rho = 1$, then $r^* = 0$ and there is no UE exploitation in the international economy. As Veneziani [28, 29] has argued, once the dynamic nature of the global economy is fully taken into account, UE exploitation and classes are not driven *entirely* by different levels of wealth and development.

The same conclusion holds if one analyses the long run dynamics of the economy focusing on a more general set of equilibria with stationary capital but a time-varying price vector. This is interesting for two reasons. First, it is well known that persistent accumulation may lead to the disappearance of UE exploitation and classes by making capital abundant relative to labour. As argued by Veneziani [28, 29], it is more surprising that a similar result may hold even in economies *without accumulation*. Second, although initial aggregate endowments $\omega_0$ may not be equal to the optimal level, $\omega^*$, such that an IRS with stationary capital exists, well-known turnpike results (McKenzie [17]) suggest that there is a sufficiently high $\rho' \in (0,1]$ such that for any $\rho \in [\rho', 1]$, any optimal path of capital stocks starting from a suitably restricted initial capital stock 'converges' to a 'neighbourhood' of the stationary optimal capital stock $\omega^*$, as $T$ tends to infinity.

If we focus on equilibria with stationary capital, we have the following:

**Theorem A2:** *Let $T \to +\infty$ and let $\left((\mathbf{p}, \mathbf{r}), (\xi^\nu)_{\nu \in \mathcal{N}}\right)$ be an IRS for $E(\Omega_0)$ such that $\omega_t = \omega_0$ and $1 + r_t > \max_i \frac{p_{it}}{p_{it-1}}$ for all $t$. Suppose that the sequence of equilibrium interest rates $\{r_t\}_{t=0}^{\infty}$ is convergent. Then $r_{T-1} \to r^* = \frac{1-\rho}{\rho}$, as $T \to +\infty$.*

**Proof.** 1. Suppose, contrary to the statement, that $\lim_{T \to \infty} r_{T-1} = r' > r^*$. Then, there exists a sufficiently large $t' > 0$ such that for any $t > t'$, $r_t > r^*$. Take any such $t > t'$. By Lemma 2, at an IRS, we have $\frac{\phi'(L-\Lambda_t^*)}{w_t^{\max}} = \rho(1 + r_{t+1})\frac{\phi'\left(L-\Lambda_{t+1}^*\right)}{w_{t+1}^{\max}}$, all $t$, and therefore, for any $T - 1 > t > t'$,

$$\frac{\phi'\left(L - \Lambda_t^*\right)}{w_t^{\max}} = \rho^{T-t-1} \prod_{k=1}^{T-t-1} (1 + r_{t+k}) \frac{\phi'\left(L - \Lambda_{T-1}^*\right)}{w_{T-1}^{\max}}. \tag{7}$$

Because $r_t > r^*$ for all $t > t'$, we have $\lim_{T\to\infty} \rho^{T-t-1} \prod_{k=1}^{T-t-1} (1 + r_{t+k}) = \infty$. Therefore, since at any IRS, $0 < \frac{\phi'(L-\Lambda_t^*)}{w_t^{\max}} < \infty$ at all $t$, equation (7) can hold only if $\lim_{T\to\infty} \frac{\phi'\left(L-\Lambda_{T-1}^*\right)}{w_{T-1}^{\max}} = 0$. We show that this is not possible at an IRS, yielding the desired contradiction.

2. First we prove that $\lim_{T\to\infty} w_{T-1}^{\max} < \infty$. At an IRS with $\omega_t = \omega_0$ and $1 + r_t > \max_i \frac{p_{it}}{p_{it-1}}$ for all $t$, by Proposition 1 and Lemma 2, at all $T-1$,

$$\begin{aligned} w_{T-1}^{\max} &= \frac{p_{T-1}\left(\overline{\alpha}_{T-1}^{\mathbf{p,r}} + \overline{\beta}_{T-1}^{\mathbf{p,r}}\right) - (1 + r_{T-1})\, p_{T-2}\left(\underline{\alpha}_{T-1}^{\mathbf{p,r}} + \underline{\beta}_{T-1}^{\mathbf{p,r}}\right)}{N\left(\alpha_{lT-1}^{\mathbf{p,r}} + \beta_{lT-1}^{\mathbf{p,r}}\right)} \\ &= \frac{p_{T-1}\left(\overline{\alpha}_{T-1}^{\mathbf{p,r}} + \overline{\beta}_{T-1}^{\mathbf{p,r}}\right) - (1 + r_{T-1})\, p_{T-2}\omega_0}{N\Lambda_{T-1}^*}. \end{aligned} \tag{8}$$

To see that the numerator of (8) is bounded above, note, first, that $1 + r_{T-1} > \max_i \frac{p_{iT-1}}{p_{iT-2}}$ holds for all $T$, and $\{r_t\}_{t=0}^{\infty}$ is convergent. Next, we show that $\left\{\left(\overline{\alpha}_t^{\mathbf{p,r}} + \overline{\beta}_t^{\mathbf{p,r}}\right)\right\}_{t=0}^{\infty}$ is bounded above. Let

$$P_{\omega_0,NL} \equiv \{\alpha' \in P \mid \underline{\alpha}' = \omega_0 \;\&\; \alpha'_l \in [0, NL]\}.$$

By A1 and the cone property of $P$, $P_{\omega_0,NL}$ is bounded and so compact. Note that $\{(\alpha_t^{\mathbf{p,r}} + \beta_t^{\mathbf{p,r}})\}_{t=0}^{\infty} \subseteq P_{\omega_0,NL}$. For any $\alpha \in P_{\omega_0,NL}$, let $F(\alpha) \equiv \max_{i=1,\dots,n} \frac{\overline{\alpha}_i}{\alpha_l}$. By the nontriviality of the IRS, $F(\alpha_t^{\mathbf{p,r}} + \beta_t^{\mathbf{p,r}}) > 0$, all $t$.

Suppose, by way of contradiction, that $\left\{\left(\overline{\alpha}_t^{\mathbf{p,r}} + \overline{\beta}_t^{\mathbf{p,r}}\right)\right\}_{t=0}^{\infty}$ is unbounded above. Then $\lim_{T\to\infty} \sup_{\alpha \in \left\{\left(\alpha_t^{\mathbf{p,r}} + \beta_t^{\mathbf{p,r}}\right)\right\}_{t=0}^{T-1}} F(\alpha) = \infty$. Without loss of generality, let $\sup_{\alpha \in \left\{\left(\alpha_t^{\mathbf{p,r}} + \beta_t^{\mathbf{p,r}}\right)\right\}_{t=0}^{T-1}} F(\alpha) = F\left(\alpha_{T-1}^{\mathbf{p,r}} + \beta_{T-1}^{\mathbf{p,r}}\right)$. Then,

$$\lim_{T\to\infty} \sup_{\alpha \in \left\{\left(\alpha_t^{\mathbf{p,r}} + \beta_t^{\mathbf{p,r}}\right)\right\}_{t=0}^{T-1}} F(\alpha) = F(\alpha') = \infty$$

for $\alpha' = \lim_{T\to\infty}\left(\alpha_{T-1}^{\mathbf{p,r}} + \beta_{T-1}^{\mathbf{p,r}}\right)$. By the boundedness of $P_{\omega_0,NL}$, $F(\alpha') = \infty$ implies $\alpha'_l = 0$. However, since $P$ is closed, $\alpha' \in P$, which contradicts A1. Thus, $\left\{\left(\overline{\alpha}_t^{\mathbf{p,r}} + \overline{\beta}_t^{\mathbf{p,r}}\right)\right\}_{t=0}^{\infty}$ is bounded from above.

To see that the denominator of (8) is bounded away from zero, note that $\left\{\left(\overline{\alpha}_t^{\mathbf{p},\mathbf{r}}+\overline{\beta}_t^{\mathbf{p},\mathbf{r}}\right)\right\}_{t=0}^{\infty}$ is also bounded from below, since $\overline{\alpha}_t^{\mathbf{p},\mathbf{r}}+\overline{\beta}_t^{\mathbf{p},\mathbf{r}} > \omega_0 \geq \mathbf{0}$ holds at all $t$. Therefore, $\lim_{T\to\infty} N\Lambda_{T-1}^* > 0$ by A1.

In summary, we have $\lim_{T\to\infty} w_{T-1}^{\max} < \infty$.

3. Next, observe that $\lim_{T\to\infty} N\Lambda_{T-1}^* > 0$ and the assumptions on $\phi$ imply that $\lim_{T\to\infty} \phi'\left(L-\Lambda_{T-1}^*\right) > 0$. But then $\lim_{T\to\infty} \frac{\phi'\left(L-\Lambda_{T-1}^*\right)}{w_{T-1}^{\max}} > 0$, which yields the desired contradiction. Hence, $\lim_{T\to\infty} r_{T-1} = r' \leqq r^*$.

4. A similar argument proves that $\lim_{T\to\infty} r_{T-1} = r' \geqq r^*$, and therefore $\lim_{T\to\infty} r_{T-1} = r^*$, as desired. ■

Theorem A2 states that at any dynamic equilibrium in which aggregate capital remains constant, if the sequence of equilibrium interest rates is convergent, it converges to the rate $r^* = \frac{1-\rho}{\rho}$ that supports a SRS.[23] Therefore, in the long run, unless $\rho < 1$, UE exploitation and the correspondence between $WP$ class and exploitation status cease to exist. If $\rho = 1$, the exploitation and class structure of the dynamic international economy is not persistent, even if inequalities in wealth and development remain unchanged over time.

To be sure, it may be argued that a strictly positive rate of time preference, $\rho < 1$, is empirically reasonable and a standard assumption in neoclassical growth theory, and so UE exploitation and classes are indeed explained by wealth inequalities. Yet, as argued by Veneziani [28, 29], from a normative perspective, a theory of unequal exchange between countries that crucially relies on time preference does not seem fully satisfactory. Rather, we interpret the above results as suggesting that in a perfectly competitive context wealth inequalities are necessary for the emergence of UE exploitation and classes, but not sufficient for their persistence. Wealth inequalities are fundamental to understand, and normatively evaluate, unequal exchange between countries, but they do not provide the full picture. In the international arena, some other asymmetries in power, technology, access to credit, and so on, are likely to play a fundamental role in explaining the persistence of UE exploitation and class. But a proper analysis of these factors is beyond the scope of the present paper.

[23] It can be proved that there exists a stationary price vector of commodities $p^*$ corresponding to $r^* = \frac{1-\rho}{\rho}$ such that the iteration path of the vector $(p^*, r^*)$ supports a SRS for an appropriately selected $\omega_0$. See the Addendum for a thorough analysis.

# References


[1] Amin, S. (1976): *Unequal Development: An Essay on the Social Formations of Peripheral Capitalism*, Monthly Review, New York.

[2] Bacha, E.L. (1978): "An Interpretation of Unequal Exchange from Prebisch-Singer to Emmanuel," *Journal of Development Economics* 5, 319-330.

[3] Ben-David, D. (1993): "Equalizing Exchange: Trade Liberalization and Income Convergence," *Quarterly Journal of Economics* 108, 653-679.

[4] Chen, Z. (1992): "Long-run Equilibria in a Dynamic Heckscher-Ohlin Model," *Canadian Journal of Economics* 92, 923-943.

[5] Duménil, G. (1980): *De la Valeur aux Prix de Production*, Economica, Paris.

[6] Duménil, G. (1984): "The So-Called 'Transformation Problem' Revisited: A Brief Comment," *Journal of Economic Theory* 33, 340-348.

[7] Duménil, G., Foley, D.K., and D. Lévy (2009): "A Note on the Formal Treatment of Exploitation in a Model with Heterogeneous Labor," *Metroeconomica* 60, 560-567.

[8] Emmanuel, A. (1972): *Unequal exchange*, Monthly Review Press, New York.

[9] Frank, A.G. (1978): *Dependent accumulation and underdevelopment*, Monthly Review Press, New York.

[10] Foley, D.K. (1982): "The Value of Money, the Value of Labor Power, and the Marxian Transformation Problem," *Review of Radical Political Economics* 14, 37-47.

[11] Foley, D.K. (1982): "Realization and Accumulation in a Marxian Model of the Circuit of Capital," *Journal of Economic Theory* 28, 300-319.

[12] Griffin, K., and J. Gurley (1985): "Radical Analyses of Imperialism, The Third World, and the Transition to Socialism: A Survey Article," *Journal of Economic Literature* 23, 1089-1143.

[13] Krugman, P. (1981): "Trade, Accumulation, and Uneven Development," *Journal of Development Economics* 8, 149-161.

[14] Luxemburg, R. (1951): *The Accumulation of Capital*, Routledge, London.

[15] Marx, K. (1968): *Theories of Surplus Value, Vol.III*, Lawrence & Wishart, London.

[16] Matsuyama, K. (2004): "Financial Market Globalization, Symmetry-breaking and Endogenous Inequality of Nations," *Econometrica* 72, 853-884.

[17] McKenzie, L.W. (2002): *Classical General Equilibrium Theory*, The MIT Press, Cambridge.

[18] Morishima, M. (1969): *Theory of Economic Growth*, Oxford University Press, Oxford.

[19] Morishima, M. (1974): "Marx in the Light of Modern Economic Theory," *Econometrica* 42, 611-632.

[20] Nolan, P. and J. Zhang (2010): "Global Competition after Financial Crisis," *New Left Review* 64, 97-108.

[21] Raj, D. (2010): "Uneven Growth: A Framework for Research in Development Economics," *Journal of Economic Perspectives* 24, 45-60.

[22] Roemer, J.E. (1982): *A General Theory of Exploitation and Class*, Harvard University Press, Cambridge.

[23] Roemer, J.E. (1983): "Unequal Exchange, Labor Migration, and International Capital Flows: A Theoretical Synthesis," in: Desai, P. (Ed.), *Marxism, Central Planning and the Soviet Economy*, MIT Press, Cambridge.

[24] Roemer, J.E. (1986): *Value, exploitation, and class*, Harwood Academic Publishers, New York.

[25] Roemer, J.E. and R. Veneziani (2004): "What we owe our children, they their children...," *Journal of Public Economic Theory* 6, 637-654.

[26] Silvestre, J. (2007): "Intergenerational equity and human development," in: Roemer, J. E. and K. Suzumura (Eds.), *Intergenerational Equity and Sustainability*, MacMillan, London.

[27] Slaughter, M.J. (2001): "Trade Liberalization and per capita Income Convergence: a Difference-in-differences Analysis," *Journal of International Economics* 55, 203-228.

[28] Veneziani, R. (2007): "Exploitation and Time," *Journal of Economic Theory* 132, 189-207.

[29] Veneziani, R. (2013): "Exploitation, Inequality, and Power," *Journal of Theoretical Politics* 25, 526-545.

[30] Veneziani, R. and N. Yoshihara (2014): "One million miles to go: taking the axiomatic road to defining exploitation," DP 10, UMass Amherst.

[31] Veneziani, R. and N. Yoshihara (2015): "Exploitation in Economies with Heterogeneous Preferences, Skills and Assets: An Axiomatic Approach," *Journal of Theoretical Politics* 27, 8-33.

[32] Yoshihara, N. (2010): "Class and Exploitation in General Convex Cone Economies," *Journal of Economic Behavior & Organization* 75, 281-296.

[33] Yoshihara, N. (2016): "A Progress Report on Marxian Economic Theory: On the Controversies in Exploitation Theory since Okishio (1963)," forthcoming in *Journal of Economic Surveys*.

[34] Yoshihara, N. and R. Veneziani (2009): "Exploitation as the Unequal Exchange of Labour: An Axiomatic Approach," WP No.655, Queen Mary University of London.

[35] Yoshihara, N. and R. Veneziani (2011): "Strong Subjectivism in the Theory of Exploitation: A Critique," *Metroeconomica* 62, 53-68.

# Addendum to: "Globalisation and Inequality: A Dynamic General Equilibrium Model of Unequal Exchange"

Roberto Veneziani* and Naoki Yoshihara†

April 15, 2016

**Abstract**

This addendum proves the existence of an equilibrium for the economies analysed in the paper.

*School of Economics and Finance, Queen Mary University of London, Mile End Road, London E1 4NS, UK. E-mail: r.veneziani@qmul.ac.uk

†(Corresponding author) Department of Economics, University of Massachusetts Amherst, 200 Hicks Way, Amherst, MA 01003, USA; The Institute of Economic Research, Hitotsubashi University, Kunitachi, Tokyo 186-0004, Japan; and School of Management, Kochi University of Technology, Kami-city, Kochi 782-8502, Japan. E-mail: n_yoshihara_1967@yahoo.co.jp

# 1 Existence of a SRS

In this Addendum, we prove the existence of a RS with constant prices, interest rate, and aggregate capital. Formally, a *Stationary Reproducible Solution* (SRS) is a RS such that at all $t$, $p_{t+1} = p_t$, $r_{t+1} = r_t$ and $\omega_{t+1}^{\nu} = \omega_t^{\nu}$ for all $\nu \in \mathcal{N}$.

By Proposition 1 and Lemma 1, for any given intertemporal price vector $(\mathbf{p}, \mathbf{r}) = \{(p, r_t)\}_{t=0}^{T-1}$, for all $\nu$, the set of individually optimal solutions $\mathcal{O}^{\nu}(\mathbf{p}, \mathbf{r})$ always contains vectors $\xi^{\nu} = (\mathbf{0},\ \beta^{\nu},\ z^{\nu}, \delta^{\nu}, c^{\nu}, \omega^{\nu})$ such that, at all $t$, $w_t^{\max}\beta_{lt}^{\nu} + r_t z_t^{\nu} = p\left(c_t^{\nu} + \omega_{t+1}^{\nu} - \omega_t^{\nu}\right)$ with $z_t^{\nu} = p\omega_t^{\nu}$, $w_t^{\max} \equiv \max_{\alpha \in P} \frac{p\widehat{\alpha} - r_t p\underline{\alpha}}{\alpha_l}$, and - given that we shall focus on equilibria with $r_t \geqq 0$, - $\delta_t^{\nu} = \mathbf{0}$. Thus, problem $MP^{\nu}$ can be reduced to the following form:

$$\max_{\xi^{\nu}} \sum_{t=0}^{T-1} \rho^t u\left(c_t^{\nu}, L - \Lambda_t^{\nu}\right),$$

subject to

$$\begin{aligned} w_t^{\max}\beta_{lt}^{\nu} + r_t z_t^{\nu} &= p\left(c_t^{\nu} + \omega_{t+1}^{\nu} - \omega_t^{\nu}\right), \\ z_t^{\nu} &= p\omega_t^{\nu}, \\ \beta_t^{\nu} &\in P_t^{w}(\mathbf{p}, \mathbf{r}),\ \beta_{lt}^{\nu} = \Lambda_t^{\nu} \leqq L, \\ p\omega_T^{\nu} &\geqq p\omega_0^{\nu}. \end{aligned}$$

Then, by Lemma 2, in order to prove the existence of a SRS, it suffices to focus on the Euler equation $\frac{p}{w_t^{\max}}\phi'\left(L - \Lambda_t^{*}\right) = \rho(1 + r_{t+1})\frac{p}{w_{t+1}^{\max}}\phi'\left(L - \Lambda_{t+1}^{*}\right)$ for all $t$. Let $\Delta \equiv \left\{q_t \in \mathbb{R}_{+}^{n} \mid \sum_{i=1}^{n} q_{it} = 1\right\}$ and $\Delta_{+} \equiv \left\{q_t \in \mathbb{R}_{++}^{n} \mid \sum_{i=1}^{n} q_{it} = 1\right\}$.

To begin with, consider the function $w^{(p,r^*)} \equiv \max_{\alpha \in P} \frac{p\widehat{\alpha} - r^* p\underline{\alpha}}{\alpha_l}$, where $r^* = \frac{1-\rho}{\rho}$. Define $\widetilde{w}(r) = \min_{p \in \Delta} \max_{\alpha \in P} \frac{p\widehat{\alpha} - rp\underline{\alpha}}{\alpha_l}$. Note that $\widetilde{w}(0) > 0$ and that $\widetilde{w}(r)$ is continuous. This implies that there exists an interval $I^r = [0, \overline{r}]$ such that for all $r \in I^r$, $w_t^{\max} = \max_{\alpha \in P} \frac{p_t\widehat{\alpha} - r_t p_t\underline{\alpha}}{\alpha_l} \geqq 0$ for all $p \in \Delta$. Let $\overline{\rho} \equiv \frac{1}{1+\overline{r}}$: if $\rho \in [\overline{\rho}, 1]$, then $r^* = \frac{1-\rho}{\rho} \geqq 0$ guarantees $w^{(p,r^*)} \geqq 0$ for all $p \in \Delta$. In what follows, we assume $\rho \in (\overline{\rho}, 1]$.[1] Therefore, if $p \in \Delta_{+}$ and $r_t = r^* \geqq 0$

[1] This is without loss of generality because Proposition 1 rules out the possibility that $w_t^{\max} = w_{t+1}^{\max} = w^{(p,r^*)} = 0$ at an IRS.

for all $t$, then $w_t^{\max} = w_{t+1}^{\max} = w^{(p,r^*)} > 0$ all $t$, and the Euler equation is well-defined with $\Lambda_t^* = \Lambda_{t+1}^* = \Lambda^*$ all $t$.

Moreover, by the assumptions on $u$, $w_t^{\max} = w_{t+1}^{\max} = w^{(p,r^*)} > 0$ and $p_{t+1} = p_t$, all $t$, imply that the structure of individual consumption demand will be constant over time, with $c_t^\nu = k_t^\nu c\left(\frac{p}{w^{(p,r^*)}}\right)$ for some $k_t^\nu \geqq 0$, all $t$, and all $\nu \in \mathcal{N}$. Finally, note that if $(\mathbf{p}, \mathbf{r})$ is a SRS such that $r_t = r^*$ for all $t$, then $w_t^{\max} = w_{t+1}^{\max}$ for all $t$, implies that $\widehat{\beta}_t = \widehat{\beta}_{t+1}$ and $c_t = c_{t+1} = k^* c\left(\frac{p}{w^{(p,r^*)}}\right)$ for all $t$, where $k^* = \sum_{\nu \in \mathcal{N}} k^\nu$. Let $P^{w^{(p,r^*)}} = \left\{\alpha \in P \mid w^{(p,r^*)} = \frac{p\widehat{\alpha} - r^* p\underline{\alpha}}{\alpha_l}\right\}$. Because $\beta_t, \beta_{t+1} \in P^{w^{(p,r^*)}}$, $\widehat{\beta}_t = \widehat{\beta}_{t+1}$ and $\underline{\beta}_t = \underline{\beta}_{t+1}$ imply $\beta_t = \beta_{t+1}$ for all $t$.

In order to show the existence of an SRS, we first show the existence of a one-period *temporary reproducible solution*, which focuses on resource allocation in period $t = 0$. In this model, in fact, the one-period individual optimisation programmes can be reduced to a social planner's problem. Formally, given $p \in \Delta_+$, let $f^{\min}(p) \equiv \min_{\beta_0 \in P^{w^{(p,r^*)}}} \frac{p\underline{\beta}_0}{\beta_{l0}}$ and $f^{\max}(p) \equiv \max_{\beta_0 \in P^{w^{(p,r^*)}}} \frac{p\underline{\beta}_0}{\beta_{l0}}$. Then, given $p \in \Delta_+$, the planner will solve the following optimisation problem $SMP$: for any given $f(p) \in \left[f^{\min}(p), f^{\max}(p)\right]$,

$$\max_{\left(\beta_0, \left(c_0^\nu\right)_{\nu \in \mathcal{N}}, \left(\Lambda_0^\nu\right)_{\nu \in \mathcal{N}}\right)} \sum_{\nu \in \mathcal{N}} u\left(c_0^\nu, L - \Lambda_0^\nu\right)$$

subject to

$$\begin{aligned} w^{(p,r^*)}\beta_{l0} + r^* p\underline{\beta}_0 &= \sum_{\nu \in \mathcal{N}} pc_0^\nu,\ \beta_0 \in P^{w^{(p,r^*)}}, \\ \beta_{l0} &= \sum_{\nu \in \mathcal{N}} \Lambda_0^\nu \leqq NL, \\ \frac{p\underline{\beta}_0}{\beta_{l0}} &= f(p). \end{aligned}$$

Denote the set of solutions to $SMP$ at $p \in \Delta_+$ for a given $f(p) \in \left[f^{\min}(p), f^{\max}(p)\right]$ by

$$\mathcal{O}(p, r^*; f(p)) \equiv \left\{\left(\beta_0, \left(c_0^\nu\right)_{\nu \in \mathcal{N}}, \left(\Lambda_0^\nu\right)_{\nu \in \mathcal{N}}\right) \text{ solving } SMP \text{ for a given } f(p)\right\}.$$

Moreover, define the set

$$\mathcal{O}(p, r^*) = \cup_{f(p) \in [f^{\min}(p), f^{\max}(p)]} \mathcal{O}(p, r^*; f(p))$$

with generic element $\left(\beta_0\left(p, r^*\right), \left(c_0^\nu\left(p, r^*\right)\right)_{\nu \in \mathcal{N}}, \left(\Lambda_0^\nu\left(p, r^*\right)\right)_{\nu \in \mathcal{N}}\right)$.

We show the existence of $p^* \in \Delta_+$ such that there exists

$$\left(\beta_0\left(p, r^*\right), \left(c_0^\nu\left(p, r^*\right)\right)_{\nu \in \mathcal{N}}, \left(\Lambda_0^\nu\left(p, r^*\right)\right)_{\nu \in \mathcal{N}}\right) \in \mathcal{O}\left(p^*, r^*\right)$$

with the property that $\widehat{\beta}_0\left(p^*, r^*\right) \geqq \sum_{\nu \in \mathcal{N}} c_0^\nu\left(p^*, r^*\right)$. Let $E(P, \mathcal{N}, u, \rho)$ denote the economy with technology $P$, agents $\mathcal{N}$, and welfare function $u$ with discount factor $\rho$, where both aggregate productive endowments and their distribution are left unspecified. Formally, define the following solution concept.

**Definition 1A:** A *temporary quasi-reproducible solution* (TQRS) *for* $E(P, \mathcal{N}, u, \rho)$ is a $p \in \Delta_+$ and an associated $(c_0, \beta_0) \in \mathbb{R}_+^n \times P$ such that

(i) there exists a profile $\left(\beta_0\left(p, r^*\right), \left(c_0^\nu\left(p, r^*\right)\right)_{\nu \in \mathcal{N}}, \left(\Lambda_0^\nu\left(p, r^*\right)\right)_{\nu \in \mathcal{N}}\right) \in \mathcal{O}\left(p, r^*\right)$ such that $\beta_0\left(p, r^*\right) = \beta_0$, $\sum_{\nu \in \mathcal{N}} c_0^\nu\left(p, r^*\right) = c_0$, and $\sum_{\nu \in \mathcal{N}} \Lambda_0^\nu\left(p, r^*\right) = \beta_{0l}$;

(ii) $\widehat{\beta}_0 \geqq c_0$.

Definition 1A is called a *quasi*-reproducible solution because it states that the social consumption and production vectors are optimal and aggregate output is sufficient to replace inputs and to satisfy consumption, but it imposes no constraint on aggregate social endowments. In order to analyse the existence of a TQRS, for all $p \in \Delta_+$, let us define:

$$B(p, r^*) \equiv \left\{(c_0, \beta_0) \in \mathbb{R}_+^n \times P^{w^{(p,r^*)}} \mid pc_0 = r^* p\underline{\beta}_0 + w^{(p,r^*)}\beta_{l0};\ 0 \leqq \beta_{l0} \leqq NL\right\}.$$

Moreover, let $f^* \equiv \max_{p \in \Delta} \max_{\beta \in P^{w^{(p,r^*)}}} \frac{p\underline{\beta}}{\beta_l}$ and let $W_0 \equiv f^* NL$. The following set can also be defined:

$$\overline{B}(p, r^*) \equiv \left\{(c_0, \beta_0) \in \mathbb{R}_+^n \times P \mid pc_0 = r^* p\underline{\beta}_0 + w^{(p,r^*)}\beta_{l0};\ p\underline{\beta}_0 \leqq W_0;\ 0 \leqq \beta_{l0} \leqq NL\right\}.$$

Note that $B(p, r^*) \subseteq \overline{B}(p, r^*)$ holds for any $p \in \Delta_+$. Then:

**Lemma 1A:** *Assume* $\rho \in (\overline{\rho}, 1]$. *The correspondence* $B$ *is non-empty, compact-valued and convex-valued, and upper hemi-continuous on* $\Delta_+$. *Moreover,* $\overline{B}$ is *non-empty, closed-valued, convex-valued, and continuous on* $\Delta_+$.

**Proof.** It is easy to see that $B(p, r^*)$ is non-empty, closed, and convex for all $p \in \Delta_+$. We now prove that it is also bounded and so compact-valued.

Let $\mathbb{B}_0(p, r^*)$ be the set of $\beta_0 \in P^{w^{(p,r^*)}}$ such that $0 \leqq \beta_{l0} \leqq NL$. Since $w^{(p,r^*)} > 0$, then for any $\beta_0 \in P^{w^{(p,r^*)}}$, $p\overline{\beta}_0 > 0$ holds, which implies that its corresponding $\beta_{l0}$ is positive by A1. Then, because of $0 \leqq \beta_{l0} \leqq NL$ and the convex cone property of $P$, $\mathbb{B}_0(p, r^*)$ is bounded. Next, let $\mathbb{C}_0(p, r^*)$ be the set of $c_0 \in \mathbb{R}^n_+$ such that $pc_0 \leqq r^* W_0 + w^{(p,r^*)} NL$. Clearly, $\mathbb{C}_0(p, r^*)$ is bounded for any $p \in \Delta_+$. Therefore, $\mathbb{C}_0(p, r^*) \times \mathbb{B}_0(p, r^*)$ is bounded for any $p \in \Delta_+$. Since $B(p, r^*)$ is a subset of $\mathbb{C}_0(p, r^*) \times \mathbb{B}_0(p, r^*)$, it is also bounded for any $p \in \Delta_+$. Hence, $B(p, r^*)$ is compact for any $p \in \Delta_+$.

It is obvious that $\overline{B}$ is non-empty, closed-valued, and convex-valued.

Let us prove the continuity of $\overline{B}$. To show this, note that since $w^{(p,r^*)} > 0$ then $r^* W_0 + w^{(p,r^*)} NL > 0$ for all $p \in \Delta_+$. The latter property, together with the fact that $r^* W_0 + w^{(p,r^*)} NL$ is continuous on $p \in \Delta_+$, implies that $\overline{B}(p, r^*)$ is continuous at each $p \in \Delta_+$.

Finally, let us show that $B$ is upper hemi-continuous on $\Delta_+$. Let $p^\upsilon \to p$, $(c_0^\upsilon, \beta_0^\upsilon) \in B(p^\upsilon, r^*)$, and $(c_0^\upsilon, \underline{\beta}_0^\upsilon) \to (c_0, \beta_0)$. We need to show $(c_0, \beta_0) \in B(p, r^*)$. Since $B(p^\upsilon, r^*) \subseteq \overline{B}(p^\upsilon, r^*)$ and $\overline{B}$ is upper hemi-continuous, $(c_0, \beta_0) \in \overline{B}(p, r^*)$. Suppose $(c_0, \beta_0) \notin B(p, r^*)$. This implies $\beta_0 \notin P^{w^{(p,r^*)}}$. Then, there exists $(c_0', \beta_0') \in \overline{B}(p, r^*)$ such that $\frac{p\widehat{\beta}_0' - r^* p\underline{\beta}_0'}{\beta_{l0}'} > \frac{p\widehat{\beta}_0 - r^* p\underline{\beta}_0}{\beta_{l0}}$. Since $\overline{B}$ is lower hemi-continuous, there exists a sequence $\{(c_0'^\upsilon, \beta_0'^\upsilon)\}$ such that for each $p^\upsilon$, $(c_0'^\upsilon, \beta_0'^\upsilon) \in \overline{B}(p^\upsilon, r^*)$ and $(c_0'^\upsilon, \beta_0'^\upsilon) \to (c_0', \beta_0')$ as $p^\upsilon \to p$. Then, for $p^\upsilon$ which is sufficiently close to $p$, $\frac{p^\nu \widehat{\beta}_0'^\upsilon - r^* p^\nu \underline{\beta}_0'^\upsilon}{\beta_{l0}'^\upsilon} > \frac{p^\nu \widehat{\beta}_0^\upsilon - r^* p^\nu \underline{\beta}_0^\upsilon}{\beta_{l0}^\upsilon}$. However, this is a contradiction, since $\beta_0^\upsilon \in P^{w^{(p^\upsilon, r^*)}}$. Thus, $(c_0, \beta_0) \in B(p, r^*)$. ■

Given Lemma 1A, we can prove some important properties of $\mathcal{O}(p, r^*)$.

**Lemma 2A:** *Assume $\rho \in (\overline{\rho}, 1]$. The correspondence $\mathcal{O}$ is non-empty, compact-valued, convex-valued, and upper hemi-continuous on $\Delta_+$.*

**Proof.** Given $(p, r^*)$ with $p \in \Delta_+$, $w^{(p,r^*)} > 0$ is uniquely specified. Therefore, by the assumptions on $u$, the profile of optimal labour supply $\left(\Lambda_0^\nu(p, r^*)\right)_{\nu \in \mathcal{N}}$ is uniquely determined, with $\Lambda_0^\nu(p, r^*) = \Lambda_0^*(p, r^*)$ for all $\nu \in \mathcal{N}$ and $0 < \Lambda_0^*(p, r^*) < L$. Moreover, by the linear homogeneity and strict quasi-concavity of $v$, there exists a unique consumption vector $c_0^{(p,r^*)}$ satisfying $\frac{v_i'\left(c_0^{(p,r^*)}\right)}{v_j'\left(c_0^{(p,r^*)}\right)} = \frac{p_i}{p_j}$ for all $i, j$, and $pc_0^{(p,r^*)} = w^{(p,r^*)} N \Lambda_0^*(p, r^*)$. Take any profile $(\lambda^\nu)_{\nu \in \mathcal{N}} \in \mathbb{R}^N_{++}$ with $\sum_{\nu \in \mathcal{N}} \lambda^\nu = 1$. Then, $\left(\beta_0, (c_0^\nu)_{\nu \in \mathcal{N}}, (\Lambda_0^\nu)_{\nu \in \mathcal{N}}\right) \in \mathcal{O}(p, r^*)$ if and only if

$\Lambda_0^\nu(p,r^*) = \Lambda_0^*(p,r^*)$ for all $\nu \in \mathcal{N}$, and for some $f(p) \in \left[f^{\min}(p), f^{\max}(p)\right]$, $c_0^\nu = \lambda^\nu \left(\frac{r^* f(p)}{w^{(p,r^*)}} + 1\right) c_0^{(p,r^*)}$ for each $\nu \in \mathcal{N}$, and $\beta_0 \in P^{w^{(p,r^*)}}$ such that $\beta_{l0} = N\Lambda_0^*(p,r^*)$ and $p\underline{\beta}_0 = f(p) N\Lambda_0^*(p,r^*)$. Therefore $\mathcal{O}(p,r^*)$ is non-empty for each $p \in \Delta_+$. The convexity of $\mathcal{O}(p,r^*)$ for each $p \in \Delta_+$ can be proved in the standard manner. Since $B(p,r^*)$ is compact-valued by Lemma 1A and $\mathcal{O}(p,r^*)$ is closed-valued, $\mathcal{O}(p,r^*)$ is compact for any $p \in \Delta_+$. We need to prove upper hemi-continuity.

Let $\mathcal{F} : \Delta_+ \twoheadrightarrow \mathbb{R}_+$ be such that for each $p \in \Delta_+$, $\mathcal{F}(p) \equiv \left[f^{\min}(p), f^{\max}(p)\right]$: $\mathcal{F}$ is easily shown to be upper hemi-continuous. Then, since $w^{(p,r^*)}$ and $c_0^{(p,r^*)}$ are continuous at every $p \in \Delta_+$, we have the following property: if $p^\upsilon \to p$, then $\lambda^\nu \left(\frac{r^* f(p^\upsilon)}{w^{(p^\upsilon,r^*)}} + 1\right) c_0^{(p^\upsilon,r^*)} \to \lambda^\nu \left(\frac{r^* f(p)}{w^{(p,r^*)}} + 1\right) c_0^{(p,r^*)}$ holds for each $\nu \in \mathcal{N}$, where $f(p^\upsilon) \in \mathcal{F}(p^\upsilon)$ for each $p^\upsilon$ and $f(p) \in \mathcal{F}(p)$. Also, since $\Lambda_0^*(p,r^*)$ is continuous at every $p \in \Delta_+$ and $P^{w^{(p,r^*)}}$ is upper hemi-continuous at every $p \in \Delta_+$, if $p^\upsilon \to p$, $\beta_0^\upsilon \to \beta_0$, where $\beta_0^\upsilon \in P^{w^{(p^\upsilon,r^*)}}$ such that $\beta_{l0}^\upsilon = N\Lambda_0^*(p^\upsilon,r^*)$ and $p^\upsilon \underline{\beta}_0^\upsilon = f(p^\upsilon) N\Lambda_0^*(p^\upsilon,r^*)$ for each $p^\upsilon$, then $\beta_0 \in P^{w^{(p,r^*)}}$ such that $\beta_{l0} = N\Lambda_0^*(p,r^*)$ and $p\underline{\beta}_0 = f(p) N\Lambda_0^*(p,r^*)$. These arguments ensure that $\mathcal{O}$ is upper hemi-continuous on $\Delta_+$. ■

Given Lemmas 1A and 2A, we can now prove the existence of a TQRS.

**Lemma 3A:** *Assume $\rho \in (\overline{\rho}, 1]$. Then, a* TQRS *exists for $E(P, \mathcal{N}, u, \rho)$.*

**Proof.** 1. For any $p \in \Delta_+$, let us define:

$$Z(p) \equiv \left\{ \sum_{\nu \in \mathcal{N}} c_0^\nu - \widehat{\beta}_0 \mid \left(\beta_0, (c_0^\nu)_{\nu \in \mathcal{N}}, (\Lambda_0^\nu)_{\nu \in \mathcal{N}}\right) \in \mathcal{O}(p,r^*) \right\}.$$

It is easy to check that by Lemma 2A, the correspondence $Z$ is non-empty, compact-valued and convex-valued, and upper hemi-continuous on $\Delta_+$. Moreover, for any $z(p) \in Z(p)$, $pz(p) = 0$ holds. This is because

$$\begin{aligned} pz(p) &= p\left[\sum_{\nu\in\mathcal{N}} c_0^\nu(p,r^*) - \widehat{\beta}_0(p,r^*)\right] = \sum_{\nu\in\mathcal{N}} pc_0^\nu(p,r^*) - p\widehat{\beta}_0(p,r^*) \\ &= \sum_{\nu\in\mathcal{N}} pc_0^\nu(p,r^*) - \left[\frac{p\widehat{\beta}_0(p,r^*) - r^*p\underline{\beta}_0(p,r^*)}{\beta_{l0}(p,r^*)}\beta_{l0}(p,r^*) + r^*p\underline{\beta}_0(p,r^*)\right] \\ &= \sum_{\nu\in\mathcal{N}} pc_0^\nu(p,r^*) - \left[w^{(p,r^*)}\beta_{l0}(p,r^*) + r^*p\underline{\beta}_0(p,r^*)\right] \text{ by } \beta_0(p,r^*) \in P^{w^{(p,r^*)}} \\ &= 0 \text{ by } \left(\beta_0(p,r^*), (c_0^\nu(p,r^*))_{\nu\in\mathcal{N}}, (\Lambda_0^*(p,r^*))_{\nu\in\mathcal{N}}\right) \in \mathcal{O}(p,r^*). \end{aligned}$$

**2.** Let us prove that for every sequence $q^m \to q$, $q \in \Delta \setminus \Delta_+$ and $z^m \in Z(q^m)$, there is a $p \in \Delta_+$ - which may depend on $\{q^m\}$ - such that $p \cdot z^m > 0$ for infinitely many $m$. Consider any price vector $q \in \Delta \setminus \Delta_+$, such that $q_i = 0$ for one $i$. Then one may choose $p \in \Delta_+$ such that $p_j = \varepsilon > 0$ for sufficiently small $\varepsilon$ and for all $j \neq i$, and $p_i = 1-(n-1)\varepsilon$. By the strict monotonicity of $u$, and noting that $\widehat{\beta}_0(q^m)$ is bounded from below by zero as well as bounded from above by $\beta_{l0}(q^m) \leqq NL$, it follows that $z_i^m > 0$ for $q^m$ sufficiently close to $q$. Thus, there exists a neighbourhood $B(q,\delta)$ of $q$ such that $p \cdot z^m > 0$ for all $q^m \in B(q,\delta) \cap \Delta_+$. A similar argument holds if $q \in \Delta \setminus \Delta_+$, with $q_i = 0$, for more than one $i$.

**3.** Given steps 1 and 2, it is possible to use Lemma 1 in Grandmont [1], which establishes that there exists $p^* \in \Delta_+$ such that there exists $z^*(p^*) \in Z(p^*)$ such that $z^*(p^*) = \mathbf{0}$. Thus, $\widehat{\beta}_0^*(p^*,r^*) = c_0^*(p^*,r^*) \equiv \sum_{\nu\in\mathcal{N}} c_0^{*\nu}(p^*,r^*)$ holds for $\left(\beta_0^*(p^*,r^*), (c_0^{*\nu}(p^*,r^*))_{\nu\in\mathcal{N}}, (\Lambda_0^*(p^*,r^*))_{\nu\in\mathcal{N}}\right) \in \mathcal{O}(p^*,r^*)$. Thus, $(p^*; c_0^*(p^*,r^*), \beta_0^*(p^*,r^*))$ is a TQRS. ■

The existence of a Stationary Reproducible Solution can now be proved.

**Theorem 1A:** *Assume $\rho \in (\overline{\rho}, 1]$. There exists an aggregate capital endowment $\omega_0 \in \mathbb{R}_+^n$ such that for any profile $(\omega_0^\nu)_{\nu\in\mathcal{N}} \in \mathbb{R}_+^{nN}$ with $\sum_{\nu\in\mathcal{N}} \omega_0^\nu = \omega_0$, there exists a* SRS *for the economy $E(P,\mathcal{N},u,\rho,(\omega_0^\nu)_{\nu\in\mathcal{N}})$.*

**Proof.** By Lemma 3A, there exists $(p^*; (c_0^*(p^*,r^*), \beta_0^*(p^*,r^*)))$ that is a TQRS for $E(P,\mathcal{N},u,\rho)$. Let $(\mathbf{p}^*,\mathbf{r}^*)$ be such that $p_t^* = p^*$ and $r_t^* = r^*$, all $t$. We shall prove that if $\omega_0 = \underline{\beta}_0^*(p^*,r^*)$, then for any profile of capital endowments $(\omega_0^\nu)_{\nu\in\mathcal{N}} \in \mathbb{R}_+^{nN}$ such that $\sum_{\nu\in\mathcal{N}} \omega_0^\nu = \omega_0$, there exists a suitable profile of actions $(\xi^\nu)_{\nu\in\mathcal{N}}$ such that $\left((\mathbf{p}^*,\mathbf{r}^*), (\xi^\nu)_{\nu\in\mathcal{N}}\right)$ is a SRS for the economy $E(P,\mathcal{N},u,\rho,(\omega_0^\nu)_{\nu\in\mathcal{N}})$.

1. Let $(\omega_0^\nu)_{\nu\in\mathcal{N}} \in \mathbb{R}_+^{nN}$ be any profile of capital endowments such that $\sum_{\nu\in\mathcal{N}} \omega_0^\nu = \omega_0 = \underline{\beta}_0^*(p^*, r^*)$. Then for each $\nu \in \mathcal{N}$, consider the vector $\xi^\nu = (\alpha^\nu, \beta^\nu, z^\nu, \delta^\nu, c^\nu, \omega^\nu)$ constructed as follows: at all $t$, $\alpha_t^\nu = \mathbf{0}$, $\beta_t^\nu \in P^{w(p^*,r^*)}$ with $\beta_t^\nu = \frac{1}{N}\beta_0^*(p^*, r^*)$, $\beta_{lt}^\nu = \Lambda_0^*(p^*, r^*)$, $z_t^\nu = p^*\omega_0^\nu$, $\delta_t^\nu = \mathbf{0}$, $c_t^\nu = \frac{r^*p^*\omega_0^\nu + w(p^*,r^*)\Lambda_0^*(p^*,r^*)}{p^*\widehat{\beta}_0^*(p^*,r^*)} c_0^*(p^*, r^*)$, and $\omega_{t+1}^\nu = \omega_t^\nu$. Note that, by construction, $c_t^\nu = k_t^\nu c\left(\frac{p^*}{w(p^*,r^*)}\right)$ and $\beta_{lt}^\nu = \Lambda_0^*(p^*, r^*) < L$ for all $t$, $\nu$. Given $(\mathbf{p}^*, \mathbf{r}^*)$, it follows from Theorem A1 that $\xi^\nu \in \mathcal{O}^\nu(\mathbf{p}^*, \mathbf{r}^*)$ for all $\nu \in \mathcal{N}$ and Definition 1(i) is satisfied.

2. By construction, it follows that, at all $t$, $\sum_{\nu\in\mathcal{N}} \beta_t^\nu = \beta_0^*(p^*, r^*)$, $\sum_{\nu\in\mathcal{N}} c_t^\nu = c_0^*(p^*, r^*)$, $\sum_{\nu\in\mathcal{N}} \left(\omega_{t+1}^\nu - \omega_t^\nu\right) = \mathbf{0}$, and $\sum_{\nu\in\mathcal{N}} z_t^\nu = p^*\underline{\beta}_0^*(p^*, r^*)$. By Lemma 3A, this implies that parts (ii) and (iv) of Definition 1 are also satisfied. Since $\sum_{\nu\in\mathcal{N}} \alpha_t^\nu = \mathbf{0}$ for all $t$, then Definition 1(iii) is satisfied by the assumption $\omega_0 = \underline{\beta}_0^*(p^*, r^*)$. And Definition 1(v) is clearly satisfied.

Therefore $\left((\mathbf{p}^*, \mathbf{r}^*), (\xi^\nu)_{\nu\in\mathcal{N}}\right)$ is a SRS for $E(P, \mathcal{N}, u, \rho, (\omega_0^\nu)_{\nu\in\mathcal{N}})$. ■

# References


[1] Grandmont, J.M. (1977): "Temporary General Equilibrium Theory", *Econometrica* 45, 535-68.