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Naoki Yoshihara
Department of Economics, University of Massachusetts Amherst
Institute of Economic Research, Hitotsubashi University
School of Economics and Management, Kochi University of Technology

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School of Economics and Management
Research Center for Social Design Engineering
Kochi University of Technology

A Progress Report on Marxian Economic Theory: On the Controversies in Exploitation Theory since Okishio (1963)*

Naoki Yoshihara†

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Abstract

This report explores the development of exploitation theory in mathematical Marxian economics by reviewing the main controversies surrounding the proper definition of exploitation since the contribution of Okishio (1963). The report first examines the debates on the Fundamental Marxian Theorem and Class-Exploitation Correspondence Principle, developed mainly in the 1970s and 1980s, followed by the property relation theory of exploitation by Roemer (1982). Then, the more recent exploitation theory proposed by Vrousalis (2013) and Wright (2000) is introduced. Finally, the report introduces and comments on recent axiomatic studies of exploitation by focusing on the work of Veneziani and Yoshihara (2015a).

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†Department of Economics, University of Massachusetts Amherst, Amherst, MA, 01003 USA; School of Management, Kochi University of Technology, Tosayamada, Kami-city, Kochi 782-8502, Japan; and Institute of Economic Research, Hitotsubashi University, Tokyo, 186-8603, Japan, E-mail: n_yoshihara_1967@yahoo.co.jp

1 Introduction

Given the recent and common trends of growing disparity in income and wealth and the increase in poverty among advanced countries, the issue of the long-run distributional feature of wealth and income in the capitalist economy should be at the heart of economic analysis, as Piketty (2014) emphasizes. Piketty (2014) also suggests that divergence in the distribution of wealth and income is explained by the significant inequality between the earnings rate of financial assets and the growth rate, since the former represents the increase in rewards for capital holders while the latter does the increase of the real wage rate. Although controversial, his argument reminds us of Marx's view of the capitalist economy as a conflicting distributional relationship between capitalists and workers.

Marx recognized the conflicting distributional relationship as *exploitative*, and argued that an exploitative relation between capitalists and workers is generic and persistent in the capitalist economy. Since then, the notion of exploitation has been one of the prominent concepts relevant to capitalist economic systems, particularly in a number of debates and analyses of labor relations, especially focusing on the weakest segments of the labor force (see, e.g., ILO, 2005a,b).

However, the nature of exploitation in the capitalist economy is unclear, while it is a matter of observation that feudal lords exploit serfs in the feudal system in that the serf spends part of his/her time working for him/herself and another part on uncompensated work for the lord. Marx argued that a wage worker is also forced to give up part of his/her life to the capitalist. That is, while the serf must work for the lord according to feudal law and usage, the relationship between capitalists and workers is mediated by a market contract that the worker is formally free not to enter. Nonetheless, the worker cannot but spend part of his/her time working for a given capitalist by entering into a contract, since otherwise he/she could not procure his/her necessities because of a lack of access to means of production.

Given these background arguments, the *unequal exchange of labor* (UE) may well be considered as a *descriptive feature* of exploitation in the capitalist economy, in that exploitative relations involve systematic differences between the amount of labor that individuals contribute to the economy *in some relevant sense* and the amount of labor they receive *in some relevant sense* through their income. Exploitation as UE (hereafter, UE exploitation) may also have some *normative features* relevant to a diagnosis of the capitalist economy. For instance, it captures certain inequalities in the distribution of material well-being and free hours[1] that are —at least *prima facie*— of normative relevance. Indeed, they are relevant for the *inequalities of well-being freedom* as discussed by Rawls (1971) and Sen (1985) because material well-being and free hours are two key determinants of individual well-being freedom.[2] Second, an UE

[1] See Fleurbaey (2014) for this point.

[2] The notion of well-being freedom emphasizes an individual's ability to pursue the life he/she values. According to Rawls–Sen theory, inequalities in the distribution of well-being freedom are formulated as *inequalities of capabilities*, whereas they are formulated as *inequal-*

exploitation-free allocation coincides with the so-called *proportional solution*, a well-known fair allocation rule whereby every agent's income is proportional to his/her contribution to the economy (Roemer and Silvestre, 1993).

Although the notion of UE exploitation seems to be intuitive, the application of this notion to the capitalist economy involves a fundamental difficulty: unlike the case of feudal exploitation, the division of a worker's labor into working for him/herself and working for a capitalist is not a matter of observation since the market contract between buyers and sellers of labor power is simply observed as an equal exchange of labor. Therefore, the existence of UE due to the exploitative relationship in the capitalist economy should be measured through economic analysis. To promote such an analysis, one of the central issues in exploitation theory is to stipulate a suitable operational method to measure the difference between the labor expended and the labor received by an individual via his/her income, without which we cannot credibly diagnose the capitalist economy as an exploitative economic system.

As such a measure, Okishio (1963) provides a formal definition of exploitation to diagnose the capitalist economy as an exploitative economic system under the so-called *Fundamental Marxian Theorem* (FMT). Although Okishio's (1963) definition of exploitation is essentially faithful to the classical labor theory of value and theory of surplus value, it intrinsically relies on the assumption of two-class economies with simple Leontief production technology, homogeneous labor, and homogeneous preferences for consumption and leisure. Indeed, outside of these simple economies, the appropriate definition of the labor "contributed to" and "received by" agents is no longer obvious; moreover, the core diagnoses of the capitalist economy such as the FMT no longer hold under Okishio's (1963) definition of UE exploitation, which has led to several proposals for alternative definitions.

This report examines the development of exploitation theory since the contribution of Okishio (1963) by reviewing the main controversies regarding the proper formal definition of UE exploitation.[3] In the first place, assuming homogeneous labor and homogeneous preferences for consumption and leisure, we review Okishio's definition of exploitation and its generalized versions such as Morishima (1974) and Roemer (1982) through a debate on the FMT in economies with more general production technology. Given that the FMT holds with Okishio's (1963) definition in economies with simple Leontief production technology, if the FMT with any generalization of this definition is not preserved under more general production technology, this would suggest the *incoherence* of such a definition rather than the limitation of the basic Marxian perception of capitalist economies as exploitative. Indeed, it seems to be plausible to think that a purely technological matter such as the complexity of production technology should not be essential to determine the position of each agent in the

ities of (comprehensive) resources in Dworkin's (2000) theory.

[3] Because of this limited purpose, we mainly refer to the literature relevant to the proper formal definition of UE exploitation in mathematical Marxian economics, although many other influential works exist in that field such as the "macro-monetary" approach of Fred Moseley and the "temporal single-system" framework of Andrew Kliman and others.

exploitative relation. Similarly, we also discuss the validity of alternative generalizations of Okishio's (1963) definition from the viewpoint of the so-called *Class-Exploitation Correspondence Principle* (CECP), which was first shown by Roemer (1982) by using Okishio's (1963) definition in economies with simple Leontief production technology. We summarize these arguments and suggest the limitation and noneligibility of these classical definitions.

Second, we review the *property relation definition of exploitation* (PR exploitation), proposed by Roemer (1982, 1994) as an alternative to UE exploitation. Again assuming homogeneous labor and identical preferences for consumption and leisure, the definition of PR exploitation is a mathematical extension of Okishio's definition. Moreover, it is generally true that under this definition, the capitalist economy can be conceived of as exploitative. However, the PR theory of exploitation denies the relevance of exploitation as a primary normative concern: Roemer (1994) argues that the primary normative concern should be the injustice of the unequal distribution of productive assets rather than UE exploitation *per se*. Given this criticism of the notion of UE exploitation, we also review some arguments in political philosophy and sociology such as those proposed by Cohen (1995), Wright (2000), and Vrousalis (2013), which criticize the PR theory of exploitation and encourage the revival of the UE theory of exploitation. Based on their arguments, Roemer's claim that the theory of exploitation is reduced to a theory of distributive injustice can be invalidated.

Third, allowing heterogeneous labor skills, heterogeneous preferences for consumption and leisure, and general production technology, we review the recent development of an axiomatic theory of exploitation. Among the works of this approach, this report focuses on the *Profit-Exploitation Correspondence Principle* (PECP) (Veneziani and Yoshihara, 2015a). Then, an extension of the exploitation form à la the New Interpretation (NI) originally introduced by Duménil (1980) and Foley (1982) is shown to be uniquely eligible among the main definitions provided in the literature.

The remainder of this paper is organized as follows. Section 2 discusses the robustness and economic implications of the debates on the FMT initiated by Okishio (1963) and developed mainly in the 1970s and 1980s, followed by a discussion of the CECP. Section 3 introduces the criticism of UE exploitation from the standpoint of PR exploitation, and section 4 provides the counterarguments to PR exploitation by Cohen (1995), Wright (2000), and Vrousalis (2013). Section 5 provides an overview of recent axiomatic studies of exploitation. Finally, section 6 concludes the report and provides a perspective on the remaining subjects of exploitation theory.

2 The main developments in mathematical Marxian economics from the 1970s until the 1990s

In this section, we provide an overview of the main arguments regarding a proper definition of exploitation in mathematical Marxian economics developed

until the 1990s. We begin with the significant contribution by Nobuo Okishio, known for the Fundamental Marxian Theorem, and then discuss the successive developments and relevant debates on this theorem, mainly initiated by Michio Morishima and John Roemer during the 1970s and 1980s.

2.1 The formulation of labor exploitation by Okishio and the Fundamental Marxian Theorem

An economy comprises a set of agents, $\mathcal{N} = \{1, .., N\}$, with generic element $\nu \in \mathcal{N}$.[4] Denote the cardinal number of this set by N. Similarly, the cardinal number for any subset, $\mathcal{S} \subseteq \mathcal{N}$, is denoted by S. There are n types of (purely private) commodities that are transferable in markets.

Production technology, commonly accessible by any agent, is represented by a production possibility set $P \subseteq \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+$ with generic element $\alpha \equiv (-\alpha_l, -\underline{\alpha}, \overline{\alpha})$, where $\alpha_l \in \mathbb{R}_+$ is the *effective* labor input; $\underline{\alpha} \in \mathbb{R}^n_+$ are the inputs of the produced goods; and $\overline{\alpha} \in \mathbb{R}^n_+$ are the outputs of the n goods. The net output vector arising from α is denoted as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. P is assumed to be closed and convex-cone such that (i) $\mathbf{0} \in P$; (ii) for any $\alpha \in P$ with $\overline{\alpha} \geq \mathbf{0}$, $\alpha_l > 0$ holds; and (iii) for any $c \in \mathbb{R}^n_+$, there exists $\alpha \in P$ such that $\widehat{\alpha} \geqq c$. Property (ii) implies that labor is indispensable for the production of a positive amount of a commodity, while property (iii) implies that any non-negative vector of commodities can be produced as a net output. A specific type of production technology P is of a *Leontief type* if there exists a pair (A, L), where A is an $n \times n$ non-negative square matrix of material input coefficients and L is a $1 \times n$ positive vector of labor input coefficients, such that P is represented by the following form:

$$P_{(A,L)} \equiv \left\{\alpha \equiv (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists x \in \mathbb{R}^n_+ : \alpha \leqq (-Lx, -Ax, x)\right\}.$$

Here, A is assumed to be productive and indecomposable. Another specific type of production technology P is of a *von Neumann type* if there exists a profile (A, B, L) such that P is represented by the following form:

$$P_{(A,B,L)} \equiv \left\{\alpha \equiv (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists x \in \mathbb{R}^m_+ : \alpha \leqq (-Lx, -Ax, Bx)\right\}.$$

where A is an $n \times m$ matrix, the generic component of which, $a_{ij} \geqq 0$, represents the amount of commodity i used as an input to operate one unit of the j-th production process; B is an $n \times m$ matrix, the generic component of which, $b_{ij} \geqq 0$, represents the amount of commodity i produced as an output by operating one unit of the j-th production process; and L is a $1 \times m$ positive row vector of direct labor input coefficients. In the following discussion, we sometimes use the notation A_i (resp. B_i) to refer to the i-th row vector of A (resp. B).

[4] Let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}_+$ (*resp.* $\mathbb{R}_-$) the set of non-negative (*resp.* non-positive) real numbers. For all $x, y \in \mathbb{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, n)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; and $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, n)$. For any set, X and Y, $X \subseteq Y$ if and only if for any $x \in X$, $x \in Y$; $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$; and $X \subsetneq Y$ if and only if $X \subseteq Y$ and $X \neq Y$.

In this and the next sections, assume that for each production period, the maximal amount of labor supply by every agent is equal to unity and there is no difference in labor skills (human capital) among agents. Let $b \in \mathbb{R}^n_+$ be the basic consumption bundle, which is the minimum consumption necessary for every agent when supplying one unit of labor. Let $\omega \in \mathbb{R}^n_+ \setminus \{\mathbf{0}\}$ be the social endowments of commodities.

Assuming a private ownership economy, let ω^ν be the initial endowment of commodities owned by agent $\nu \in \mathcal{N}$. In the following discussion, let $\mathcal{W} \equiv \{\nu \in \mathcal{N} \mid \omega^\nu = \mathbf{0}\}$ be the set of propertyless agents. Typically, $\mathcal{W}$ would represent the set of workers who own no material means of production. In summary, one *capitalist economy* is described by a profile $\langle \mathcal{N}; P; (\omega^\nu)_{\nu\in\mathcal{N}} \rangle$.

Given a capitalist economy $\langle \mathcal{N}; P_{(A,L)}; (\omega^\nu)_{\nu\in\mathcal{N}} \rangle$, let v represent a vector of each commodity's labor value. Note that, according to classical economics and Marx, the labor value of commodity i, v_i, is defined as the sum of the amount of labor directly and/or indirectly input to produce one unit of this commodity. Therefore, this value is mathematically formulated by the solution of the system of equations, $v = vA + L$. Here, since matrix A is productive and vector L is positive, $v \in \mathbb{R}^n_{++}$ is the unique solution of the system of equations. Then, the labor value of any commodity vector $c \in \mathbb{R}^n_+$ is given by $vc \geqq 0$.

Let $w \in \mathbb{R}_+$ represent a wage rate. Assume that any agent, $\nu \in \mathcal{W}$, can purchase the consumption vector, b, with wage revenue, w, per working day. Moreover, let $p \in \mathbb{R}^n_+ \setminus \{\mathbf{0}\}$ represent a vector of market prices for n types of commodities. Then:

Definition 1: A *balanced-growth equilibrium (BGE) for a capitalist economy* $\langle \mathcal{N}; P_{(A,L)}; (\omega^\nu)_{\nu\in\mathcal{N}} \rangle$ is a profile $(p, w) \in \mathbb{R}^{n+1}_+ \setminus \{\mathbf{0}\}$ that satisfies the following:

$$p = (1 + \pi)\,[pA + wL] \quad \& \quad w = pb,$$

where the scalar $\pi \geqq 0$ represents the equal profit rate.[5]

Definition 2 (Okishio, 1963): In a capitalist economy $\langle \mathcal{N}; P_{(A,L)}; (\omega^\nu)_{\nu\in\mathcal{N}} \rangle$, *labor exploitation exists* if and only if $vb < 1$.

That is, within one working day, normalized to unity, vb corresponds to the necessary labor hours for each $\nu \in \mathcal{W}$, so that $1 - vb$ represents the surplus labor hours. Therefore, the existence of labor exploitation is none other than the existence of positive surplus labor.

Under Definition 2, Okishio proves the validity of the basic Marxian view, which conceives the capitalist economy as exploitative, as the following theorem shows:

[5] The condition of $w = pb$ means that the wage rate is so low that any propertyless agent cannot but expend all of the wage revenue for the basic consumption bundle, which corresponds to the case where the aggregate labor supply is excessive with respect to the aggregate labor demand in the equilibrium.

Fundamental Marxian Theorem (FMT) (Okishio, 1963): Let (p, w) be a BGE associated with an equal profit rate π for capitalist economy $\langle \mathcal{N}; P_{(A,L)}; (\omega^{\nu})_{\nu \in \mathcal{N}} \rangle$. Then,

$$\pi > 0 \Leftrightarrow vb < 1.$$

Since Okishio (1963), numerous studies have examined the robustness of the FMT. Of these, we review the works of Morishima (1974) and Roemer (1980), which discuss the generalization of the FMT to a model of the von Neumann type, in order to show the robustness of the FMT in economies with fixed capital, joint production, and the possibility of technical choices.[6]

Let $x_j \geqq 0$ represent an activity level of the j-th production process, so that a profile of social production activities is represented by a non-negative $m \times 1$ column vector, $x \equiv (x_j)_{j=1,...,m}$. For a von Neumann capitalist economy, $\langle \mathcal{N}; P_{(A,B,L)}; (\omega^{\nu})_{\nu \in \mathcal{N}} \rangle$, we can respectively define the notions of BGEs, labor values, and labor exploitation as follows:

Definition 3: A *balanced-growth equilibrium* ***(BGE)*** *for a capitalist economy* $\langle \mathcal{N}; P_{(A,B,L)}; (\omega^{\nu})_{\nu \in \mathcal{N}} \rangle$ is a profile of non-negative and non-zero vectors, $((p, w), x) \in \mathbb{R}^{n+1}_{+} \times \mathbb{R}^{m}_{+}$, that satisfy the following:

$$pB \leqq (1+\pi)[pA + wL]; \; Bx \geqq (1+\pi)[A + bL]x; \; pBx > 0; \; \& \; w = pb.$$

Definition 4 (Morishima, 1974): Given a capitalist economy, $\langle \mathcal{N}; P_{(A,B,L)}; (\omega^{\nu})_{\nu \in \mathcal{N}} \rangle$, the *labor value of a consumption bundle*, $c \in \mathbb{R}^{n}_{+}$, is the solution, Lx^{c}, of the following constrained optimization program:

$$\min_{x \geqq \mathbf{0}} Lx \quad s.t. \; [B - A]x \geqq c.$$

Then, *labor exploitation* is said to exist if and only if $Lx^{b} < 1$.

Definition 4 extends Definition 2 into von Neumann capitalist economies.

Morishima (1974) shows that, under the BGE, the equivalence between the existence of labor exploitation and a positive equal profit rate is preserved, even in a von Neumann capitalist economy:

Generalized Fundamental Marxian Theorem (GFMT) (Morishima, 1974): Let $((p, w), x)$ be a BGE associated with the equal profit rate, π, for capitalist economy $\langle \mathcal{N}; P_{(A,B,L)}; (\omega^{\nu})_{\nu \in \mathcal{N}} \rangle$. Then,

$$\pi > 0 \Leftrightarrow Lx^{b} < 1.$$

[6] Note that generalizations of the FMT to a Leontief economy with *heterogeneous labor* have also been examined by Morishima (1973), Bowles and Gintis (1977, 1978), and Krause (1981). This line of research focuses on solving the reduction problem of heterogeneous labor into one common unit, and/or the dilemma of the heterogeneity of labor and the respective rates of exploitation. Thus far, the robustness of the FMT in this line of generalization has remained firm.

By contrast, Roemer (1980) defines an alternative equilibrium notion, called a *reproducible solution*, which is to preserve its coherency with the profit-maximizing behavior of every capital owner, $\nu \in \mathcal{N}\backslash\mathcal{W}$. He then examines the robustness of the FMT under this equilibrium. That is,

Definition 5 (Roemer, 1980): A *reproducible solution* ***(RS)*** *for a capitalist economy* $\left\langle \mathcal{N}; P_{(A,B,L)}; (\omega^{\nu})_{\nu\in\mathcal{N}} \right\rangle$ is a profile of non-negative and non-zero vectors, $((p^*, w^*), x^*) \in \mathbb{R}^{n+1}_{+} \times \mathbb{R}^{m}_{+}$, that satisfies the following:[7]
(a) $x^{\nu *} \in \arg\max_{x^{\nu} \geqq \mathbf{0}} p^* B x^{\nu} + (p^* \omega^{\nu} - [p^* A + w^* L] x^{\nu})$, such that $[p^* A + w^* L] x^{\nu} \leqq p^* \omega^{\nu}$ $(\forall \nu \in \mathcal{N}\backslash\mathcal{W})$, where $x^* \equiv \sum_{\nu\in\mathcal{N}\backslash\mathcal{W}} x^{\nu *}$;
(b) $(B - [A + bL]) x^* + \omega \geqq \omega$;
(c) $w^* = p^* b$;
(d) $[A + bL] x^* \leqq \omega$.

In this definition, (a) requires that every capital owner maximizes the monetary value of her capital for the next period, given the equilibrium price vector; (b) requires the *reproducibility* of the economy itself, in that the aggregate capital stock at the beginning of the next period, $(B - [A + bL]) x^* + \omega$, is at least as much as that at the beginning of the present period, ω;[8] (c) stipulates the labor market equilibrium; (d) is the feasibility condition of social production. An RS is a Walrasian equilibrium with an additional condition of the reproducibility.

Roemer's Fundamental Marxian Theorem (RFMT) (Roemer, 1980): For any capitalist economy, $\left\langle \mathcal{N}; P_{(A,B,L)}; (\omega^{\nu})_{\nu\in\mathcal{N}} \right\rangle$, and any RS, $((p^*, w^*), x^*)$, the following two statements are equivalent:
(1) $p^* [B - A] x^* - w^* L x^* > 0 \Leftrightarrow L x^b < 1$;
(2) $\forall x, x' \geqq \mathbf{0}$, $Lx = Lx', [\exists i \in \{1, \ldots, n\} : (B_i - A_i) x > (B_i - A_i) x'] \Rightarrow \exists x'' \geqq \mathbf{0} : Lx'' = Lx', (B_i - A_i) x'' = (B_i - A_i) x', \& \exists i' \in \{1, \ldots, n\} : (B_{i'} - A_{i'}) x'' > (B_{i'} - A_{i'}) x$.

In the above theorem, statement (1) implies the equivalence between the positivity of total profit, $p^* [B - A] x^* - w^* L x^*$, at the RS and the existence of labor exploitation in terms of Definition 4. By contrast, statement (2) characterizes the necessary and sufficient condition for statement (1) to hold. Suppose two production activities, say x and x', have the same corresponding labor inputs. Then, according to statement (2), if the net output of some commodity, say i, via activity x is strictly greater than that via activity x', then there is another commodity, say i', such that the net output of i' via some suitable production

[7] As Roemer (1980) explicitly shows, there is essentially no difference between the BGE (given in Definition 1) and the RS in capitalist economies with Leontief production technology. However, these two notions of equilibria differ whenever a more general model of capitalist economies is considered.

[8] As Roemer (1980, p.507) states, "What equilibrium or solution concept is adopted in Marxian-Sraffian analysis? The concern is with whether the economic system can reproduce itself: whether it can produce enough output to replenish the inputs used, and to reproduce the workers for another period of work."

activity x'', which may be identical to or different from x', is strictly greater than that via x. This statement is named the *independence of production* by Roemer (1980). If it fails, there is a commodity that can be produced only as a joint product in fixed proportions with another commodity. Thus, the RFMT implies that, in any capitalist economy, the equivalence relationship between positive profits and the existence of labor exploitation holds for any RS if and only if no commodity is produced only as a joint product of another good (in terms of condition (2)) in this economy.

To see the difference between the GFMT and RFMT, consider the following example.

Example 1: Consider a von Neumann economy, $\left\langle \mathcal{N}; P_{(A,B,L)}; (\omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$, such that

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix},\ L = (1,1),\ b = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ \text{and } \omega = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

In this economy, condition (2) of the RFMT is violated, because $B - A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $L = (1,1)$, which implies that while the second production process is superior to the first production process in the production of commodity 1, no production process is superior to the second production process in the production of commodity 2.

Note that in this economy, the set of BGEs is characterized by

$$\left\{ ((p,w),x) \in \{((0,1),1)\} \times \mathbb{R}^2_+ \mid x \neq \mathbf{0} \right\},$$

where all BGEs are associated with $\pi = 0$. By contrast, the set of RSs is characterized by

$$\left\{ ((p^*,w^*),x^*) \in \mathbb{R}^2_+ \times \{1\} \times \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \mid p_1^* + p_2^* = 1 \right\},$$

where, if $p_1^* > 0$, then $\pi^* = \frac{p_1^*}{2p_1^* + p_2^*} > 0$; while, if $p_1^* = 0$, then $\pi^* = 0$.

Next, in this economy, the labor value of the commodity bundle b is $Lx^b = 1$, where x^b is any non-negative vector satisfying $x_1^b + x_2^b = 1$. Thus, according to Definition 5, there is no exploitation in this economy.

Therefore, the GFMT holds in this economy, since in any BGE, the corresponding profit rate is $\pi = 0$, while there is no exploitation. However, we can find an RS $((p^*,w^*),x^*)$, with $p_1^* > 0$, whose corresponding profit rate is $\pi^* > 0$. Thus, if the economy arrives at this equilibrium, then positive profits are generated in conjunction with no exploitation, which violates condition (1) of the RFMT. This contrast between the GFMT and RFMT can be observed when the economy does not satisfy condition (2) of the RFMT. ■

Okishio–Morishima's proposal for the formulation of labor exploitation given in Definitions 2 and 4 is consistent with the basic perception of the labor theory

of value, since it is formulated completely independently of price information. Given this formulation, however, Example 1 suggests that the equivalence between positive profits and the existence of exploitation no longer holds for RSs in a general economic environment with the possibility of fixed capital, joint production, and technical choices. Firstly, the extension of the equilibrium notion from the BGE to the RS seems to be reasonable whenever we view a capitalist economy as a resource allocation mechanism working via the capitalists' profit-seeking motivation under market competition. Secondly, there is no reason to eliminate such an economy as in Example 1 from the subject of our analysis. However, this negative result may suggest that Okishio–Morishima's definition of labor exploitation is inappropriate rather than that the basic Marxian perception of the capitalist economy as exploitative is not confirmed, since it seems strange that such a purely technological condition as the existence of a *non-independently produced commodity* makes the capitalist economy non-exploitative.

There is another, even more serious criticism of Okishio–Morishima's definition, given by Bowles and Gintis (1981), Roemer (1982), and Samuelson (1982). To see this, we return to a Leontief capitalist economy, $\left\langle \mathcal{N}; P_{(A,L)}; (\omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$. Then, take any commodity, k, and let $v_i^{(k)}$, for each commodity i, be the aggregate amount of commodity k directly and/or indirectly input to produce one unit of the commodity i. Let $v^{(k)} \equiv \left(v_i^{(k)} \right)_{i \in \{1,\ldots,n\}}$ be a vector of *commodity k-values*. Analogical to the case of the vector of labor values, $v^{(k)}$ can be defined as the solution of the following system of equations:

$$v^{(k)} = v^{(k)} \left[A + bL \right] + \left(1 - v_k^{(k)} \right) \left[A_k + b_k L \right],$$

where A_k is the k-th row vector of matrix A. Then,

Definition 6: In a capitalist economy, $\left\langle \mathcal{N}; P_{(A,L)}; (\omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$, the *exploitation of commodity k exists* if and only if $v_k^{(k)} < 1$.

Generalized Commodity Exploitation Theorem (GCET) (Bowles and Gintis, 1981; Roemer, 1982; Samuelson, 1982): Let (p, w) be a BGE associated with the equal profit rate, π, for capitalist economy $\left\langle \mathcal{N}; P_{(A,L)}; (\omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$. Then,

$$\pi > 0 \Leftrightarrow vb < 1 \Leftrightarrow v_k^{(k)} < 1.$$

Establishing the GCET leads us to see Okishio–Morishima's definition of labor exploitation as representing the productiveness of an overall economic system. This is because the existence of commodity k's exploitation is the exact numerical representation of the productiveness of an overall economic system if we select commodity k as the numéraire, in that the overall economic system is productive enough to guarantee the possibility of surplus products via the efficient use of commodity k as a factor of production. Analogically, we can interpret the existence of labor exploitation in terms of Definition 2 as the

numerical representation of the productiveness of an overall economic system by selecting labor as the numéraire. Therefore, the equivalence between the FMT and GCET indicates that the necessary and sufficient condition for positive profits is that the whole economic system is sufficiently productive to guarantee the possibility of surplus products, which is a trivial proposition. This view prompted criticism of Okishio's original motivation and interpretation of the FMT, in that it may simply affirm the productiveness of the capitalist economy, rather than the Marxian perception of the capitalist economy as an exploitative system.[9] However, the criticism should be directed to the suitability of Okishio–Morishima's formulation, since such a definition fails to present the intrinsic feature of UE exploitation, which should not be reduced to the productiveness of an overall economic system.

2.2 Class-Exploitation Correspondence Principle

Although the FMT has been criticized as mentioned above, there is another well-known analysis to validate the Marxian perception of the capitalist economy as an exploitative system: that is, the Class-Exploitation Correspondence Principle (CECP) proposed by Roemer (1982).

2.2.1 A general model

We first set up a more general economic model than that presented in section 2.1 and its corresponding equilibrium notion. These settings are also used in section 5.

Given $\mathcal{N}$ and P defined in section 2.1, agents can be heterogeneous in terms of their capital endowments $(\omega_t^\nu)_{\nu\in\mathcal{N}}$ in each period t. Moreover, for each $\nu \in \mathcal{N}$, $s^\nu > 0$ represents his/her skill level. Let $C \subseteq \mathbb{R}^n_+ \times [0,1]$ be the consumption space common to all agents, and for each $\nu \in \mathcal{N}$, let $u^\nu : C \to \mathbb{R}_+$ be his/her welfare function. All available welfare functions are assumed to be strongly increasing in consumption bundles and decreasing in the supply of labor hours. Thus, one capitalist economy is defined by the list $\mathcal{E} \equiv \left\langle \mathcal{N}; P; (u^\nu, s^\nu, \omega_0^\nu)_{\nu\in\mathcal{N}} \right\rangle$.

As in Roemer (1980, 1982), the time structure of production is explicitly considered and production activities are financed with current wealth. Agent ν's wealth, at the beginning of period t, is given by $p_{t-1}\omega_t^\nu$: this is fixed at the end of period $t-1$ given market prices p_{t-1}. Thus, given a price system $\langle \{p_{t-1}, p_t\}, w_t \rangle$ in period t, each agent $\nu \in \mathcal{N}$ engages in an optimal choice of production plan $\alpha_t^\nu \in P$. Here, each agent (i) purchases a bundle of capital goods $\underline{\alpha_t^\nu}$ at price p_{t-1} under his/her wealth constraint, $p_{t-1}\omega_t^\nu$, and employs labor power, α_{lt}^ν, at the beginning of this period; (ii) purchases an optimal amount of commodity

[9] For a more detailed discussion on the implications of the GCET, see Roemer (1982) and Yoshihara and Veneziani (2010a, b). Studies such as Fujimoto and Fujita (2008) and Matsuo (2009) criticize the GCET, supporting Okishio–Morishima's definition of labor exploitation. Such critics emphasize that the coefficients L and b are not simply technologically determined but reflect the state of class struggle. This point is true, but it does not deny that Okishio–Morishima's definition is mainly used to represent the productiveness of an overall economic system.

bundle δ_t^ν at price p_{t-1} under budget constraint $p_{t-1}\left(\omega_t^\nu - \underline{\alpha}_t^\nu\right)$ for speculative purposes, to be sold at the end of the period with an expected price p_t; and (iii) chooses an optimal labor supply and consumption plan, $(c_t^\nu, l_t^\nu) \in C$, where c_t^ν will be purchased at the end of this period with an expected price (p_t, w_t) under the budget constraint of his/her revenue from both production and speculation. This choice behavior is determined as a solution to the optimization problem (MP_t^ν), as follows:

$$MP_t^\nu : \max_{(c_t^\nu, l_t^\nu)\in C;\ \delta_t^\nu \in \mathbb{R}^n_+\ ;\ \alpha_t^\nu \in P} u^\nu\left(c_t^\nu, l_t^\nu\right)$$

$$\begin{aligned} \text{s.t.} \quad [p_t\overline{\alpha}_t^\nu - w_t\alpha_{lt}^\nu] + w_t\Lambda_t^\nu + p_t\delta_t^\nu &\geqq p_t c_t^\nu + p_t\omega_{t+1}^\nu, \text{ where } \Lambda_t^\nu \equiv s^\nu l_t^\nu; \\ p_{t-1}\delta_t^\nu + p_{t-1}\underline{\alpha}_t^\nu &\leqq p_{t-1}\omega_t^\nu; \\ p_t\omega_{t+1}^\nu &\geqq p_{t-1}\omega_t^\nu. \end{aligned}$$

Then, denote the set of solutions to the problem (MP_t^ν) by $\boldsymbol{O}_t^\nu\left(\{p_{t-1}, p_t\}, w_t\right)$.

We focus on the stationary equilibrium price vector, $p^* = p_{t-1} = p_t\ (\forall t)$. Moreover, we focus on the non-trivial equilibrium satisfying $\pi \equiv \max_{\alpha' \in P} \frac{p^*\overline{\alpha}' - p^*\underline{\alpha}' - w_t\alpha_l'}{p^*\underline{\alpha}'} \geqq 0$. In this case, according to the monotone increasing characteristic of u^ν at c_t^ν, there always exists an optimal solution having $\delta_t^\nu = \mathbf{0}$. By focusing on this optimal solution, we can remove the description of δ_t^ν without loss of generality. Therefore, we consider the following equilibrium notion:

Definition 7: For a capitalist economy, $\mathcal{E}$, a *reproducible solution (RS)* is a profile $\left(\left(p^*, w_t^*\right); \left(\left(c_t^{*\nu}, l_t^{*\nu}\right); \alpha_t^{*\nu}\right)_{\nu\in\mathcal{N}}\right)$ of a price system and economic activities in each period, t, satisfying the following conditions:
(i) $\left(\left(c_t^{*\nu}, l_t^{*\nu}\right); \alpha_t^{*\nu}\right) \in \boldsymbol{O}_t^\nu\left(p^*, w_t^*\right)\ (\forall t)$ (each agent's optimization);
(ii) $\sum_{\nu\in\mathcal{N}} \widehat{\alpha}_t^{*\nu} \geqq \sum_{\nu\in\mathcal{N}} c_t^{*\nu}\ (\forall t)$ (demand-supply matching at the end of each period);
(iii) $\sum_{\nu\in\mathcal{N}} \alpha_{lt}^{*\nu} = \sum_{\nu\in\mathcal{N}} \Lambda_t^{*\nu}\ (\forall t)$ (labor market equilibrium);
(iv) $\sum_{\nu\in\mathcal{N}} \underline{\alpha}_t^{*\nu} \leqq \sum_{\nu\in\mathcal{N}} \omega_t^\nu\ (\forall t)$ (social feasibility of production at the beginning of each period).

Henceforth, we assume the stationary state on economic activities of agents and delete the time description, t.

2.2.2 Class and exploitation

Although the above definitions of MP_t^ν and the RS are the most general, let us focus on a specific case of such a general form of capitalist economies within this subsection. First, we assume that all agents have the same level of skill, so that $s^\nu = 1$ for each $\nu \in \mathcal{N}$, without loss of generality. Therefore, $\Lambda_t^\nu = l_t^\nu$ holds for each $\nu \in \mathcal{N}$. Second, we focus on the following case: for each $\nu \in \mathcal{N}$, $u^\nu(c, l) \equiv \psi^\nu(c)$. That is, all agents are indifferent to the increase in leisure. In this case, for any economy $\mathcal{E} = \left\langle \mathcal{N}; P; \left(\psi^\nu, 1, \omega^\nu\right)_{\nu\in\mathcal{N}} \right\rangle$ and any RS

$\left((p,w)\,;((c^\nu,l^\nu)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$, $l^\nu=1$ and $pc^\nu=\pi p\omega^\nu+w$ hold for any $\nu\in\mathcal{N}$. Let $\Pi^\nu(p,w)\equiv\pi p\omega^\nu+w$ be the net revenue of $\nu\in\mathcal{N}$ at RS-prices (p,w).

Based on Roemer (1982), we define the class structure of the capitalist economy as follows:

Definition 8: For a capitalist economy, $\mathcal{E}=\left\langle\mathcal{N};P;(\psi^\nu,1,\omega^\nu)_{\nu\in\mathcal{N}}\right\rangle$, let $\left((p,w)\,;((c^\nu,1)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$ be an RS. Then, for each $\nu\in\mathcal{N}$:
(i) ν is a member of the *capitalist class*, $\mathcal{C}^1(\subseteq\mathcal{N})$ if and only if $\alpha_l^\nu>1$;
(ii) ν is a member of the *petit bourgeois class*, $\mathcal{C}^2(\subseteq\mathcal{N})$ if and only if $\alpha_l^\nu=1$;
(iii) ν is a member of the *quasi-proletariat class*, $\mathcal{C}^3(\subseteq\mathcal{N})$ if and only if $\alpha_l^\nu<1$; and
(iv) ν is a member of the *pure proletariat class*, $\mathcal{C}^4(\subseteq\mathcal{N})$ if and only if $\alpha_l^\nu=0$.

Under this definition, $\alpha_l^\nu>1$ implies that agent ν should employ others' labor in the equilibrium, meaning that such an agent is an employer; $\alpha_l^\nu=1$ implies that agent ν should be self-employed in the equilibrium; and $\alpha_l^\nu<1$ implies that agent ν could spend a part of his/her labor in his/her own shop but must sell his/her remaining labor to others. Finally, $\alpha_l^\nu=0$ implies that this agent can optimize only by selling all of his/her labor to others.

Based on Okishio–Morishima's definition of exploitation, Roemer (1982) provides a more comprehensive definition of exploitative relations as follows. First, for any bundle $c\in\mathbb{R}_+^n$, let us denote the production possibility set to produce c as a net output by $\phi(c)\equiv\{\alpha\in P\mid\widehat{\alpha}\geqq c\}$. Then:

Definition 9: For a capitalist economy, $\mathcal{E}=\left\langle\mathcal{N};P;(\psi^\nu,1,\omega^\nu)_{\nu\in\mathcal{N}}\right\rangle$, let $\left((p,w)\,;((c^\nu,1)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$ be an RS. Then, for each $\nu\in\mathcal{N}$:
(i) ν is *exploited* if and only if $1>\max_{f^\nu\in\mathbb{R}_+^n:\ pf^\nu=\Pi^\nu(p,w)}\min_{\alpha\in\phi(f^\nu)}\alpha_l$;
(ii) ν is an *exploiter* if and only if $1<\min_{f^\nu\in\mathbb{R}_+^n:\ pf^\nu=\Pi^\nu(p,w)}\min_{\alpha\in\phi(f^\nu)}\alpha_l$.

Note that when $P=P_{(A,B,L)}$, then $\alpha^\nu=(-Lx^\nu,-Ax^\nu,Bx^\nu)$ and so $\min_{\alpha\in\phi(f^\nu)}\alpha_l=Lx^{f^\nu}$, where Lx^{f^ν} is given by Definition 4. Likewise, when $P=P_{(A,L)}$, then $\alpha^\nu=(-Lx^\nu,-Ax^\nu,x^\nu)$ and so $\min_{\alpha\in\phi(f^\nu)}\alpha_l=vf^\nu$.

Given these formulations of class structure and exploitation, Roemer (1982) shows the following:

Theorem of the Class-Exploitation Correspondence Principle (CECP): For any $\mathcal{E}=\left\langle\mathcal{N};P_{(A,L)};(\psi^\nu,1,\omega^\nu)_{\nu\in\mathcal{N}}\right\rangle$ and any RS $\left((p,w)\,;((c^\nu,1)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$ with $\pi>0$,
(i) every agent $\nu\in\mathcal{C}^1$ is an exploiter;
(ii) every agent $\nu\in\mathcal{C}^3\cup\mathcal{C}^4$ is exploited; and
(iii) for any $k=1,2,3$ and any $\nu\in\mathcal{C}^k$ and any $\mu\in\mathcal{C}^{k+1}$, $p\omega^\nu>p\omega^\mu$ holds.

Given Okishio–Morishima's definition of exploitation, the CECP establishes that for any capitalist economy with Leontief production technology and any RS with a positive maximal profit rate, every member of the capitalist class $\mathcal{C}^1$ is an exploiter while every member of the working class $\mathcal{C}^3\cup\mathcal{C}^4$ is exploited. Moreover,

every member of the capitalist class is richer than every member of the working class. This states that in the equilibrium class membership and exploitation status emerge endogenously: the wealthy can rationally choose to belong to the capitalist class among other available options and become an exploiter, while the poor have no other option than being in the working class and are exploited. Thus, it provides a more comprehensive analysis of the capitalist economy as an exploitative system than the FMT, since the latter is only concerned with the (average rate) of exploitation for propertyless agents in $\mathcal{W}$.

Unfortunately, as in the case of the FMT, the robustness of the CECP is also problematic if a more general production technology is considered: given Definition 9, the CECP does not hold in an economy with $P_{(A,B,L)}$. This finding can be confirmed by checking the economy presented in Example 1.[10] From Definition 8(iv), any agent in $\mathcal{W}$ belongs to $\mathcal{C}^4$. Therefore, to verify the CECP, such an agent must be exploited in terms of Definition 9(i). However, in Example 1, at an RS (p^*, w^*) with $p_1^* > 0$ and $w^* > 0$, $p^* b = \Pi^\nu (p^*, w^*)$ holds for any $\nu \in \mathcal{W}$, and $Lx^b = 1$. Therefore, since $\max_{f^\nu \in \mathbb{R}^n_+ :\ p^* f^\nu = \Pi^\nu (p^*, w^*)} Lx^{f^\nu} \geqq Lx^b = 1$ for any $\nu \in \mathcal{W}$, no agent in $\mathcal{C}^4$ is exploited. This means the violation of the CECP.

Given this negative result, Roemer (1982, chapter 5) rightly argues that this is the most serious criticism of Okishio–Morishima's definition of exploitation, rather than the failure of the basic Marxian perception of the capitalist economy as an exploitative system. This is because, taking the CECP as a guiding postulate even though it is a theorem, "we learn something about what the formal model must look like" (Roemer, 1982, p. 152).

Roemer (1982, chapter 5) also provides an alternative to Okishio–Morishima's definition. For any price system $(p, w) \in \mathbb{R}^{n+1}_+$ and any $c \in \mathbb{R}^n_+$, let $\phi\,(c; p, w) \equiv \left\{ \alpha \in \arg\max_{\alpha' \in P} \frac{p\overline{\alpha}' - w\alpha'_l}{p\underline{\alpha}'} \mid \widehat{\alpha} \geqq c \right\}$. Then,

Definition 10: For a capitalist economy, $\mathcal{E} = \left\langle \mathcal{N}; P; (\psi^\nu, 1, \omega^\nu)_{\nu \in \mathcal{N}} \right\rangle$, let $\left((p, w)\,; ((c^\nu, 1)\,; \alpha^\nu)_{\nu \in \mathcal{N}} \right)$ be an RS. Then, for each $\nu \in \mathcal{N}$:
(i) ν is *exploited* if and only if $1 > \max_{f^\nu \in \mathbb{R}^n_+ :\ p f^\nu = \Pi^\nu (p,w)} \min_{\alpha \in \phi(f^\nu; p, w)} \alpha_l$;
(ii) ν is an *exploiter* if and only if $1 < \min_{f^\nu \in \mathbb{R}^n_+ :\ p f^\nu = \Pi^\nu (p,w)} \min_{\alpha \in \phi(f^\nu; p, w)} \alpha_l$.

Unlike Definitions 2, 4, and 9, Definition 10 formulates the labor value of a commodity vector as a function of equilibrium prices. Therefore, it is not faithful to the labor theory of value. Roemer (1982, chapter 5) claims that given Definition 10, the CECP holds in the general economy, and relying on this claim, he also criticizes the labor theory of value, by arguing that to preserve the CECP, labor values must be properly defined as "logically posterior" to equilibrium prices.

[10] Although the economy in Example 1 violates condition (2) of the RFMT, this is not essential for deriving a negative result. Indeed, as Yoshihara (2010, Corollary 1) explicitly shows, we can develop essentially the same proof, even for an economy without an inferior process.

Unfortunately, his claim is wrong in that, given Definition 10, the CECP still fails in an economy with $P_{(A,B,L)}$. This point again can be verified by the economy in Example 1.[11] At an RS (p^*, w^*) with $p_1^* > 0$ and $w^* > 0$, it is verified that $\phi(b; p^*, w^*) = \{\lambda\alpha^* \mid \lambda \geqq 1\}$ for $\alpha^* \equiv (-L\mathbf{e}_2, -A\mathbf{e}_2, B\mathbf{e}_2)$ where $\mathbf{e}_2 \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Therefore, $\min_{\alpha\in\phi(b;p^*,w^*)} \alpha_l = 1 \leqq \max_{f^\nu\in\mathbb{R}^n_+:\ p^* f^\nu = w^*} \min_{\alpha\in\phi(f^\nu;p^*,w^*)} \alpha_l$, which implies that no agent in $\mathcal{C}^4$ is exploited in terms of Definition 10. Thus, although Roemer's (1982) view about the epistemological role of the CECP is correct, his proposal about the proper definition of UE exploitation cannot be validated.

3 The property relation definition of exploitation by Roemer (1982)

Roemer (1994) argues that UE exploitation should be replaced with exploitation as the distributional consequences of an unjust inequality in the distribution of productive assets and resources. What constitutes unjust inequality in capitalist societies? Roemer (1994) argues that this is the unequal distribution of alienable assets.[12]

Based on this view, Roemer (1994) proposes the *property relation definition of exploitation* (*PR exploitation*). Namely, *a group or individual* (*capitalistically*) *exploits another group or individual* if and only if the following three conditions hold: (i) were the latter to withdraw from society, endowed with his/her per capita share of social alienable goods and own labor skill, then his/her welfare would improve compared with under the present allocation; (ii) were the former to withdraw under the same conditions, then his/her welfare would worsen compared with under the present allocation; and (iii) were the latter to withdraw from society, endowed with his/her own endowments, then the former would be worse off than at present.

Such a definition can be formulated within the framework of cooperative game theory. Let $(V^1, \ldots, V^N) \in \mathbb{R}^N_+$ be a profile of each agent's welfare level in society. Let $P(\mathcal{N})$ be the power set of $\mathcal{N}$ and let $K : P(\mathcal{N}) \to \mathbb{R}_+$ be a *characteristic function* of society, which assigns to every coalition $\mathcal{S} \subseteq \mathcal{N}$, with S agents, an aggregate payoff, $K(\mathcal{S})$, if it withdraws from the economy.

The types of features that characteristic function K would have to include as a welfare allocation rule of the alternative society depends on the nature of the alternative society. For instance, in a capitalist society, function K would be defined in terms of the welfare allocation implementable from the equal distribution of alienable assets. Consider an economy $\mathcal{E} = \langle \mathcal{N}; P_{(A,L)}; (u, 1, \omega^\nu)_{\nu\in\mathcal{N}} \rangle$, where u is the common welfare function of all agents. Define a feasible allocation for this economy as a profile $\left((c^\nu, l^\nu)_{\nu\in\mathcal{N}}, x\right) \in C^N \times \mathbb{R}^n_+$ satisfying (i)

[11] Again, the violation of condition (2) of the RFMT in the economy in Example 1 is not essential for this negative result. For this point, see Yoshihara (2010, Corollary 2).

[12] Alienable assets are typically financial assets and/or material capital goods. By contrast, inalienable assets are typically talents and/or skills immanent in individuals.

$Ax \leqq \omega$; (ii) $Lx = \sum_{\nu \in \mathcal{N}} l^{\nu}$; and (iii) $(I - A)\, x \geqq \sum_{\nu \in \mathcal{N}} c^{\nu}$. If a feasible allocation $\left(\left(c^{*\nu}, l^{*\nu} \right)_{\nu \in \mathcal{N}}, x^{*} \right)$ is implemented as an RS for the capitalist economy $\mathcal{E}$, then its corresponding welfare allocation is denoted by $(V^{*1}, \ldots, V^{*N})$, where $V^{*\nu} \equiv u\left(c^{*\nu}, l^{*\nu} \right)$ for each $\nu \in \mathcal{N}$.

Denote the welfare allocation rule of an alternative society by $K^{CE} : P(\mathcal{N}) \to \mathbb{R}_{+}$. For each coalition, $\mathcal{S} \subseteq \mathcal{N}$, then consider the following optimization program (CE):

$$\max_{\left(\left(c^{\nu}, l^{\nu} \right)_{\nu \in \mathcal{S}}, x \right)} \sum_{\nu \in \mathcal{S}} u\left(c^{\nu}, l^{\nu} \right)$$

$$\text{s.t. } (I - A)\, x \geqq \sum_{\nu \in \mathcal{S}} c^{\nu};\ Lx = \sum_{\nu \in \mathcal{S}} l^{\nu} \leqq S;\ \&\ Ax \leqq \frac{S}{N} \omega. \qquad (CE)$$

Denote the solution of program (CE) by $\left(\left(c^{**\nu}, l^{**\nu} \right)_{\nu \in \mathcal{S}}, x^{\mathcal{S}} \right)$. Then, the characteristic function, K^{CE}, is defined by $K^{CE}(\mathcal{S}) \equiv \sum_{\nu \in \mathcal{S}} u\left(c^{**\nu}, l^{**\nu} \right)$ for each $\mathcal{S} \subseteq \mathcal{N}$.

The program (CE) presumes a counterfactual situation in which group $\mathcal{S}$ withdraws from the capitalist society to form a *commune* comprising the members of this group, and then investigates the expected sum of the welfare levels achievable in that alternative society. That is, the program maximizes the aggregate of the welfare levels attainable by group $\mathcal{S}$ endowed with its accessible aggregate capital stock, $\frac{S}{N}\omega$. Here, $\frac{S}{N}\omega$ is the sum of the capital stocks of all members in $\mathcal{S}$ derived from the counterfactual equal distribution of the overall material means of production, ω. The solution to this program constitutes the value $K^{CE}(\mathcal{S})$ as the total payoff attainable by group $\mathcal{S}$ if it forms a communal society by withdrawing from the present society. Following Roemer (1982), the **property-relation exploitation of a capitalist society** (**capitalist PR exploitation**) is defined by means of this K^{CE}, as follows:

Definition 11 (Roemer, 1982): At a welfare allocation $(V^{*1}, \ldots, V^{*N})$ of a capitalist economy, $\left\langle \mathcal{N}; P_{(A,L)}; (u, 1, \omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$, *coalition* $\mathcal{S} \subseteq \mathcal{N}$ *is capitalistically exploited (resp. capitalistically exploiting)* if and only if the complement $\mathcal{T} \equiv \mathcal{N} \backslash \mathcal{S}$ is in a relation of dominance to $\mathcal{S}$, and the following two conditions hold:
(i) $\sum_{\nu \in \mathcal{S}} V^{*\nu} < K^{CE}(\mathcal{S})$ (resp. $\sum_{\nu \in \mathcal{S}} V^{*\nu} > K^{CE}(\mathcal{S})$);
(ii) $\sum_{\nu \in \mathcal{T}} V^{*\nu} > K^{CE}(\mathcal{T})$ (resp. $\sum_{\nu \in \mathcal{T}} V^{*\nu} < K^{CE}(\mathcal{T})$).

That is, condition (i) of Definition 11 states that a capitalistically exploited coalition is worse off in terms of its attainable payoff in the capitalist society than in the communal society endowed with an equal distribution of material means of production. Moreover, condition (ii) of Definition 11 states that the complement of the capitalistically exploited coalition would be better off in terms of its attainable payoff in the capitalist society than in the communal society of this complement. It would be expected that a capitalistically exploiting coalition would exist within this complement. In addition to the definition given in Roemer (1982), Roemer (1994) introduces a third condition: (iii) the aggregate

welfare of group $\mathcal{T}$ would be worse off if group $\mathcal{S}$ withdraws, taking $\omega^{\mathcal{S}} \equiv \sum_{\nu \in \mathcal{S}} \omega^{\nu}$ with it from the capitalist society.[13] This condition would naturally follow whenever the welfare allocation $(V^{*1}, \ldots, V^{*N})$ is derived from the RS in our setting of the Leontief capitalist economy.

A non-exploitative society in terms of Definition 11 can be formulated as a society without an unequal distribution of material capital goods, as confirmed by the following definition.

Definition 12 (Roemer, 1982)]: For any Leontief production economy, $\left\langle \mathcal{N}; P_{(A,L)}; (u, 1, \omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$, a *welfare allocation* $(V^{*1}, \ldots, V^{*N})$ *lies in a communal core* if and only if any coalition $\mathcal{S} \subseteq \mathcal{N}$ is not capitalistically exploited by the allocation.

Definition 12 implies that the core property of a communal society is equivalent to the non-existence of capitalist exploitation in terms of Definition 11.

What types of feasible allocations can a communal core contain? The welfare allocation lies in the communal core if (i) it is generated from the situation in which all individuals in $\mathcal{N}$ constitute a communal society, (ii) all individuals engage in a cooperative production activity using the overall set of material capital goods, ω, and (iii) all individuals share the reward of the activity equally. Such an allocation is a non-exploitative allocation in terms of Definition 11.[14]

Unlike the traditional Marxian theory of exploitation, the capitalist PR exploitation formulated in Definition 11 never refers to UE. Rather, it straightforwardly refers to the unequal distribution of material means of production as the basic feature of exploitation in the capitalist economy. However, Definition 11 extends Okishio–Morishima's definition of UE exploitation, as pointed out by Roemer (1982). Indeed, given the RS $((p^*, w^*), x^*)$ in the capitalist economy $\left\langle \mathcal{N}; P_{(A,L)}; (u, 1, \omega^{\nu})_{\nu \in \mathcal{N}} \right\rangle$, if any worker, $\nu \in \mathcal{W}$, is identified as an exploited agent by Okishio–Morishima's definition of exploitation, then he/she would be a member of an exploited coalition in terms of Definition 11. Furthermore, Definition 11 allows us to identify all exploited agents beyond the members of $\mathcal{W}$ as well as all members of the exploiters. Henceforth, the PR theory of exploitation provides a finer definition of exploitation than do UE theories. In summary,

[13] The condition that $\mathcal{T}$ is in a relation of dominance to $\mathcal{S}$ in Definition 11 is not formally specified by Roemer (1982). This condition is, firstly, to ensure the existence of economic interactions between $\mathcal{T}$ and $\mathcal{S}$. Secondly, it is to eliminate certain perverse cases such as an invalid supported with costly medication by the rest of society, who would be worse off and the rest of society better off after their respective withdrawal according to K^{CE}. Indeed, because of this dominance condition, the relationship between the invalid and the rest of society is not exploitative even though Definition 11(i) and Definition 11(ii) are satisfied: they are not in a relation of dominance (see Roemer (1994, p.21, footnote 4)).

However, the condition of dominance *alone* may be insufficient to define exploitation. For instance, as discussed in section 4.9.2, we may say that the European settlers and indigenous people in North America were in a relation of dominance, but they were not in an exploitative relation even if Definition 11(i) and Definition 11(ii) were satisfied. In this respect, the third condition (iii) by Roemer (1994) requests that in order to identify capitalist PR exploitation, $\mathcal{T}$ depends on $\mathcal{S}$ in the situation for its fortune to flourish. By adding this condition, the relationship between the European settlers and indigenous people was not PR exploitative.

[14] For a more detailed discussion, see Veneziani and Yoshihara (2015b, section 4.4).

whenever we are interested in exploitation as a feature of social relations, Roemer (1994) concludes that we should discuss it based on the PR definition rather than the UE definition of exploitation.

Given this alternative definition, Roemer (1994) questions whether the issue of exploitation is an intrinsic normative problem worth discussing in the context of contemporary societies. He argues that exploitation *per se* is at best a morally secondary phenomenon. Instead, he believes that the normatively primary concern that we should be addressing is the injustice of property relations. For instance, according to Definition 11, capitalist PR exploitation exists in the RS $((p^*, w^*), x^*)$ of the economy $\langle \mathcal{N}; P_{(A,L)}; (u, 1, \omega^\nu)_{\nu \in \mathcal{N}} \rangle$, whenever alienable capital goods are unequally distributed.[15] However, although inequality in the distribution of alienable resources could be conceived of as unjust when all agents are homogeneous in their welfare functions and skills, the issue is less straightforward when these functions and skills are heterogeneous and diverse. Given that the heterogeneity and diversity of agents are generic features of contemporary societies, it seems to be necessary for us to develop a more comprehensive theory of distributive justice, which should be the normatively primary concern in contemporary societies, rather than the development of exploitation theory.

Therefore, what types of theories of distributive justice should be addressed? As a solution, Roemer (1994, 1998) develops a *theory of equality of opportunity*, based on the debates on equality by Dworkin (2000), Arneson (1989), and Cohen (1989). His theory can be summarized by the following axiom:

Principle of voluntary disadvantage: The distribution of alienable resources between any agents, $\nu \in \mathcal{N}$ and $\nu' \in \mathcal{N}$, is *just* if and only if any difference in ν's and ν''s enjoyment of the resources reflects a difference in their choices, desserts, or faults.

Any inequality violating this principle implies *involuntary disadvantage*, which should be deemed to be distributive injustice.

Note that involuntary disadvantage implies disadvantages due to circumstantial factors for which individuals should not be deemed to be responsible, such as those due to household environments, native talents, disaster, and so on. It is reasonable to regard an agent's disadvantage in the private ownership of material capital goods as involuntary, at least in his/her initial stage of economic activities. For instance, in the above-mentioned capitalist economy $\langle \mathcal{N}; P_{(A,L)}; (u, 1, \omega^\nu)_{\nu \in \mathcal{N}} \rangle$, there is supposedly no difference in agents' native talents, and the possibility of disaster is not considered. Therefore, the inequality in the private ownership of material capital goods is the sole source of involuntary disadvantages in this economy. In this respect, an equilibrium allocation in the economy $\langle \mathcal{N}; P_{(A,L)}; (u, 1, \omega^\nu)_{\nu \in \mathcal{N}} \rangle$ implies involuntary disadvantages if and only if it entails capitalist PR exploitation in terms of Definition 11.

In summary, given the above arguments, the existence of exploitation à la Roemer's theory of PR exploitation is equivalent to distributive injustice à la

[15] The additional condition (iii) of capitalist PR exploitation is met under the RS, as mentioned above.

Roemer's theory of equality of opportunity, at least in any Leontief capitalist economy with no heterogeneity or diversity of agents. Hence, in such homogeneous societies, it is sufficient to argue distributive injustice in terms of the theory of equality of opportunity. Moreover, the theory of equality of opportunity can diagnose allocations of alienable resources as unjust, even in societies with heterogeneity and/or diversity among agents. Therefore, the issue of exploitation can be replaced with, or be reduced to, the issue of distributive injustice because of the theory of equality of opportunity. It is sufficient that we diagnose societies using the theory of equal opportunity, which is the main message derived from Roemer's PR theory of exploitation in conjunction with the theory of equality of opportunity.

4 Recent trends of exploitation theory in political philosophy and sociology

Roemer's PR theory of exploitation was so influential that, while the Marxian theory of exploitation was almost dismissed in economics, given the absence of substantial studies in this field since Roemer (1994), many counterarguments were developed, particularly in political philosophy and sociology. Among these, this section reviews Cohen's (1995) criticism of PR exploitation and the recent works of Vrousalis (2013) and Wright (2000) on reviving UE exploitation theory.

4.1 Cohen's (1995) criticism of PR exploitation

Cohen (1995) criticizes Roemer's (1994) claim that normatively fundamental injustice is the maldistribution of assets rather than their unreciprocated flow (i.e., the UE transfer of products). Let us consider agents ν and μ, who are equal in talent and external assets but who have different preferences for income and leisure in that ν is an idler and μ is a workaholic. Then, ν let μ work on ν's means of production after μ has finished working on his own. As a consequence, a part of μ's product derived from her working on ν's means of production goes to ν. In this case, there is nothing unjust since there is no unjust extraction.

Based on this argument, Roemer concludes that the UE transfer *per se* is not unjust and that unequal asset distribution is normatively fundamental injustice. By contrast, according to Cohen, the injustice of such a distribution is *normatively derivative*. Indeed, Cohen (1995) argues that an unreciprocal transfer of products is unjust if and only if it occurs for the wrong reason. The transfer is unjust when it is caused not by *different preferences* but by an *unequal asset endowment*. Moreover, an unequal asset endowment is unjust because of its tendency to induce an unjust product flow. Therefore, although it is causally primary in the explanation of the possibility and occurrence of unjust transfers, it remains normatively secondary unjust. Based on this argument, the UE transfer in the above example is simply not unjustly exploitative.[16]

[16] Consider also the case that ν and μ have dissimilar external assets but the same preference

Note that Cohen (1995) also thinks that taking the UE transfer itself as coherently unjust and exploitative causes difficulties. The position to regard the UE transfer as such implicitly affirms at least a strongly qualified version of the self-ownership principle.[17] Such a position cannot criticize *cleanly generated capitalism*,[18] since such a brand of capitalism was generated without violating the self-ownership principle.

By contrast, based on Cohen's (1995) argument, cleanly generated capitalism can be criticized: even in such a capitalist economy, the worker μ is exploited by the capitalist ν, since ν gets some of what μ produces (for no return) by virtue of the differential ownership of means of production. Further, since such asset inequality causes ν to get some of what μ produces, it is unjust that ν would get it. Moreover, the unequal distribution of means of production is unjust because it tends to cause such an unjustly unreciprocal transfer.

4.2 A conceptual definition of Vrousalis (2013) in political philosophy

Vrousalis (2013) offers the following argument for the conceptual definition of exploitation in capitalist economies:

Definition 13 (Vrousalis, 2013): Agent ν *economically exploits* agent μ if and only if ν and μ are embedded in a systematic relationship in which, (a) ν *instrumentalizes* μ's *economic vulnerability to* ν in order to (b) *appropriate (the fruits of)* μ*'s labor*.

To clarify this definition, we examine each concept in Definition 12 individually.[19]

First, the *instrumentalization* of a subject implies that the subject is being used as a means to an end. Note that, according to Vrousalis (2013), neither unfairness nor the intentionality of instrumentalization is necessary for the definition of exploitation. As we will see, Vrousalis (2013) provides examples of the "non-unfair" utilization of others' attributes, which is still deemed to be

for income and leisure. Because of these different endowments, suppose that ν must get less product than μ for the same labor input if each of them respectively chooses to work autarchically. Thus, an inequality in income and leisure is derived from unequal asset endowments. Cohen (1995) acknowledges that this situation is not exploitative, although it does represent the injustice generated by unequal asset endowments.

[17] That is, a person should be sovereign with respect to what he/she will do with his/her energies.

[18] That is, a form of capitalism in which a capital-lacking worker is on one side and a capital-endowed capitalist is on the other, that does not arise from "primitive accumulation" through massacre, plunder, forced extraction, or, more generally, by transgressing some norm of distributive justice. Rather, it arises from "clean" social interactions: a laborer, starting from equality of external assets, manages to accumulate significant quantities of capital through toil and savings, thereby turning him/herself into a capitalist.

[19] Here, we mainly summarize Vrousalis's own account without necessarily endorsing it, although some of his claims may need more careful discussion based on economic theory, as mentioned in footnote 23.

exploitative. Vrousalis (2013) also discusses that one can unintentionally or unknowingly instrumentalize another's vulnerability and thereby exploit that person.

Second, before discussing the notion of *economic vulnerability*, let us mention that Vrousalis (2013) describes two types of vulnerability: absolute and relational. An agent suffers *absolute vulnerability* when he/she is at substantial risk of a significant loss in the relevant metric (welfare, resources, capabilities, etc.). The absence of absolute vulnerability is guaranteed by security, which implies such losses will not occur. However, absolute vulnerability does not refer to an agent's power over another person. By contrast, the notion of *relational vulnerability* is defined as follows: μ *is relationally vulnerable to* ν if ν has some sort of power over μ in that (i) μ lacks something that he/she wants/needs, F, that is a requirement for μ to flourish; (ii) μ can only obtain F from ν; and (iii) ν has it within his/her discretion to withhold F from μ.[20]

Now, the notion of *economic vulnerability* is defined as follows: μ *is economically vulnerable to* ν if and only if μ is relationally vulnerable to ν by virtue of μ's position relative to ν in the relations of production. Here, it refers to the systematic relations of effective ownership in that ν's ownership of a means of production and μ's lack thereof (or, μ's ownership is substantially less than ν's), as a result of which ν has *economic power over* μ in the sense that ν has the relevant ability and opportunity to get μ to do something by virtue of his/her control over a greater share of resources than μ.

In summary, if ν instrumentalizes μ's economic vulnerability to ν, then in doing so, ν takes advantage of his/her economic power over μ. Under capitalism, if μ has no means of production but ν does, or μ owns substantially less than ν, then μ is economically vulnerable to ν. In other words, ν is given economic power over μ and can get μ to supply his/her labor power to ν. For instance, assuming an equal distribution of internal resources,[21] the wealth owned by capitalists (or agent ν) systematically gives them a decisive bargaining advantage over workers (or agent μ), which means capitalists take advantage of their economic power over workers, but never the other way around.[22][23]

Finally, condition (b) of Definition 13 needs clarification: ν appropriates μ's labor when μ toils for H hours, and ν appropriates a use value of $H - G$ hours

[20] Vrousalis (2013) does not consider condition (iii) of relational vulnerability to be a necessary condition for exploitation, since there is nothing contradictory in the thought that ν is forced to exploit μ and therefore lacks the said discretion.

[21] Internal resources imply talents and/or skills inherent in individuals. By contrast, any other types of resources that are transferrable are often called external resources. For a more detailed argument on these concepts, see Cohen (1995).

[22] Therefore, economic vulnerability in the definition of economic exploitation refers to the relations of production that must be *unilateral* in nature as Vrousalis (2013, p. 137) states: "there can be no "reciprocal" economic power-over: if Bill Gates and Warren Buffett own approximately the same amount of wealth, then neither power-overs the other economically." Note that the general notion of relational vulnerability also allows the case of two parties that are *mutually* relationally vulnerable to each other in that one party's resources are a necessity of the other and vice versa.

[23] To logically ensure this claim, we would need to develop a more detailed, step-by-step argument, based upon economic theory, which is beyond the scope of this paper.

of toil, where G can be any number satisfying $H > G \geqq 0$.[24]

In Definition 13, UE, which is represented by condition (b), is simply a necessary condition for economic exploitation, since conditions (a) and (b) of Definition 13 together constitute economic exploitation. For instance, gift-giving implies UE, but no one thinks of (even systematic) gift-giving as exploitative. If one party freely decides to pass on a large part of whatever use value he/she creates (with his/her own labor power) to another party of society, the resulting inequality in the consumption of (surplus) labor need not be objectionable.

4.3 A conceptual definition of exploitation by Wright (2000) in sociology

Wright (2000) defines exploitation as follows:

Definition 14 (Wright, 2000): Exploitation exists if the following three criteria are satisfied:
(1) *The inverse interdependent welfare principle*: The material welfare of exploiters causally depends upon the reduction of the material welfare of the exploited;
(2) *The exclusion principle:* This inverse interdependence of the welfare of exploiters and the exploited depends upon the exclusion of the exploited from access to certain productive resources; and
(3) *The appropriation principle:* The exclusion generates a material advantage to exploiters because it enables them to appropriate the labor effort of the exploited.

In a market economy, both parties to an exchange gain relative to their condition before making the exchange: both workers and capitalists gain when an exchange of labor power for a wage occurs. While such mutual gains from trade can occur, the magnitude of the gain by one party may still be at the expense of another party.[25] Thus, criterion (1) should be satisfied and, according to

[24] Note that although the notion of economic exploitation only refers to the extraction of labor, any other form of extraction from the exploited can be argued in the general notion of exploitation, such as the case of sexual exploitation, which is also discussed by Vrousalis (2013).

[25] For this point, Wright (2000, pp. 1566–1567) explains as follows: "Let us examine the three criteria for exploitation specified above in a capitalist economy with perfect competition in which there are only two categories of economic actors: capitalists who own the means of production—and thus have the effective power to exclude others from access to those assets—and workers who own only their labor power. .... Are the inverse interdependent welfare principle and the exclusion principle satisfied in this case? Is the material welfare of capitalists causally dependent upon the exclusion of workers from access to capital assets? The test here is whether or not it is the case that workers would be better off and capitalists worse off if property rights were redistributed so that workers would no longer be "excluded" from capital. It seems hard to argue that this is not the case: in the initial condition capitalists have a choice of either consuming their capital or investing it, as well as the choice of whether or not they will work for earnings. Workers only have the latter choice. To be sure, they can borrow capital (and in a world of perfect information they would not need collateral to do so

Wright (2000), we should not assume that market exchanges do not satisfy (1) because of mutual gains from trade.

Wright (2000) argues that exploitation is the process through which certain inequalities in income are generated by inequalities in rights and powers over productive resources. Such inequalities in income occur by the ways in which exploiters, by virtue of their exclusionary rights and powers over productive resources, are able to appropriate the labor effort of the exploited.

Before closing this subsection, it is worth noting that Definition 14 is insufficient as a definition of exploitation, and nor is it as elaborate a conceptual configuration as Definition 13. Definition 14 simply lists the indispensable principles of exploitation as its essential features, although the three principles are intuitively appealing and well acknowledged. Moreover, it is easy to check that Definition 13 satisfies all three principles in Definition 14. Indeed, the appropriation principle is obviously satisfied, and the exclusion principle is satisfied by the definition of economic vulnerability. Finally, Definition 13 also satisfies the inverse interdependence welfare principle as long as the fruit of labor is defined as a use value contributing to human welfare.

4.4 Relations of exploitation with economic oppression and distributive injustice

This subsection examines the logical relation of exploitation to similar notions of economic oppression and/or distributive injustice using the conceptual definition of exploitation developed by Vrousalis (2013) and Wright (2000).

4.4.1 Exploitation and distributive injustice

Based on the notion of economic exploitation in Definition 13, Roemer's claim that the issue of exploitation can be reduced to that of distributive injustice is not valid. To argue this point, Vrousalis (2013) applies the notion of cleanly generated capitalism defined in section 4.1 and provides us with the following example:

Example of Grasshopper and Ant: Grasshopper spends the summer months singing, whereas Ant spends all her time working. When the winter comes, Grasshopper needs shelter, which he presently lacks. Ant has three options:
(i) She can do nothing to help Grasshopper, in which case, the corresponding payoff allocation, (V^{*An}, V^{*Gh}) is $(V^{*An}, V^{*Gh}) = (10, 1)$;
(ii) She can offer Grasshopper her own shelter on the condition that he signs a sweatshop contract to pay the rent, in which case, $(V^{*An}, V^{*Gh}) = (12, 2)$; and
(iii) She can offer Grasshopper her own shelter rent-free, where the cost of maintenance is equal to -1, then $(V^{*An}, V^{*Gh}) = (9, 3)$.

since there would be no transaction costs, no monitoring costs, no possibility of opportunism), but still workers would be better off owning capital outright than having to borrow it."

Now, it is plausible to think that Ant has an obligation to help Grasshopper. However, one need not have a view on this to believe that (ii) is morally worse than (iii), in part because the choice of (ii) constitutes exploitation. Indeed, according to the Roemerian principle of voluntary disadvantages discussed in the last section, (i), (ii), and (iii) are equally acceptable. This fact implies that even if it is agreed that option (ii) involves exploitation, it cannot be condemned as distributive injustice by means of Roemer's theory of equality of opportunity.

The above argument suggests that Roemer's claim that exploitation implies distributive injustice cannot be validated as long as Definition 13 is presumed. The reason why exploitation survives in the absence of distributive injustice is that, according to Definition 13, the notion of exploitation aims to diagnose the structure of an economic transaction involving an asymmetric power relation that systematically generates an unequal exchange of labor. In other words, exploitation constitutes a procedural injury to status, which is not reducible to distributive injury.

4.4.2 Exploitation and non-exploitative economic oppression

Exploitation is nothing but a category of *economic oppression*. Generally speaking, economic oppression could be conceived of as social relations satisfying the inverse interdependence welfare principle and the exclusion principle in Definition 14. According to Wright (2000), various forms of economic oppression can be categorized into the following two notions: exploitation and *non-exploitative economic oppression*.

In non-exploitative oppression, the advantaged group does not itself need the excluded group. Although the welfare of the advantaged does depend on the exclusion principle, there is no ongoing interdependence between their activities and those of the disadvantaged. However, in exploitation, exploiters depend upon the effort of the exploited for their own welfare. Hence, exploiters depend upon and need the exploited.

We can find a sharp contrast between these two notions by considering the difference in the treatment of indigenous people in North America (non-exploitative economic oppression) and South Africa (exploitation) by European settlers. First, in both cases, we can find a causal relationship between the material advantage to the settlers and the material disadvantage to the indigenous people. This fact implies that both cases satisfy the inverse interdependence welfare principle. Second, in both cases, this causal relation is rooted in processes by which indigenous people were excluded from a crucial productive resource, namely land. Hence, both cases satisfy the exclusion principle.

However, in South Africa, the settlers appropriated the fruits of labor of the indigenous population, first as agricultural labor and later as mine workers. This finding implies that the relation between the settlers and indigenous people in South Africa was exploitative.

By contrast, in North America, the labor effort of the indigenous people was generally not appropriated. The indigenous people were simply excluded from the capitalistic economic activities developed by the settlers. This finding

implies that the settlers in North America could adopt a strategy of genocide in response to the conflict generated by this exclusion, because they did not need the labor effort of Native Americans. Thus, the relation between the settlers and indigenous people in North America is an example of non-exploitative economic oppression.

5 Recent developments of exploitation theory in economics: an axiomatic approach

According to Cohen (1995), Vrousalis (2013), and Wright (2000), exploitation should be conceptualized as the systematic structure of economic transactions, in which some of the fruits of the labor of the exploited agents is appropriated by the exploiters under the institutional framework of asymmetric power relations resulting from private ownership. Thus, while the UE theory of exploitation is conceptually sophisticated and well motivated by these works, the issue of proper formal definitions of UE exploitation has yet remained unresolved, as we saw in section 2.

Note that if a definition of UE exploitation is appropriate, it should point out the existence of a transfer mechanism by which UE is mediated: UE occurs by a mechanism that transfers (a part of) the productive fruits of the exploited to the exploiter. In perfectly competitive markets, neglecting the issue of rent, net outputs are distributed into wage income and profit income. Moreover, every party receives an equal wage per unit of (effective) labor. Therefore, the appropriation of more of the productive fruits by exploiters must be explained as a source of income other than wages, that is, profits. In other words, a valid formal definition of UE exploitation should be able to verify the correspondence between UE and profits.

Summarizing the above argument leads to the following logical implication as our desideratum:

(a) The formal definition of UE exploitation is valid $\Rightarrow$ (b) in any economic equilibrium, the generation of positive profits must imply a UE transfer from each propertyless worker and vice versa, according to the presumed definition of exploitation.

Statement (b) is referred to as the **Profit-Exploitation Correspondence Principle (PECP)**.

The PECP looks similar to the FMT, but they are both conceptually and formally different. Conceptually, the FMT, in general, refers to the (average) rate of exploitation (i.e., the rate of surplus value) for the working class as a whole.[26] By contrast, the PECP requires equivalence between the generation

[26] As shown by Yoshihara and Veneziani (2012), in a von Neumann economy with the heterogeneity of propertyless workers' welfare functions, the positivity of the average rate of exploitation may coexist with the non-exploitation of some propertyless workers, simply because of their consumption choices. This fact implies that even if the FMT holds in such

of positive profits and situation in which each propertyless worker is identified as exploited, for any capitalist economy. Formally, the PECP and the FMT are logically independent, as discussed below.

Veneziani and Yoshihara (2015a) axiomatically characterize the definitions of UE exploitation that satisfy the PECP, shedding new light on the debate about the proper definition of UE exploitation. Firstly, they propose a general model of capitalist economies that allows for heterogeneity in each agent's preferences for consumption goods and leisure, heterogeneity in their endowments of material and human capital, and a general closed-convex cone type of production set. Secondly, given such a general model, they axiomatically characterize the formal definitions of UE exploitation in which the PECP is preserved in any equilibrium. As a result, few definitions of exploitation proposed in the literature preserve the PECP, with only the definition à la the New Interpretation (Duménil, 1980; Foley, 1982) being an exception.

5.1 Alternative definitions of exploitation and the domain axiom of admissible definitions of exploitation

Recall that the model of capitalist economies considered in sections 2.1 and 2.2.2 assumes no difference in agents' labor skills or preferences for leisure. In this section, we assume a more general model of a capitalist economy, $\mathcal{E} = \left\langle \mathcal{N}; P_{(A,B,L)}; \left(u^\nu, s^\nu, \omega_0^\nu\right)_{\nu\in\mathcal{N}} \right\rangle$, that includes the heterogeneity of labor skills and preferences for consumption bundles and leisure. Here, we discuss the axiom proposed by Veneziani and Yoshihara (2015a), which represents the minimal necessary condition for admissible definitions of UE exploitation. Then, we introduce alternative definitions of exploitation proposed in the literature on mathematical Marxian economics.

As a preliminary step, given any P, we define the set of production activities feasible with k units of labor inputs by $P\left(\alpha_l = k\right) \equiv \left\{\left(-\alpha_l', -\underline{\alpha}', \overline{\alpha}'\right) \in P \mid \alpha_l' = k\right\}$. Moreover, given $c \in \mathbb{R}_+^n$, we define the set of efficient production activities to produce c as a net output by $\partial\underline{\phi}\left(c\right) \equiv \left\{\alpha \in \phi\left(c\right) \mid \forall \alpha' \in \phi\left(c\right),\ \left(-\alpha_l' > -\alpha_l \Rightarrow \exists i : -\underline{\alpha}_i' \leqq -\underline{\alpha}_i < 0\right)\right\}$.[27]

Any definition of exploitation should be able to identify, associated with each equilibrium allocation, the set of exploiting agents, $\mathcal{N}^{ter} \subseteq \mathcal{N}$, and the set of exploited agents, $\mathcal{N}^{ted} \subseteq \mathcal{N}$, such that $\mathcal{N}^{ter} \cap \mathcal{N}^{ted} = \varnothing$ holds. Moreover, it should capture the feature of UE as the difference between the amount of labor supplied by each agent and the amount of labor "received" through each agent's income. In particular, the supplied labor amount should be greater than the received labor amount for each exploited agent. Such properties should be preserved as a core feature of exploitation regardless of the way in which UE exploitation is measured.

Note that for the capitalist economies considered herein, each agent's supply of labor is identified by Λ^ν. By contrast, how to formulate the labor amount

economies, it may be that some propertyless workers are not exploited.

[27] By this definition, for the frontier of the production possibility set P, $\partial P \equiv \left\{\alpha \in P \mid \nexists \alpha' \in P : \alpha' > \alpha\right\}$, we have $\partial\underline{\phi}\left(c\right) \subseteq \partial P \cap \left\{\alpha \in \phi\left(c\right) \mid \widehat{\alpha} \not> c\right\}$.

that each agent can "receive" through his/her earned income remains open to debate. Based on the forms of "received" labor, a number of possible definitions of exploitation exist.

Summarizing the above arguments, Veneziani and Yoshihara (2015a) propose an axiom that represents the minimal necessary condition for any definition of exploitation, whenever it is deemed to be admissible as the form of UE:

Labor Exploitation (LE) (Veneziani and Yoshihara, 2015a): Given any definition of exploitation, for any capitalist economy $\mathcal{E}$ and any RS $\left((p,w)\,;((c^\nu,l^\nu)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$, the set of exploited agents, $\mathcal{N}^{ted}\subseteq\mathcal{N}$, should have the following property: there exists a profile of commodity bundles, $(c_e^\nu)_{\nu\in\mathcal{W}}\in\mathbb{R}_+^{nW}$, such that, for any $\nu\in\mathcal{W}$, $pc_e^\nu=w\Lambda^\nu$ holds, and for some production point, $\alpha^{c_e^\nu}\in\partial\underline{\phi}\,(c_e^\nu)$:

$$\nu\in\mathcal{N}^{ted}\Leftrightarrow\alpha_l^{c_e^\nu}<\Lambda^\nu.$$

That is, axiom **LE** requires that any admissible definition of UE exploitation must identify whether each propertyless agent is exploited for each RS under any economy. More specifically, the axiom stipulates that the set of propertyless exploited agents be identified as follows: according to each specific admissible definition, there should be a profile, $(c_e^\nu)_{\nu\in\mathcal{W}}$, for each propertyless agent's commodity bundle affordable by that agent's revenue, and its corresponding profile $\left(\alpha^{c_e^\nu}\right)_{\nu\in\mathcal{W}}$ of production activities, where each $\alpha^{c_e^\nu}$ can produce the corresponding commodity bundle c_e^ν as a net output in a technologically efficient way. Then, the exploitation status of each propertyless agent can be identified by comparing his/her amount of labor supply Λ^ν with the amount of labor input $\alpha_l^{c_e^\nu}$ that he/she is able to "receive" through his/her income $w\Lambda^\nu$.

Axiom **LE** is a rather weak condition in that it only refers to the exploitation status of propertyless agents in each RS. This should be reasonable as a minimal necessary condition for the admissible domain. In other words, a definition of exploitation is not necessarily deemed to be proper, even if it satisfies **LE**. In fact, there may be infinitely many definitions of exploitation that satisfy **LE**, and all the main definitions proposed in the mathematical Marxian economics literature satisfy this axiom.[28]

To see the last point, let us consider three main definitions under general economies with possibly heterogeneous agents. First, the following two definitions are respectively natural extensions of Definitions 4 and 10 to economies with possibly heterogeneous agents:

Definition 15 (Morishima, 1974)**:** For any capitalist economy, $\mathcal{E}$, and any $\nu\in\mathcal{W}$, who supplies Λ^ν and purchases $c^\nu\in\mathbb{R}_+^n$, $\nu\in\mathcal{N}^{ted}$ if and only if $\Lambda^\nu>\min_{\alpha\in\phi(c^\nu)}\alpha_l$.

[28] Of course, this does not imply that the axiom **LE** is trivial. For instance, the definition proposed by Matsuo (2008) does not satisfy **LE.**

Definition 16 (Roemer, 1982, chapter 5)**:** For any capitalist economy, $\mathcal{E}$, any RS, $\left((p,w)\,;((c^\nu,l^\nu)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$, and any $\nu\in\mathcal{W}$, who supplies Λ^ν and purchases $c^\nu\in\mathbb{R}^n_+$, $\nu\in\mathcal{N}^{ted}$ if and only if $\Lambda^\nu>\min_{\alpha\in\phi(c^\nu;p,w)}\alpha_l$.

Finally, for any capitalist economy, $\mathcal{E}$, and any RS, $\left((p,w)\,;((c^\nu,l^\nu)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$, let $\alpha^{p,w}\equiv\sum_{\nu\in\mathcal{N}}\alpha^\nu$. Moreover, for any $c\in\mathbb{R}^n_+$, we define a non-negative number, $\tau^c\in\mathbb{R}_+$, as satisfying $\tau^c p\widehat{\alpha}^{p,w}=pc$. Then:

Definition 17 (Veneziani and Yoshihara, 2015a)**:** For any capitalist economy, $\mathcal{E}$, any RS, $\left((p,w)\,;((c^\nu,l^\nu)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$, and any $\nu\in\mathcal{W}$, who supplies Λ^ν and can purchase $c^\nu\in\mathbb{R}^n_+$, $\nu\in\mathcal{N}^{ted}$ if and only if $\Lambda^\nu>\tau^{c^\nu}\alpha_l^{p,w}$.

In Definition 17, for each $\nu\in\mathcal{W}$, τ^{c^ν} represents ν's share of national income, and thus $\tau^{c^\nu}\alpha_l^{p,w}$ is the share of social labor that this agent receives through the wage income sufficient to purchase c^ν. It is conceptually related to the New Interpretation (NI) definition of exploitation à la Duménil (1980) and Foley (1982), which was originally defined in Leontief economies with homogeneous agents. In the NI, *the value of money* is defined by the labor amount per unit of national income,[29] and the wage multiplied by the value of money is the value of labor power, as Foley (1986, p. 43) states: "the amount of average social labor workers receive a claim to in the wage for each hour they actually work—that is, as the average wage multiplied by the value of money." In Definition 7, for each $\nu\in\mathcal{W}$, $\tau^{c^\nu}\alpha_l^{p,w}=w\Lambda^\nu\frac{\alpha_l^{p,w}}{p\widehat{\alpha}^{p,w}}$ holds by $w\Lambda^\nu=pc^\nu$. Since $w\Lambda^\nu$ is ν's wage income and $\frac{\alpha_l^{p,w}}{p\widehat{\alpha}^{p,w}}$ corresponds to the value of money in the NI, $\Lambda^\nu>\tau^{c^\nu}\alpha_l^{p,w}$ means that ν is exploited as "a worker expends more labor hours than he or she receives an equivalent for in wages" (Foley 1986, p.122).

5.2 Profit-Exploitation Correspondence Principle

Now, we are ready to formulate the axiom of Profit-Exploitation Correspondence Principle, given as follows:

Profit-Exploitation Correspondence Principle (PECP) [Veneziani and Yoshihara (2015a)]: For any capitalist economy, $\mathcal{E}$, and any RS, $\left((p,w)\,;((c^\nu,l^\nu)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$:

$$\left[p\widehat{\alpha}^{p,w}-w\alpha_l^{p,w}>0\Leftrightarrow\mathcal{N}^{ted}\supseteq\mathcal{W}_+\right],$$

where $\mathcal{W}_+\equiv\{\nu\in\mathcal{W}\mid\Lambda^\nu>0\}\neq\varnothing$.

That is, whatever the definition of exploitation is, it must follow that for any capitalist economy and any RS, total profits are positive if and only if any propertyless employee is exploited in terms of this definition, assuming the definition of exploitation is deemed appropriate. This is required by **PECP**.

[29] In other words, the ratio of aggregate direct labor time to aggregate money value added.

For the available class of capitalist economies considered here, there is no requirement of a restriction that excludes the existence of fixed capital goods, the possibility of joint production, or of technical changes. In addition, unlike in condition (2) of the RFMT discussed in section 2, there is no restriction that excludes the existence of a dependently produced commodity. Moreover, heterogeneity in agents' preferences and/or skills is also permitted. The equilibrium notion presumed here is also sufficiently general that there is no requirement of a subsistence wage condition. Therefore, the correspondence between profits and exploitation is required for a large class of economies, as assumed by the standard general equilibrium theory.

However, **PECP** *per se* is not so strong. Indeed, it even allows for a situation in which some propertyless employees are exploited in an equilibrium with zero total profit.[30] This finding implies that, at least within the class of economies with homogeneous agents, **PECP** is logically weaker than the statement of the FMT, as within such economies, the latter implies that no propertyless employee is exploited in any equilibrium with zero profit. By contrast, while the FMT implies that the rate of exploitation for the whole working class is positive in any equilibrium with positive total profits, **PECP** requests that every propertyless worker is exploited, which is a stronger claim than that of the FMT.

As noted at the start of this section, if a definition of exploitation satisfying axiom **LE** is proper, it must satisfy **PECP**. Based on this perspective, Veneziani and Yoshihara (2015a) study the necessary and sufficient condition for **PECP**, as stated in the following theorem:[31]

Theorem 1 (Veneziani and Yoshihara, 2015a): For any definition of exploitation satisfying **LE**, the following two statements are equivalent for any capitalist economy, $\mathcal{E}$, and any RS, $\left((p,w);((c^\nu,l^\nu);\alpha^\nu)_{\nu\in\mathcal{N}}\right)$:
(1) **PECP** holds under this definition of exploitation;
(2) If $p\widehat{\alpha}^{p,w}-w\alpha_l^{p,w}>0$, then for any $\nu\in\mathcal{W}_+$, there exists a production activity $\alpha_\pi^\nu\in P\left(\alpha_l=\Lambda^\nu\right)\cap\partial P$ such that $\widehat{\alpha}_\pi^\nu\in\mathbb{R}_+^n$, $p\widehat{\alpha}_\pi^\nu>w\Lambda^\nu$, and $(\alpha_{\pi l}^\nu,\underline{\alpha}_\pi^\nu,\overline{\alpha}_\pi^\nu)\geqq\eta^\nu\left(\alpha_l^{c_e^\nu},\underline{\alpha}^{c_e^\nu},\overline{\alpha}^{c_e^\nu}\right)$ hold for some $\eta^\nu>1$.

That is, condition (2) of Theorem 1 is the necessary and sufficient condition for any definition of exploitation satisfying **LE** to preserve **PECP**. Condition (2) states that if total profits are positive in the present equilibrium, then for each propertyless employee, $\nu\in\mathcal{W}_+$, there exists a suitable efficient production point, α_π^ν, activated by the present amount of labor supply, Λ^ν, which in conjunction with production activity, $\alpha^{c_e^\nu}$, can verify that this agent is being exploited. Recall that, according to axiom **LE**, production activity $\alpha^{c_e^\nu}$ is identified by the presumed definition of exploitation, and the corresponding labor input $\alpha_l^{c_e^\nu}$

[30] However, any definition of exploitation satisfying **LE** does not allow the existence of exploited propertyless employees in conjunction with zero profit.

[31] Note that, though all of the following analyses herein presume economies with a homogeneous type of labor with heterogeneous levels of skills, the completely parallel results can be obtained even if we consider economies with heterogeneous types of labor, as shown in Veneziani and Yoshihara (2014).

represents agent ν's "received" labor. Production activity $\alpha_\pi^\nu \in P\left(\alpha_l = \Lambda^\nu\right) \cap \partial P$ is defined as the proportional expansion of production point $\alpha^{c_e^\nu}$ up to the point of his/her present labor supply, Λ^ν, and that produces a non-negative net output, $\widehat{\alpha}_\pi^\nu \in \mathbb{R}_+^n$, that is non-affordable by ν at the present equilibrium because $p\widehat{\alpha}_\pi^\nu > w\Lambda^\nu$. Therefore, since $\Lambda^\nu = \alpha_{\pi l}^\nu > \alpha_l^{c_e^\nu}$ holds for such a selection of α_π^ν, we can confirm that agent $\nu \in \mathcal{W}_+$ is exploited at this RS, according to the given definition satisfying **LE**.

Theorem 1 does not provide a normative characterization of the presumed definition of exploitation, but rather a demarcation line (condition (2)) by which one can test which of infinitely many potential definitions preserves the essential relation of exploitation and profits in capitalist economies. Thus, if a definition of exploitation satisfying **LE** does not generally meet condition (2), then it will not satisfy **PECP**, which implies that it is not a proper definition of UE exploitation.

Some may criticize the methodological positions of **PECP** and Theorem 1, claiming that **PECP** should be proved as a theorem rather than treated as an axiom. In fact, as Okishio and Morishima did, the methodological standpoint of the FMT was, assuming a specific definition of exploitation, to verify that a capitalist economy can be conceived of as exploitative.

By contrast, Theorem 1 presumes a correspondence between positive profits and exploitation for every propertyless employee as an axiom and then tests the validity of each alternative definition of UE exploitation by checking whether it satisfies this axiom. Such a methodology has been implicitly adopted within debates on the FMT. Typically, whenever a counterexample has been raised against the FMT with a major definition of exploitation by generalizing the model of economies, this criticism has been resolved by proposing an alternative definition and proving that the FMT is held with this alternative form under the generalized economic model. This implicitly suggests that in the overall debate on the FMT, the validity of each form of exploitation has been tested by the robustness of the equivalence between exploitation and positive profits. However, even if such an interpretation is acceptable, the structure of the debate on the FMT could not function as such, because it may involve an infinite repetition of counterexample and alternate proposal. By contrast, by providing an axiomatic characterization such as Theorem 1, the validity of every form of UE exploitation is testable simply by checking condition (2).

Another argument justifies the treatment of **PECP** as an axiom. In any Leontief economy, regardless of whether agents are heterogeneous in preferences and/or skill levels, the equivalence of positive profits and exploitation of each propertyless employee and the equivalence of zero profit and no exploitation are preserved for any definition of exploitation, as long as it satisfies **LE**.

Theorem 2 (Veneziani and Yoshihara, 2015a): For any capitalist economy, $\left\langle \mathcal{N}; P_{(A,L)}; (u^\nu, s^\nu, \omega_0^\nu)_{\nu\in\mathcal{N}} \right\rangle$, and any RS, $\left((p,w); ((c^\nu, l^\nu); \alpha^\nu)_{\nu\in\mathcal{N}}\right)$, **PECP** holds for any definition of exploitation satisfying **LE**.

Proof. Take any definition of exploitation that satisfies **LE**. Then, for any Leon-

tief economy and any RS, (p, w), we can find a profile of reference commodity bundles, $(c_e^\nu)_{\nu\in\mathcal{W}} \in \mathbb{R}_+^{nW}$. Then, regardless of the heterogeneity of welfare functions and skills, the corresponding profile of production activities, $\left(\alpha^{c_e^\nu}\right)_{\nu\in\mathcal{W}}$, is uniquely given by

$$\alpha^{c_e^\nu} \equiv \left(-vc_e^\nu, -A\left(I-A\right)^{-1} c_e^\nu, \left[I + A\left(I-A\right)^{-1}\right] c_e^\nu\right) \text{ for each } \nu \in \mathcal{W}.$$

Thus, $\alpha_l^{c_e^\nu} = vc_e^\nu$. Let $p\widehat{\alpha}^{p,w} - w\alpha_l^{p,w} > 0$ for this RS. This finding implies that, under the Leontief economy

$$p = (1+\pi)\, pA + wL \text{ for some } \pi > 0.$$

Then, as is well known, $\frac{p}{w} > v$. Thus, by $w\Lambda^\nu = pc_e^\nu$ from **LE**, we have $\Lambda^\nu = \frac{p}{w}c_e^\nu > vc_e^\nu$, for any $\nu \in \mathcal{W}_+$. Therefore, according to **LE**, any propertyless employee is exploited in terms of the presumed definition of exploitation. ■

However, once the production technology of economies is replaced by a more general type such as the von Neumann production technology, some definitions of exploitation violate **PECP**, even if they satisfy **LE**. Does this suggest that the validity of the basic Marxian perception of capitalist economies as exploitative crucially depends on the degree of the complexity of the production technology? Or, does it suggest that such counterexamples are generated because of the *incoherency* of these definitions in that they cannot properly identify the set of exploited agents whenever a more complex production technology is applied? Veneziani and Yoshihara (2015a) take the latter view. For the complexity of production technology such as the existence of fixed capital and of alternative techniques should not be essential for the exploitation status of each agent. Rather, these counterexamples should be viewed as representing the non-validity of the presumed definitions of exploitation.

Theorem 4 does not identify a unique definition that meets **PECP**, but rather a class of definitions that satisfy condition (2). Yet, Veneziani and Yoshihara (2015a, Corollary 1) show that it has surprising implications concerning the main approaches in exploitation theory. There are economies in which, for all $\nu \in \mathcal{W}_+$, condition (2) is never satisfied if $\alpha^{c_e^\nu}$ is given by Definition 15 or 16, and thus **PECP** does not hold. By contrast, Definition 17 satisfies condition (2), and thus **PECP** holds *for all* $\mathcal{E}$ *and all* RS:

Corollary 1 (Veneziani and Yoshihara, 2015a): There exists a capitalist economy, $\mathcal{E}$, and an RS for this economy such that neither Definition 15 nor Definition 16 satisfies **PECP**.

The proof of Corollary 1 is given in section 2.2.2 by using the economy in Example 1. In that economy, assume an RS $(p^*, 1)$ with $p_1^* > 0$. Then, every agent, $\nu \in \mathcal{W}_+$, consumes $c^\nu = b$ and $\min_{\alpha\in\phi(c^\nu)} \alpha_l = \min_{\alpha\in\phi(c^\nu;p^*,1)} \alpha_l = 1 = \Lambda^{*\nu}$, while $\pi^* > 0$. This finding implies that neither Definition 15 nor Definition 16 satisfies **PECP**.

Corollary 2 (Veneziani and Yoshihara, 2015a): For any capitalist economy, $\mathcal{E}$, and any RS, Definition 17 satisfies **PECP**.

These corollaries suggest that, at least among the main competing proposals of exploitation forms, Definition 17 is the sole appropriate form.

The above arguments are sufficient to show that among the main proposals in the literature, the NI one is the only definition of UE exploitation that can be used to measure UE coherently regardless of the complexity of production technology. However, other arguments also support the NI form of UE exploitation. First, following Roemer's (1982) view on the epistemological role of the CECP, Yoshihara (2010) formulates **Class-Exploitation Correspondence Principle** (**CECP**) as an axiom that any proper definition of exploitation should meet and then shows that the NI definition is the unique one satisfying this axiom among the main definitions. This is even more supportive for the NI definition since **CECP** may provide a more comprehensive view of capitalist exploitative relations than **PECP**. Second, Yoshihara and Veneziani (2009) introduce an axiom called **Relational Exploitation (RE)** that requests that an exploiter exists if and only if an exploited agent exists as a minimal condition to capture the social relational feature of exploitation. Then, they show that any definition satisfying **RE**, together with a small number of rather weak axioms, is uniquely the NI definition.

Another interesting argument supports the NI definition. Although Definition 17 formulates exploitation as the unequal exchange of labor, it is also possible to formulate the unequal exchange of any commodity, k, in an analogical way. In this case, is an argument such as the GCET re-established by using such a definition of unequal exchange? The answer is negative, according to Yoshihara and Veneziani (2013).

Let us define exploitative relations as an unequal exchange of commodity k, analogical to Definition 17:

Definition 18 (Yoshihara and Veneziani, 2013): For any capitalist economy, $\mathcal{E}$, and any RS, $\left((p,w)\,;((c^\nu,l^\nu)\,;\alpha^\nu)_{\nu\in\mathcal{N}}\right)$, any agent, $\nu\in\mathcal{N}$, supplies some amount of commodity k, $\omega_k^\nu \geqq 0$, as a factor of production, and consumes $c^\nu \in \mathbb{R}_+^n$. Then, agent ν is k-exploited if and only if $\omega_k^\nu > \tau^{c^\nu}\underline{\alpha}_k^{p,w}$.

Yoshihara and Veneziani (2013) prove that the equivalence between positive profits and existence of k-exploited agents in terms of Definition 18 does not hold. For instance, assuming an economy with a homogeneity of welfare functions and labor skills, consider an RS with zero profit. Then, it follows that, for any $\nu\in\mathcal{N}$, $\tau^{c^\nu}=\frac{1}{N}$. By contrast, whenever the initial endowment of capital good k is unequal, there generically exists an agent, ν', endowed with $\omega_k^{\nu'} > \frac{1}{N}\omega_k$. Then, it is easy to construct an equilibrium with zero profit under which this agent is deemed to be k-exploited, which violates the equivalence of k-exploitation with positive profits in terms of Definition 18.

Summarizing these arguments, if we take the NI definition such as in Definition 17, it follows that the unequal exchange of any productive factor other than

labor and UE are not logically equivalent. Therefore, there can be no room for criticism of this definition by means of an analogical argument of the GCET, unlike the criticism of Okishio–Morishima's definition.

6 Concluding remarks

One of the most prominent contributions of Okishio (1963) is that he inspired the controversy about the proper definitions of exploitation. Although Okishio's definition of exploitation (Definition 2 in this paper) was essentially faithful to the labor theory of value and theory of surplus value, the sequence of later controversies suggests the limitation and noneligibility of such a classical definition. Instead, based on the axiomatic analysis reviewed in section 5, the NI type (Definition 17 in this paper) is deemed to be appropriate as a coherent measure of UE exploitation applicable to a broader class of economies with complex structures of production and the heterogeneity of agents.

Moreover, Definition 17 also satisfies a property of *Minimal objectivism* (Veneziani and Yoshihara, 2011) in that, unlike Definitions 15 and 16, the exploitation status of agents is determined independent of possibly arbitrary consumption decisions. Further, it has a clear empirical content by being firmly anchored to actual economic data: only actual production decisions and the social allocation of labor, income, and production activities matter.

Recent discussions such as Vrousalis (2013) and Wright (2000), through the debate over Roemer's (1982, 1994) PR exploitation theory, conceptualize UE exploitation as UE transfer under the systematic asymmetric power structure due to private ownership, by which UE exploitation is shown to be irreducible to the issue of distributive injustice. Indeed, the issue of UE exploitation refers to the asymmetric structure of production relations among *citizens with capacity*. The UE feature can be criticized from a viewpoint of distributive justice, but it is simply one aspect of injustice involved in exploitative production relations.

Contemporary theories of distributive justice typically refer primarily to the treatment of citizens who suffer disadvantages in access to suitable labor markets due to bad luck, incapacity, disability, or their social background. These citizens might be deemed to be economically oppressed but not exploited, according to Wright's (2000) terminology.

By contrast, the issue of *working poor*, which has even affected regular workers in advanced countries during recent decades, is more relevant to the issue of UE exploitation, since it is not simply a matter of the insufficiency of welfare compensation, but is more related to the power relations between capital and labor and the strength of labor unions. Note that this issue also suggests that addressing a special concern to propertyless employees is still important in UE exploitation theory, as, for instance, more than 30% of current households in Japan lack financial assets.

Overall, UE exploitation theory and theories of distributive justice play mutually complementary roles in diagnosing the present society; one does not dominate the other and nor are they mutually exclusive and substitutable. This find-

ing suggests that in some contexts, the issues of redressing distributive injustice and of improving exploitative working conditions might be traded off, given the scarce budgets of the welfare state. In such a situation, it would be necessary to develop a theory of comprehensive social welfare functions to accommodate the criteria of distributive justice and of UE exploitation, in addition to the standard criterion of economic efficiency.

In this respect, UE exploitation theory has not thus far been sufficiently cultivated; it can only identify the proper measure to diagnose the *existence of UE exploitation*. The next step to develop UE exploitation theory would be to study the severity of UE exploitation in each society. Proceeding with this line of research would require a new subject to identify a *proper measure of the degree of UE exploitation*.

Definition 17 in this paper also suggests that non-exploitative resource allocations should serve as the *proportional solution*, as proposed by Roemer and Silvestre (1993). Although reducing concerns about exploitation to concerns about distributive injustice is not legitimate, it is still an intrinsically interesting problem to study the ethical properties of non-exploitative allocations. With regard to this point, Roemer (2010, 2015) recently shows that the proportional solution, that is, the allocation rule of non-exploitation, would be implementable in a moral state of society in which every citizen behaves in accordance with the Kantian categorical imperative. Such a moral state of society is formulated by Roemer (2010) as a social state of the Kantian equilibrium. This line of research would be interesting for Marxian economists to study further.

Lastly, this paper mainly discussed the generation of UE exploitation in a perfectly competitive economy. However, we have not addressed the persistency of exploitative relations[32] nor the generation of exploitative relations under capitalist economies with imperfect labor contracts.[33] The former problem would be relevant, in a broader sense, to the controversies over the Okishio Theorem (Okishio, 1961), another significant contribution by Nobuo Okishio. We leave this point to future research.

7 References

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[32] For the current standpoint of this subject, refer to Veneziani (2007, 2013) and Veneziani and Yoshihara (2015b; section 4).

[33] For more information on this line of research, refer to Yoshihara (1998) and Skillman (2014).

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