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A Referendum Experiment with Participation Quorums*

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Abstract

This paper analyzes a *yes*/*no* referendum in which the outcome is valid only if the voter turnout is greater than a predetermined level. Such a participation quorum is argued to induce the minority group of voters to abstain strategically. Such abstention is intended to adversely affect the outcome by achieving a low voter turnout. We first construct a game-theoretic model to derive a theoretical prediction about the relationship between quorums and voting outcomes. It is shown that there exist multiple equilibria, and that strategic abstention *can* happen if such a participation quorum is imposed. To examine which type of outcome is more likely to be realized, we then conduct a laboratory experiment. We observe that (*i*) if the quorum is small, all voters go to the polls, and (*ii*) if the quorum is large, voters in the *ex-ante* majority group go to the polls, whereas voters in the *ex-ante* minority group tend to abstain. As a result, it is less likely that the *ex-post* minority group wins the referendum. However, when the quorum is large, it frequently happens that the outcome is made invalid because of low voter turnout.

Keywords: referendum, participation quorum, voter turnout, strategic abstention, laboratory experiment

JEL Classification: D72, C92

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1 Introduction

Imposition of participation quorums has been observed in national referendums of, for example, Italy, Portugal, Romania, Slovenia, and Slovakia (mentioned by Côrte-Real and Pereira, 2004) and local referendums of Japan and the U.S. In most of these referendums, the voter turnout is required to be at least 50% for the outcomes to be valid. The main idea behind such a quorum requirement is statistical; that is, the vote distribution realized in a referendum is a fair sample of the opinion of the whole population only when the voter turnout is sufficiently large. However, this statement is true only when voters behave sincerely. In fact, theoretical works that assume strategic voters in non-cooperative games, such as Aguiar-Conraria and Magalhães (2010b) and Hizen and Shinmyo (2011), show that imposing such a quorum requirement can induce strategic abstention in order to try and spoil the outcome, rather than going to the polls to lose the referendum. Thus, such behaviors may distort the outcome in favor of the minority.[1] Empirical works, such as Murata (2006) and Aguiar-Conraria and Magalhães (2010a), confirm that imposing participation quorums decreases the voter turnout.

Following these theoretical and empirical works, in this study, we conduct an experiment to examine the effect of participation quorums on voting behaviors and outcomes. Laboratory experiments, in which all other factors are controlled, enable us to observe directly how institutional rules work. In particular, our experiment enables us to obtain data regarding not only the 50% participation quorum used in most actual referendums, but also other levels of quorums. Hence, we can analyze not only the effect of the *presence* of participation quorums, but also the relationship between the *level* of quorum and voter turnout.

Our experiment is related to the following two groups of voting experiments that analyze voter turnout and vote coordination, respectively. The literature on voter turnout describes an election as a costly participation game between two groups (see Schram and Sonnemans (2008) for a survey). The effect of voting rules is examined by Schram and Sonnemans (1996). They compare voter turnout in the plurality rule and proportional representation.

The literature on vote coordination analyzes a three-way race among one minority and two majority candidates, and considers the kind of information that helps two split-

[1] Other theoretical approaches include an axiomatic approach (Côrte-Real and Pereira, 2004), a group-based model (Herrera and Mattozzi, 2010), and a non-strategic voter model (Zwart, 2010). Maniquet and Morelli (2011) also assume strategic voters under population uncertainty. All of them obtain negative results for participation quorums. Laruelle and Valenciano (2011, 2012) formally describe various types of voting rules used in parliaments as combinations of majority rules and quorum rules.

majority voters to coordinate with each other against one minority group (see Rietz (2003) for a survey). The effect of voting rules is examined by Gerber, Morton, and Rietz (1998). They compare how often the minority candidate wins in straight voting and cumulative voting.

In this paper, we deal with the voting rule of participation quorums in referendums. We focus on how it affects voter turnout and, as a result, how often the outcome is made invalid or minority voters win. In our experiment, subjects are divided into two groups randomly. Furthermore, the expected number of members is greater for one group (called *ex-ante majority*) than the other (called *ex-ante minority*). Each subject knows her own group, but does not know which group the other subjects belong to.

To prepare for the experiment, we first construct a game-theoretic model of a referendum with participation quorums. Our model yields multiple equilibria, including a full-turnout equilibrium, full-abstention equilibrium, equilibria in which one group goes to the polls whereas the other group abstains, and mixed-strategy equilibria. Our experiment works as a device of equilibrium selection, in that it tells us which type of equilibrium outcome is more likely to be realized for each level of quorum.

We observe that (*i*) if the quorum is small, all voters go to the polls and (*ii*) if the quorum is large, voters in the *ex-ante* majority go to the polls, whereas voters in the *ex-ante* minority tend to abstain. As a result, it is less likely that the *ex-post* minority wins the referendum. However, when the quorum is large, it frequently happens that the voting outcome is made invalid because of low voter turnout. Therefore, when politicians design referendums with participation quorums, the possibility of large quorums inducing strategic abstention must be taken into account.

Recently, in a laboratory experiment, Aguiar-Conraria, Magalhães, and Vanberg (2013) also observe decreases in participation rates for a specific participation quorum. Their experimental design is based on Palfrey and Rosenthal's (1985) incomplete-information game. In this game, each voter receives her own voting cost privately from a uniform distribution and then chooses whether to vote or abstain. In order to focus on the comparison among three different quorum restrictions (that is, participation quorum, approval quorum, and no quorum), they keep the participation quorum fixed at the level where the status-quo outcome is equally likely in the participation quorum and approval quorum. In our experiment, in contrast, each voter is privately given her preference regarding outcomes, whereas voting costs are excluded. We deal only with participation quorums. We impose one of seven levels of the participation quorum in each round, in order to analyze the effects of different levels on voting behaviors and outcomes.

This paper is organized as follows. We present the model in Section 2 and derive the equilibria in Section 3. In Section 4, we describe our experimental design and provide the experimental observations in Section 5. In Section 6, we present our concluding remarks. The Appendix includes the instructions used in our experiment.

2 The Model

In this section, we describe a yes/no referendum with a participation quorum as a static game of incomplete information. Our experimental design is based on this model.

The basic structure of our model is similar to that of Hizen and Shinmyo (2011), but invalid outcomes due to low voter turnout are dealt with differently. They assume that, if the outcome is invalid, alternative *no* is selected; that is, the status quo is *no*. Under this assumption, voters who prefer alternative *yes* do not have an incentive to spoil the outcome by abstaining. Hence, the authors' analysis is focused on whether voters who prefer alternative *no* go to the polls or abstain. In this paper, on the other hand, we assume symmetry between alternatives, but introduce asymmetry only in the expected number of members in the two groups, as explained below.

2.1 Basic Structure

A yes/no referendum is held among m (> 0) voters. At the beginning of the game, a preference, either "*yes*" for or "*no*" against the subject of the referendum, is randomly and independently given to each voter. Specifically, each voter's preference is *yes* with probability $s \in (0, 1)$ and *no* with probability $1 - s$. This preference is private information for each voter; hence, she does not know the realized preferences of the other voters. On the other hand, the number of voters m and the probability s are common knowledge among voters.

Each voter has one vote. After their preferences are determined, voters simultaneously and non-cooperatively vote for alternative *yes* or *no*, or abstain. We can describe the pure strategy of each voter as a function from her preference to her action, $\{y, n\} \rightarrow \{y, n, a\}$, where y, n, and a represent *yes*, *no*, and abstention, respectively. Each voter may also choose mixed strategies.

The outcome of the referendum is valid only if the voter turnout is greater than or equal to a predetermined level. Let m_i $(i = y, n)$ denote the number of voters who have chosen i.

Then, the validity condition is written as $m_y + m_n \geq [rm]$, where $r \in [0, 1]$ is the turnout rate required for the validity of the outcome, and the brackets $[x]$ represent the smallest integer greater than or equal to x. In addition, if $m_y > m_n$ ($m_y < m_n$) holds, alternative *yes* (*no*) is selected as the outcome of the referendum. If $m_y = m_n$ holds, either *yes* or *no* is selected with the same probability. The following two assumptions are imposed on the parameters to make the calculation of pivot probabilities easier:

Assumption 1 *m is an odd number;*

Assumption 2 *$[rm]$ is an odd number.*

Each voter whose preference is *yes* (*no*) enjoys a benefit of 1 if alternative *yes* (*no*) is selected as the outcome, whereas the benefit is 0 if alternative *no* (*yes*) is selected. Since only the relative relationship between the benefits matters for voters' decisions, this normalization of benefits (that is, 0 and 1) does not affect the equilibrium analysis, but makes calculations easier. If the outcome is invalid, on the other hand, each voter receives a benefit of $v \in (0, 1)$. That is, the invalid outcome is worse than her preferred outcome but is better than the non-preferred outcome. We also assume that going to the polls incurs no costs. Under this assumption, the only reason why voters abstain is to spoil the outcome by decreasing the voter turnout. Each voter chooses her strategy to maximize her expected benefit by considering how her vote affects the outcome.

Our analysis focuses on symmetric Bayesian Nash equilibria in which nobody uses weakly dominated strategies. An equilibrium is called *symmetric* if all the voters choose the same strategy. The weakly dominated strategy for each *yes*-voter (*no*-voter) is to vote for alternative *no* (*yes*), because voting for *yes* has the same effect as voting for *no* on the turnout rate. However, voting for *yes* changes the winner from *no* to *yes* if her vote is pivotal, whereas voting for *no* changes it from *yes* to *no*.

2.2 Pivot Probabilities

To derive optimal strategies for voters, we first describe the pivot probabilities for each vote. Since we focus on symmetric equilibria, we express the probability that a voter with preference i ($i = y, n$) chooses action i by σ_i. Then, $1 - \sigma_i$ is the probability of abstention for that voter.

A vote for alternative i can affect the outcome in the following three ways. First, it validates the outcome with alternative i being selected. This happens with certainty if, except for that vote, $m_y + m_n = [rm] - 1$ and $m_i \geq m_j$ ($j \neq i$, $j = y, n$) hold, and with probability 1/2 if, except for that vote, $m_y + m_n = [rm] - 1$ and $m_i = m_j - 1$ hold. However, the latter event never occurs under Assumption 2. Hence, this probability is written as

$$p_i = \sum_{k=0}^{\frac{[rm]-1}{2}} \frac{(m-1)!}{k!([rm]-1-k)!(m-[rm])!} \pi_i^{[rm]-1-k} \pi_j^k \pi_a^{m-[rm]},$$

where $\pi_y = s\sigma_y$, $\pi_n = (1-s)\sigma_n$, and $\pi_a = 1 - \pi_y - \pi_n$.

Second, a vote for alternative i validates the outcome with alternative j being selected. This happens with certainty if, except for that vote, $m_y + m_n = [rm] - 1$ and $m_j \geq m_i + 2$ hold, and with probability 1/2 if, except for that vote, $m_y + m_n = [rm] - 1$ and $m_j = m_i + 1$ hold. The latter event never occurs under Assumption 2. Hence, this probability is written as

$$q_i = \sum_{k=0}^{\frac{[rm]-1}{2}-1} \frac{(m-1)!}{k!([rm]-1-k)!(m-[rm])!} \pi_i^k \pi_j^{[rm]-1-k} \pi_a^{m-[rm]}.$$

Finally, a vote for alternative i changes the winner from alternative j to alternative i when the outcome is valid even without that vote. This happens with probability 1/2 if, except for that vote, $m_y + m_n \geq [rm]$ and either $m_i = m_j - 1$ or $m_y = m_n$ hold. By Assumptions 1 and 2, this probability is written as

$$t_i = \frac{1}{2} \sum_{k=\frac{[rm]-1}{2}}^{\frac{m-1}{2}-1} \frac{(m-1)!}{k!(k+1)!(m-2k-2)!} \pi_i^k \pi_j^{k+1} \pi_a^{m-2k-2}$$
$$+ \frac{1}{2} \sum_{k=\frac{[rm]-1}{2}+1}^{\frac{m-1}{2}} \frac{(m-1)!}{k!k!(m-2k-1)!} \pi_y^k \pi_n^k \pi_a^{m-2k-1}.$$

Given the symmetric strategies of other voters, each voter calculates the above three probabilities. Voters with preference i vote for alternative i only if

$$(1-v)p_i + t_i \geq v q_i. \tag{1}$$

3 Theoretical Analysis

In this section, we derive symmetric Bayesian Nash equilibria for the above model, in which nobody uses weakly dominated strategies. We first deal with a benchmark case, where quorums are not imposed or are small enough to be ineffective. Then, we examine how quorums affect voting behaviors if they are sufficiently large.

3.1 Ineffectively Small Quorums

If participation quorums are not imposed (that is, $r = 0$), the outcome is always valid. Then, the only possibility for each vote to be pivotal is to change the winner (that is, $p_i = q_i = 0$ and $t_i > 0$, $i = y, n$). Hence, inequality (1) holds for $i = y, n$.

For $r \in (0, 1/m]$, one vote is sufficient to validate the outcome. In other words, a vote changes the outcome from invalid to valid only when all other voters abstain. Therefore, when a vote for i validates the outcome, j ($j \neq i$) is never the outcome (that is, $q_i = 0$), so that inequality (1) holds for $i = y, n$. We obtain

Proposition 1 *For $r \leq 1/m$, the unique Bayesian Nash equilibrium is $(\sigma_y = 1, \sigma_n = 1)$.*

Because all voters go to the polls, the *ex-post* majority always wins for such negligibly small quorums.

3.2 Effectively Large Quorums

Next, let us consider effectively large quorums. We can divide the range of effectively large quorums into two intervals, $r \in (1/m, (m-1)/m]$ and $r \in ((m-1)/m, 1]$. We first consider the interval $r \in (1/m, (m-1)/m]$. Subsequently, the latter interval, in which a full turnout is required for the outcome to be valid, is examined.

3.2.1 Interval $r \in (1/m, (m-1)/m]$

For a quorum in this range, one vote is not sufficient to validate the outcome, nor is full turnout required. Therefore, regardless of the behavior of one voter, the outcome is invalid if all other voters abstain, whereas the outcome is valid if all other voters go to the polls. This implies that both zero turnout, $(\sigma_y = 0, \sigma_n = 0)$, and full turnout, $(\sigma_y = 1, \sigma_n = 1)$, are realized in equilibrium.

Suppose that *yes*-voters go to the polls, whereas *no*-voters abstain (that is, $(\sigma_y = 1, \sigma_n = 0)$). Under this strategy profile, validating the outcome must be accompanied by alternative *yes* being selected as the outcome (that is, $p_y, q_n > 0$ and $p_n = q_y = 0$). In addition, any valid outcome necessarily implies that alternative *yes* is selected (that is, $t_i = 0$, $i = y, n$). Therefore, *yes*-voters go to the polls without worrying about their votes resulting in alternative *no* being selected as the outcome, whereas each *no*-voter can only spoil the outcome by abstaining. Thus, this strategy profile constitutes an equilibrium. Similarly, $(\sigma_y = 0, \sigma_n = 1)$ is also an equilibrium. The same logic suggests that neither $(\sigma_y = 0, \sigma_n \in (0,1))$ nor $(\sigma_y \in (0,1), \sigma_n = 0)$ constitutes an equilibrium, because the group members using a mixed strategy will switch to voting for their preferred alternative with certainty.

Next, let us consider the remaining strategy profiles. Suppose $(\sigma_y = 1, \sigma_n \in (0,1))$.[2] This strategy profile is incentive compatible if there exists a value of $\sigma_n \in (0,1)$ that satisfies equation (1) with either equality or inequality for $i = y$ and with equality for $i = n$. These two conditions are combined as follows:

$$\frac{p_y + t_y}{p_y + q_y} \geq \frac{p_n + t_n}{p_n + q_n} = v. \tag{2}$$

The equality (that is, the incentive constraint for *no*-voters) determines the value of σ_n as a function of four parameters, namely, m, s, v, and r. For such a value of σ_n to constitute an equilibrium, the value of σ_n must be between 0 and 1, and also must satisfy the inequality (that is, the incentive constraint for *yes*-voters). When σ_n converges to 0, p_y and q_n converge to a positive value (that is, $\frac{(m-1)!}{([rm]-1)!(m-[rm])!}s^{[rm]-1}(1-s)^{m-[rm]}$), whereas other pivot probabilities converge to 0. This implies that fraction $(p_y+t_y)/(p_y+q_y)$ converges to 1, whereas fraction $(p_n+t_n)/(p_n+q_n)$ converges to 0. Pivot probabilities are continuous in σ_n; therefore, at least for sufficiently small values of v, we can find $\sigma_n \in (0,1)$ that satisfies equation (2).

Does this type of Bayesian Nash equilibrium exist for any set of parameter values? We can show by construction that it does not. For example, suppose that s is sufficiently small. Then, fraction $(p_y+t_y)/(p_y+q_y)$ in equation (2) is greater than fraction $(p_n+t_n)/(p_n+q_n)$ only if $\sigma_n \leq s/(1-s)$ or σ_n is close to 1.[3] In the case of $\sigma_n \leq s/(1-s)$, t_n is small relative to p_n and q_n, because the expected level of voter turnout is low. Hence, fraction $(p_n + t_n)/(p_n + q_n)$ is sufficiently smaller than 1. In the case that σ_n is close to 1, on the other hand, t_n is much greater than p_n and q_n, because almost all voters are expected to

[2] By symmetry, the following argument also applies to $(\sigma_y \in (0,1), \sigma_n = 1)$.

[3] The case of $\sigma_n \leq s/(1-s)$ implies that $\pi_y \geq \pi_n$. Under such a condition, we have $p_y \geq p_n$ and $t_y \leq t_n$. For sufficiently small s, the expected level of voter turnout is low (that is, $\pi_y + \pi_n = s + (1-s)\sigma_n \leq s + (1-s)\frac{s}{1-s} = 2s$); so, p_y and p_n are much greater than t_y and t_n. On the other hand, the case in which σ_n is close to 1 implies that $\pi_y < \pi_n$ for sufficiently small s. Under such a condition, we have $p_y < p_n$ and $t_y > t_n$. Since the expected level of voter turnout is close to 100%, t_y and t_n are much greater than p_y and p_n. Both cases result in $p_y + t_y \geq p_n + t_n$. Note that the denominators of the two fractions satisfy $p_y + q_y = p_n + q_n < 1$ for any $\sigma_i \in (0,1)$ $(i = y, n)$ and parameter values.

go to the polls. Hence, fraction $(p_n+t_n)/(p_n+q_n)$ is greater than 1. As a result, if v is close to 1, equation (2) does not hold.

Finally, let us consider $(\sigma_y \in (0,1), \sigma_n \in (0,1))$. This strategy profile constitutes an equilibrium if there exists a pair $(\sigma_y, \sigma_n) \in (0,1) \times (0,1)$ that satisfies equation (1) with equality for $i = y, n$. These two conditions are combined as follows:

$$\frac{p_y+t_y}{p_y+q_y} = \frac{p_n+t_n}{p_n+q_n} = v. \tag{3}$$

As mentioned above, given a value of $\sigma_y \in (0,1)$, the convergence of σ_n to 0 leads to fraction $(p_y+t_y)/(p_y+q_y)$ converging to 1, whereas fraction $(p_n+t_n)/(p_n+q_n)$ converges to 0. The opposite is also true. That is, given a value of $\sigma_n \in (0,1)$, the convergence of σ_y to 0 leads to fraction $(p_y+t_y)/(p_y+q_y)$ converging to 0, whereas fraction $(p_n+t_n)/(p_n+q_n)$ converges to 1. Therefore, for each set of parameter values, we can find a pair $(\sigma_y, \sigma_n) \in (0,1)\times(0,1)$ that satisfies the first equality in equation (3). Then, the question is whether such a pair also satisfies the second equality for each value of v. As shown below, the answer is that it does not necessarily do so.

Suppose that v is sufficiently small. Then, the numerators of the two fractions $p_i + t_i$ $(i = y, n)$ must be sufficiently small. The convergence of the two numerators to 0 requires that both σ_y and σ_n converge to 0. However, this must also be accompanied by the convergence of the denominators to 0. Hence, we need to determine the limit of the two fractions. Suppose that we let σ_y and σ_n converge to 0 keeping either $\pi_y > \pi_n$ or $\pi_y < \pi_n$ but satisfying $|p_y - p_n| = |t_n - t_y| > 0$ so that the first equality in equation (3) holds. However, since t_i is of a higher order than p_i and q_i with respect to σ_y or σ_n, this condition does not hold for sufficiently small values of σ_y and σ_n. Hence, let σ_y and σ_n converge to 0, keeping $\pi_y = \pi_n$. Then, the first equality in equation (3) holds for any value of σ_y. In the limit, for $i = y, n$, we have

$$\lim_{\sigma_y \to 0|_{\pi_y=\pi_n}} \frac{p_i+t_i}{p_i+q_i} = \frac{\sum_{k=0}^{\frac{[rm]-1}{2}} \frac{1}{k!([rm]-1-k)!}}{2\sum_{k=0}^{\frac{[rm]-1}{2}} \frac{1}{k!([rm]-1-k)!} - \frac{1}{\left[\left(\frac{[rm]-1}{2}\right)!\right]^2}} > \frac{1}{2}.$$

Therefore, the second equality in equation (3) does not hold for sufficiently small values of v. Intuitively, if the invalid outcome is not attractive, members of at least one group will go to the polls with certainty. Thus, we have the following proposition:

Proposition 2 *For* $r \in (1/m, (m-1)/m]$,

(i) $(\sigma_y = 1, \sigma_n = 1)$, $(\sigma_y = 0, \sigma_n = 0)$, $(\sigma_y = 1, \sigma_n = 0)$, *and* $(\sigma_y = 0, \sigma_n = 1)$ *are Bayesian Nash equilibria for any parameter values;*
(ii) $(\sigma_y = 1, \sigma_n \in (0,1))$, $(\sigma_y \in (0,1), \sigma_n = 1)$, *and* $(\sigma_y \in (0,1), \sigma_n \in (0,1))$ *are Bayesian Nash equilibria for a subset of parameter values; and*
(iii) $(\sigma_y = 0, \sigma_n \in (0,1))$ *and* $(\sigma_y \in (0,1), \sigma_n = 0)$ *are never Bayesian Nash equilibria.*

3.2.2 Interval $r \in ((m-1)/m, 1]$

Finally, let us consider what happens if full turnout is required for the validity of the outcome (that is, $[rm] = m$). When even one voter can spoil the outcome by abstaining under $r > (m-1)/m$, full turnout is difficult to realize, because any voter whose preferred alternative is less likely to win will abstain. This incentive leads to the following proposition:

Proposition 3 *For* $r > (m-1)/m$,

(i) $(\sigma_y = 0, \sigma_n = 0)$, $(\sigma_y = 1, \sigma_n = 0)$, *and* $(\sigma_y = 0, \sigma_n = 1)$ *are Bayesian Nash equilibria for any parameter values;*
(ii) $(\sigma_y = 1, \sigma_n = 1)$, $(\sigma_y = 1, \sigma_n \in (0,1))$, $(\sigma_y \in (0,1), \sigma_n = 1)$, *and* $(\sigma_y \in (0,1), \sigma_n \in (0,1))$ *are Bayesian Nash equilibria for a subset of parameter values; and*
(iii) $(\sigma_y = 0, \sigma_n \in (0,1)$ *and* $(\sigma_y \in (0,1), \sigma_n = 0)$ *are never Bayesian Nash equilibria.*

Strategy profiles $(\sigma_y = 1, \sigma_n = 1)$ and $(\sigma_y \in (0,1), \sigma_n \in (0,1))$ are examined below. See the Appendix for the other strategy profiles.

There are two differences from the case of $r \in (1/m, (m-1)/m]$. First, as suggested above, under $r > (m-1)/m$, full turnout $(\sigma_y = 1, \sigma_n = 1)$ can happen only for a subset of parameter values. Let us examine this case. Since every vote is necessary for the validity of the outcome, we have $t_y = t_n = 0$ for such large values of r. Hence, the incentive constraint for full turnout is written as

$$max \left\{ \frac{q_y}{p_y}, \frac{q_n}{p_n} \right\} \leq \frac{1-v}{v}. \tag{4}$$

For what range of parameter values, v and s, is this condition easier to satisfy, given m and r? Because the right-hand side converges to infinity when v converges to 0, inequality

(4) holds for most values of s, if v is sufficiently small. Small values of v mean small benefits from invalid outcomes. This induces voters to go to the polls.

The left-hand side of inequality (4) is smaller when the values of q_y/p_y and q_n/p_n are closer to each other. The reason for this is that q_y/p_y is decreasing in s, whereas q_n/p_n is increasing in s (this comes from the fact that p_y and q_n are increasing in s, whereas p_n and q_y are decreasing in s). Therefore, suppose that $q_y/p_y = q_n/p_n$. This equality holds at $s = 1/2$. Then, for $i = y, n$, we have

$$\frac{q_i}{p_i} = 1 - \frac{1}{\left[\left(\frac{m-1}{2}\right)!\right]^2 \sum_{k=0}^{\frac{m-1}{2}} \frac{1}{k!(m-1-k)!}}. \tag{5}$$

This formula is increasing in m and converges to 1 when m converges to infinity. Therefore, inequality (4) does not hold for sufficiently large values of v. Even when $m = 3$, for example, inequality (4) does not hold for $v > 3/4$.

The second difference from the case of $r \in (1/m, (m-1)/m]$ is that the strategy profile $(\sigma_y \in (0,1), \sigma_n \in (0,1))$ constitutes an equilibrium for a measure-zero set of parameter values. The incentive constraint for this strategy profile is

$$\frac{q_y}{p_y} = \frac{q_n}{p_n} = \frac{1-v}{v}.$$

The first equality holds if and only if $\pi_y = \pi_n$. Then, equation (5) also holds for any such pair of σ_y and σ_n. Hence, this strategy profile constitutes an equilibrium only when m and v equate $(1-v)/v$ with the right-hand side of equation (5).

4 Experiment Design

In this section, we describe the design of our experiment. As shown in Section 3, there exist multiple equilibria, except for $r \leq 1/m$. We conduct a laboratory experiment to analyze which equilibrium outcome is more likely to be realized for each set of parameter values.

4.1 Parameter Values

In our experiment, we specify the parameter values of the above model as follows. The total number of voters is $m = 13$. We call alternative *yes* (*no*) as alternative A (B), and the group of voters who are given preference A (B) as group A (B). The probability of each voter being assigned to group A is either $s = 0.51$ (*close race*) or $s = 0.6$ (*A-dominance*). The benefit from the invalid outcome is $v = 0.5$. Quorums $[rm]$ are set to be 1, 3, 5, 7, 9, 11, and 13.

4.2 Theoretical Predictions

For the above parameter values, Propositions 1 and 2 lead to the following corollary:

Corollary 1 *In both cases of $s = 0.51$ and $s = 0.6$, the following hold:*
(i) For $[rm] = 1$, the unique Bayesian Nash equilibrium is that all voters go to the polls.
(ii) For $[rm] = 3, 5, 7, 9, 11$, there exist the following symmetric pure-strategy Bayesian Nash equilibria: (1) all voters go to the polls; (2) all voters abstain; (3) A-voters go to the polls whereas B-voters abstain; and (4) A-voters abstain whereas B-voters go to the polls.

Here, we examine the remaining cases, that is, symmetric pure-strategy equilibria for $[rm] = 13$ and symmetric mixed-strategy equilibria for each value of $[rm]$. In the former case, Proposition 3 says that at least the following three equilibria exist: (1) all voters abstain; (2) A-voters go to the polls whereas B-voters abstain; and (3) A-voters abstain whereas B-voters go to the polls. Now, we consider whether all voters go to the polls in equilibrium for $[rm] = 13$. Since each A-voter has the stronger incentive to go to the polls than each B-voter, we only have to examine whether a B-voter would like to go to the polls or abstain when all the other voters go to the polls. Under $[rm] = 13$, full turnout is required for the validity of the outcome. Hence, a vote for alternative B can affect the outcome in the following two ways. First, it validates the outcome and leads to B's win; the probability of this is denoted by $p_B(s)$. Second, it validates the outcome and leads to A's win; the probability of this is denoted by $q_B(s)$. We have

$$p_B(s) = \sum_{k=0}^{6} \frac{12!}{k!(12-k)!}(1-s)^{12-k}s^k,$$
$$q_B(s) = \sum_{k=0}^{5} \frac{12!}{k!(12-k)!}(1-s)^{k}s^{12-k}.$$

Each B-voter goes to the polls if

$$(1-0.5)p_B(s) + (0-0.5)q_B(s) \geq 0.$$

This is simplified as

$$p_B(s) \geq q_B(s).$$

We can calculate $p_B(0.51) \approx 0.59$, $q_B(0.51) \approx 0.41$, $p_B(0.6) \approx 0.33$, and $q_B(0.6) \approx 0.67$. Therefore, we obtain

Corollary 2 *For* $[rm] = 13$*, there exist the following symmetric pure-strategy Bayesian Nash equilibria according to the value of* s*:*
(i) For both $s = 0.51$ *and* $s = 0.6$*, (1) all voters abstain; (2) A-voters go to the polls whereas B-voters abstain; and (3) A-voters abstain whereas B-voters go to the polls.*
(ii) Only for $s = 0.51$*, all voters go to the polls.*

Part (*ii*) of Corollary 2 implies that each voter of the *ex-ante* minority group tries to spoil the outcome by abstaining, if the expected size of her group is so small that winning the referendum seems difficult.

Regarding symmetric mixed-strategy equilibria, Propositions 2 and 3 imply that there can be the following three types of equilibria according to the parameter values: (1) all voters use mixed strategies; (2) A-voters use a mixed strategy whereas B-voters go to the polls with certainty; and (3) A-voters go to the polls with certainty whereas B-voters use a mixed strategy. In the third case, as an example, the equilibrium probability of each B-voter going to the polls is calculated in Table 1.

[Table 1 here]

4.3 Procedures used in the Experiment

For the experiment, we had six sessions on November 1, 2007 at Hokkaido University, Japan. Subjects were recruited on campus. Most of them were first-year undergraduate students from various academic disciplines, without any experience of economics or political-science experiments. Each session had 13 subjects in one classroom and subjects took seats sufficiently apart from each other. Each subject joined one session only. Three sessions were held at a time, and each session lasted about 80–90 minutes.

When subjects read the instructions (given in the Appendix), a tape recording of an experimenter reading the instructions aloud was also played, so that all the subjects could read at the same pace. The instructions were written with abstract words; that is, we did not use words such as referendum, vote, win, or any others that may make the subjects feel obliged to go to the polls. After the instructions were read, subjects were given five minutes to ask questions and consider how to make decisions in the experiment. Then, 20 rounds were held. Of the six sessions, sessions 1, 2, and 3 had $s = 0.51$, whereas sessions 4, 5, and 6 had $s = 0.6$.

In each round, experimenters distributed a card to each subject, in which the subject's group (either A or B) and "the required number of subjects who choose 1" that corresponds to quorum $[rm]$, were written. The quorum $[rm]$ took numbers 1 and 13 twice; 3, 5, 9, and 11 three times; and 7 four times respectively, in random order in each round. Each subject circled either "0" or "1" printed on the card and submitted it to an experimenter. Choosing 0 or 1 in the experiment corresponds to abstaining or going to the polls in the theoretical model, respectively. Each subject also wrote her decision, 0 or 1, in her record sheet, so that she could remember her decision history. At the end of each round, experimenters collected the cards from 13 subjects and counted how many subjects chose 1 in each group and also in total. Then, the subjects' earnings were determined according to the rules of the model in Section 2. Payoffs 0, 0.5, and 1 in the model were replaced with 0, 100, and 200 yen in the experiment (1 yen equaled 0.00871 dollars on November 1, 2007). An experimenter announced how many subjects chose 1 in each group and in total, as well as the earnings for the subjects of each group. Another experimenter typed the information in a table that was projected on an overhead screen. Each subject copied the result and her payoff from the screen on her record sheet.

After the final round, the subjects answered questionnaires while experimenters prepared for payments. Then, the subjects received their payments one by one and left the classroom. Each subject's payment comprised the sum of earnings in 20 rounds. The earnings per subject ranged from $1,600$ to $3,000$ yen; the average was $2,314$ yen.

5 Results of the Experiment

In this section, we provide the results of our experiment. We focus on voter turnout, voting outcomes, and individual voting strategies.

5.1 Voter Turnout

We analyze the effects of quorums ($[rm]$), groups (A or B), and the probability of each subject being assigned to group A (that is, s) on the subjects' aggregate behaviors. Figures

1(a) and 1(b) describe the relationship between quorums and the turnout rate for each group in each session under $s = 0.51$ and $s = 0.6$, respectively. For both $s = 0.51$ and $s = 0.6$, when the quorum is 38% ($[rm] = 5$) or smaller, the turnout rate is high in both groups A and B. As the quorum becomes larger, the turnout rate decreases. However, the turnout rate of group A remains relatively high, whereas that of group B decreases to a large extent. In particular, when the quorum is 54% ($[rm] = 7$), the turnout rate of group B for $s = 0.6$ declines from $70--90\%$ to $10--30\%$. That is, the *ex-ante* minority voters abstain more aggressively when the expected number of members is sufficiently different between the two groups. We can summarize our observations as follows.

[Figure 1 here]

Observation 1

(i) A larger quorum results in a decrease in the turnout.

(ii) The turnout is greater in group A than in group B.

(iii) Group A's turnout is greater for $s = 0.6$ than for $s = 0.51$. Group B's turnout is greater for $s = 0.51$ than for $s = 0.6$.

Support

(i) We conduct a two-way analysis of variance (parametric test) and the Friedman test (nonparametric test), so that the differences between sessions with the same value of s are taken into account. We first calculate the average turnout rate of each group in each session for each quorum. Then, we obtain 12 sets (that is, two groups in each of six sessions) of seven turnout rates under each quorum. We divide them into four sets according to groups (A or B) and the value of s (0.51 or 0.6). Table 2 shows the statistics of the two tests for each set of data. Both tests show that for each data set, the differences in turnout rates between quorums are statistically significant at the 5% level or lower.

[Table 2 here]

(ii) We conduct the one-tailed t-test (parametric test) and the Wilcoxon signed-rank test (nonparametric test) for paired data. We divide our data of turnout rates calculated in Support (i) into two sets, according to the value of s (0.51 or 0.6). In each data set, we compare the turnout rates between groups A and B for each quorum of each session. Thus, there are 21 pairs (seven quorums for each of three sessions with the same value of s). The t-statistics (p-values) of the one-tailed t-test are 4.73 (0.000) for $s = 0.51$ and 6.434 (0.000) for $s = 0.6$. The Wilcoxon-statistics (p-values) are 15 (0.000) for $s = 0.51$ and 0 (0.000) for $s = 0.6$. That is, both tests show that the difference in turnout rates between groups is statistically significant at the 1% level or lower.

(iii) We conduct another one-tailed t-test and Wilcoxon signed-rank test for paired data. We first calculate the average of the turnout rates in the three sessions with the same value of s for each group and for each level of quorum. Next, we divide these data into two sets according to groups. In each data set, we compare the turnout rates between $s = 0.51$ and $s = 0.6$ for each level of quorum. The t-statistics (p-values) of the one-tailed t-test are 3.471 (0.007) for group A and 3.391 (0.007) for group B. The Wilcoxon-statistics (p-values) are 1 (0.016) for group A and 0 (0.008) for group B. That is, both tests show that the difference in turnout rates between $s = 0.51$ and $s = 0.6$ is statistically significant for both groups at the 2% level or lower.

[Table 3 here]

Observation 1 is also confirmed by random-effects probit regressions with individual data, where the fact that each subject generates 20 data is taken into account. Table 3 shows the estimated coefficients of treatment variables. The dependent variable is whether each subject voted (1) or abstained (0) in each round of each session. The variable, "quorum" takes values 0, 1, 2, 3, 4, 5, and 6 when quorums are 1, 3, 5, 7, 9, 11, and 13, respectively, in model 1. Furthermore, dummy variables are used for each level of quorum in model 2, where the quorum level of 1 is the baseline. Differences among sessions are also taken into account by session dummy variables, where session 1 is the baseline, and the dummy for session 6 was omitted because of collinearity.[4]

We can see in Table 3 that the larger quorum decreases the probability of turnout. That is, the coefficient of "quorum" is negative in model 1; furthermore, the absolute value of the coefficient of the quorum dummy variable in model 2 is increasing in the size of the quorum.

We can also see that the probability of turnout decreases when subjects are assigned to group B (that is, the coefficient of "group B dummy" is negative significantly). This tendency is strengthened if group B is more disadvantageous in its expected size (that is, the coefficient of "$s = 0.6$ dummy $\times$ group B dummy" is also negative significantly). On

[4] The coefficient of session 2 dummy is significant at 5%. In this session, subjects happened to be assigned to group B more frequently than other sessions. This might have encouraged subjects to go to the polls when they were assigned to group B.

the other hand, the coefficient of "$s = 0.6$ dummy" is positive significantly. These results imply that subjects are more likely to go to the polls under $s = 0.6$ than under $s = 0.51$ when they are assigned to group A, whereas they are less likely to do so when assigned to group B.

5.2 Voting Outcomes

Group A is the *ex-ante* majority in the sense that the expected number of members is greater for group A than group B. However, which group is the *ex-post* majority depends on how the 13 subjects are actually divided into the two groups.

[Tables 4 and 5 here]

The upper part of Table 4 describes how often the *ex-post* majority won, the *ex-post* minority won, two groups were in a tie, and the outcome was invalid, according to the probability of each subject being assigned to group A (that is, s). The lower part divides the data according to quorums. From this table, we obtain

Observation 2
(i) Ex-post minority groups hardly win.
(ii) Invalid outcomes occur frequently when the quorum is 9 *(*69%*) or larger, but rarely for smaller quorums.*

Support
There is no clear difference in the voting outcomes between $s = 0.51$ and $s = 0.6$. Therefore, let us consider the sum of all sessions. The *ex-post* minority won only 3 of 120 rounds (2.5%) in total. Table 5 shows that all the three wins by the *ex-post* minority occurred when group B had 7 members, but many of them abstained. On the other hand, referendums were made invalid in 43 of 120 rounds (35.8%). The lower part of Table 4 shows that 41 of 43 invalid outcomes occurred when the quorum was 9 (69%) or larger.

Note that these results depend on not only subjects' behaviors, but also the realizations of the random division of subjects into two groups. Nonetheless, it seems that we need to carefully consider referendums being made invalid by strategic abstention when the quorum is large. However, we do not need to consider the *ex-post* minority's win as seriously.

5.3 Individual Strategies

Finally, we examine how individual subjects behaved according to the level of quorum and the group to which they were assigned. Of the 78 subjects (6 sessions of 13 subjects), 75 subjects can be regarded to have used one of the following five strategies, according to the group that they belonged to: (1) vote under every quorum (we call this behavior *vote*); (2) vote under small quorums, but randomize between voting and abstaining under large quorums (*vote/randomize*); (3) randomize under every quorum (*randomize*); (4) randomize under small quorums, but abstain under large quorums (*randomize/abstain*); and (5) vote under small quorums, but abstain under large quorums (*vote/abstain*).

Table 6 provides some subjects' voting behaviors observed in the experiment. For example, subject 5 in session 1 is regarded to have employed *vote* when he or she was assigned to group A, whereas *randomize/abstain* was employed when the subject was assigned to group B. Even though a subject abstained several times, we regard him or her to have employed *vote* rather than *randomize*, if the observed number of abstention is so small that the null hypothesis that he or she employed *randomize* is rejected with the 5% level of significance.

[Tables 6 and 7 here]

Table 7 describes how many subjects chose each voting strategy. The row represents subjects' behaviors when they were assigned to group A, whereas the column represents those when assigned to group B. From this table, we obtain

Observation 3
(i) For $s = 0.6$, subjects tend to choose "vote" in group A and "vote/abstain" in group B.
(ii) For $s = 0.51$, subjects tend to choose "vote" in group A and "vote/randomize" and "vote/abstain" in group B.

Support
For $s = 0.6$, 21 of 38 subjects (55.3%) chose *vote* in group A and *vote/abstain* in group B. For $s = 0.51$, on the other hand, subjects' behaviors are more widely dispersed. In addition to the cell of *vote* in A and *vote/abstain* in B that has 9 subjects, the cell of *vote* in A and *vote/randomize* in B has 8 subjects.

From this observation, we can conclude the following. (i) For small quorums, full turnout is most likely to be realized. (ii) For large quorums, if group A is expected to be sufficiently larger than group B (that is, $s = 0.6$), the strategy profile in which voters vote in group A but abstain in group B is most likely to be realized. (*iii*) For large quorums, if group A is not expected to be sufficiently larger than group B (that is, $s = 0.51$), voters in group B behave asymmetrically to each other, but randomization and abstention are employed more frequently than other behaviors.

6 Conclusion

We conducted a referendum experiment with participation quorums. From our observations, we can say that large quorums induce *ex-ante* minority voters to abstain, so that referendums result in invalid outcomes frequently. Of course, the much larger number of voters in real referendums makes it difficult for each voter to affect the outcome by abstaining. Hence, a strong leadership in the minority group or a sufficiently reliable expectation about other voters' behaviors is required for strategic abstention to occur. Whether voters actually abstain in real referendums depends on the voting environment; nevertheless, our experiment shows that an incentive for strategic abstention does exist.

Appendix

Proof of Proposition 3

Here, we deal with the strategy profiles that were not examined in the main text.

The same logic as in the case of $r \in (1/m, (m-1)/m]$ implies that strategy profiles $(\sigma_y = 0, \sigma_n = 0)$, $(\sigma_y = 1, \sigma_n = 0)$, and $(\sigma_y = 0, \sigma_n = 1)$ are Bayesian Nash equilibria, whereas strategy profiles $(\sigma_y = 0, \sigma_n \in (0,1))$ and $(\sigma_y \in (0,1), \sigma_n = 0)$ are not.

Let us consider $(\sigma_y = 1, \sigma_n \in (0,1))$. This strategy profile constitutes an equilibrium if

$$\frac{q_y}{p_y} \leq \frac{q_n}{p_n} = \frac{1-v}{v}.$$

Note that q_y/p_y is increasing in π_n, whereas q_n/p_n is decreasing. When π_n converges to 0, q_y/p_y converges to 0, whereas q_n/p_n converges to infinity. Hence, for sufficiently small values of v, this incentive condition holds. If we try to make q_n/p_n as small as possible while satisfying $q_y/p_y \leq q_n/p_n$, we must have $q_y/p_y = q_n/p_n$, which is attained by $\pi_n = s/(1-s)$. For such a value of π_n, q_n/p_n is equal to the right-hand side of equation (5). Hence, this strategy profile is not an equilibrium for sufficiently large values of v. Similar logic applies to $(\sigma_y \in (0,1), \sigma_n = 0)$. *Q.E.D.*

Instructions

Next, we provide an English translation of the Japanese instructions used in the sessions with $s = 0.51$.

Instructions

Enclosures in Your Envelope

• Instructions (this booklet) • A sample of the record sheet (blue)
• A sample of card 1 (blue) • A sample of card 2 (blue)
• A piece of paper on which a number is written

Please raise your hand if any of the above is missing.

Explanation of the Experiment

This experiment is being conducted for research on decision making. The reward you receive at the end of the experiment is determined by the decisions taken by you and other participants.

Participant Number

There is a piece of paper in your envelope on which the following is written: "Your participant number is ( )." This is your participant number. This number is used when you make decisions. Please keep it at hand so that you do not lose it. Because the experiment is being held anonymously by using participant numbers, your decisions and rewards are never known to other participants.

Organization of the Experiment

The experiment consists of 20 rounds, named Round 1 through Round 20. Each round is independent of the other rounds. That is, decisions and results of the previous rounds are not carried forward to the next round.

What to Do in Each Round

Each round of the experiment proceeds as follows.

(1) Grouping and Decisions

There are 13 participants including you in this classroom. In each round, each participant is independently assigned to **group A with probability** 51% **and group** B **with probability** 49%. You are informed of your own group, but you have no information on how the other 12 participants are divided into the two groups. From the rule that "each participant is independently assigned to group A with probability 51% and group B with probability 49%," however, we can derive the probabilities about how the other 12 participants are divided into the two groups. This is shown in the following table.

[Table A1 here]

At the beginning of each round, you receive a card from an experimenter. Please look at *the sample of card 1*. It is blue in color, but we use pink ones in the experiment. The different colors help us to avoid using sample cards in the experiment.

Please look at the part below the title, "Sample of Card 1." In the first line, the following is written: "**Round 1**." This shows which of the 20 rounds is currently in progress. As the experiment proceeds, this changes to "Round 2," "Round 3," and so on, until we reach "Round 20."

In the second line, the following is written: "**Your Participant Number: ( )**." Please write your participant number in the parentheses. For practice, please write in your participant number now. Are you done? If not, please raise your hand. Hereafter, whenever you have any problem, please raise your hand. An experimenter will come to your assistance.

Please look at *the sample of the record sheet*. It is blue in color, but we use pink ones in the experiment. Again, the different colors help us to avoid using sample sheets in the experiment. In the upper-right part, the following is written: "***Your Participant Number: ( )***." Please write your participant number in the parentheses now. Have you done that?

Please look at the sample of card 1. In the third line, the following is written: "**Your group is** A." This means that you are assigned to group A in this round. If "Your group is B" is written, it means that you are assigned to group B. Please record your group name in the leftmost cell "***Your Group (***A ***or*** B***)***" of your record sheet. For practice, please record it now. Because "**Round 1**" and "**Your group is** A" are written in the sample of card 1, please write "A" in the cell "***Your Group (***A ***or*** B***)***" in the row of "***Round 1***." Have you done that?

Please look at the sample of card 1 again. In the fourth line, the following is written: "**Required Number of Participants:** 5." Please write "5" in the cell "***Required Number of Participants***" in the row of "***Round 1***" on your sample of the record sheet. Have you done that? This "5" may change in each round, or it may be the same as the previous round. We will explain what this means in a little while.

Please look at the sample of card 1 again. In the fifth line and below, the following are written: "**Your Decision (Circle** 0 **or** 1**)**" and "0 1." You need to choose and circle either 0 or 1. You also record the number you have chosen in the cell "***Your Decision (***0 ***or*** 1***)***" on your record sheet. Now suppose that you choose "0." Please circle "0" on the sample of card 1. Have you done that? Furthermore, please write "0" in the cell "***Your Decision (*0 *or* 1*)***" of the row "***Round 1***" on the sample of the record sheet. Have you done that? Next, we will explain how the earnings you receive at the end of the experiment are determined by this decision.

(2) Determining Your Earnings
After all the participants have finished writing down in the cards and the record sheets, the experimenters will collect the cards. Experimenters will now collect the samples of card 1. Please hand your cards to them. The experimenters sum up participants' decisions written on the cards, and count the following:

The number of participants who have chosen 1
(1) in group A,
(2) in group B, and
(3) in total (that is, the sum of (1) and (2)).

For example, as is written on the sample of card 1, suppose that

you are assigned to group A, and "the required number of participants" is 5.

Then, your earnings in this round are determined as follows.

Case 1: If the number of participants who have chosen 1 is
- greater than or equal to 5 in total, and
- greater in group A than group B,

then your earnings are 200 **yen**.

Case 2: If the number of participants who have chosen 1 is
- greater than or equal to 5 in total, and
- greater in group B than group A,

then your earnings are 0 **yen**.

Case 3: If the number of participants who have chosen 1 is
- greater than or equal to 5 in total, and
- the same between the two groups,

then your earnings are 100 **yen**.

Case 4: If the number of participants who have chosen 1 is
- smaller than 5 in total,

then your earnings are 100 **yen**.

When you are assigned to group B, on the other hand, your earnings are 0 yen in Case 1 and 200 yen in Case 2. That is, when the number of participants who have chosen 1 is greater than or equal to "the required number of participants," the participants of the group with more members choosing 1 than the other group earn 200 yen, whereas the participants of the other group earn 0 yen.

After the experimenters have aggregated the participants' decisions written on the cards, they will type it in an Excel table; this will be projected on the screen in front of the classroom. For example, suppose the following.

Round 1
Required Number of Participants: 5
The number of participants who have chosen 1 is 4 in group A, 3 in group B, and 7 in total.

Then, the following is projected on the screen.

[Table A2 here]

In this case, "the number of participants who have chosen 1" is 4 in group A, 3 in group B, and 7 in total. The number of total participants has reached, and even exceeded, the "required number of participants." Further, because "the number of participants who have chosen 1" is greater in group A than group B, the participants of group A earn 200 yen, whereas those of group B earn 0 yen. You look at the screen and write "4," "3," and "7," respectively, in the cells, ***Number of Participants Who Have Chosen* 1*: "Group* A*," "Group* B*," and "in Total"*** on your record sheet. Also, please write "200" in the cell, "***Your Earnings (*200*,* 100*, or* 0*)***" in the row, "***Round 1***" on your record sheet. For practice, please write them in the sample of the record sheet now. Have you done that? Note that regardless of each participant's choice, either 0 or 1, all the members of group A earn 200 yen, whereas all the members of group B earn 0 yen.

When all the participants have finished writing in their record sheets, this round ends and we proceed to the next round. The above procedures of the experiment can be summarized as follows.

Summary of What to Do in Each Round

Round (   )
Your Participant Number: (   )
Your group is (   ).
Required Number of Participants: (   )
Your Decision (Circle 0 or 1)

0   1

(1) Receive a card from an experimenter.
(2) Write your participant number in the parentheses of "**Your Participant Number: (   )**" on the card.
(3) Look at "**Your group is (   ).**" and "**Required Number of Participants: (   )**" written on the card. Copy these in the cells, "***Your Group (*A *or* B*)***" and "***Required Number of Participants***," respectively, on your record sheet.

(4) Make a decision about whether to choose 0 or 1. Based on your decision, circle "0" or "1" on the card, and record it in the cell, "***Your Decision (*0 *or* 1*)***" on your record sheet.
(5) Experimenters collect the cards and count the number of participants who have chosen 1. Then, they fill in the cells, "Number of Participants Who Have Chosen 1: Group A, Group B, and in Total" and "Earnings: Group A and Group B" on the screen.
(6) Look at the information on the screen and copy it in the cells, "***Number of Participants Who Have Chosen* 1*: Group* A*, Group* B*, and in Total***" and "***Your Earnings (*200*,* 100*,* 0*)***" on your record sheet.
(7) This round ends. The next round begins and you receive a new card. This is repeated 20 times.

Let us look at another example. Please look at *the sample of card 2*. Suppose that you have received the sample of card 2 in round 2.

First, please write your participant number in the parentheses of "**Your Participant Number: ( )**" in the second line. Have you done that? Next, please look at "**Your group is** B**.**" in the third line and "**Required Number of Participants:** 4" in the fourth line. Then, write "B" in the cell "***Your Group (*A *or* B*)***," and "4" in the cell "***Required Number of Participants***" in the row, "***Round 2***" on the sample of the record sheet. Have you done that? Note that, in any round, each participant is assigned to group A with probability 51% and group B with 49%. This way of dividing participants into the two groups will not change through the 20 rounds.

If you have done the above, it is time to make a decision. You consider whether to choose 0 or 1. Now suppose that you have decided "1." Please circle "1" at the bottom of the sample of card 2. Have you done that? At the same time, please write "1" in the cell, "***Your Decisions (*0 *or* 1*)***" on the sample of the record sheet. Have you done that? After every participant has finished writing, the experimenters collect the cards. Experimenters will now collect the samples of card 2. Please hand your cards to them.

The experimenters sum up the decisions written on the cards. Suppose that the result is as follows.

The number of participants who have chosen 1 is 1 in group A, 2 in group B, and 3 in total.

Then, an experimenter fills in the table projected on the screen as follows.

[Table A3 here]

This time, the number of participants who have chosen 1 is 3 in total. Because this is smaller than 4 ("the required number of participants"), all the participants of both groups earn 100 yen. So, on your record sheets, please write "1" in the cell "***Group*** A" of the "***Number of Participants Who Have Chosen*** 1," "2" in "***Group*** B," and "3" in "***in Total***," and "100" in "***Your Earnings (*200*,* 100*,* 0*)***." Have you done that?

Earnings

We will use this process to conduct 20 rounds. After the 20th round, you will sum up your earnings from Round 1 through Round 20, and write it in the cell "***Sum of Your Earnings from Round 1 through Round 20***" on your record sheets. This is the amount of money you will receive at the end of the experiment.

After this instruction, the experimenters will collect the samples of the record sheet and distribute the record sheet used in the experiment. Next, you have five minutes to make sure you understand the rules of the experiment and also to consider how to make decisions. Then, we start Round 1.

This is the end of the instruction. If you have any questions, please raise your hand. An experimenter will come to your assistance. Please do not talk with anyone else until the experiment ends and you leave the classroom.

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Quorum	s=0.51	s=0.6
13	0.885	NA
11	0.681	0.782
9	0.493	0.546
7	0.321	0.343
5	0.168	0.170
3	0.044	0.040

Table 1. Equilibrium Probabilities of Group B Voters Going to the Polls

Note: This table applies to symmetric mixed-strategy Bayesian Nash equilibria in which group A voters go to the polls with certainty, whereas group B voters use mixed strategies.

	Group A		Group B	
s=0.51	5.346	(0.007)	50.201	(0.000)
	12.893	(0.045)	16.036	(0.014)
s=0.6	3.082	(0.046)	73.354	(0.000)
	13.714	(0.033)	16.821	(0.010)

Table 2. Test Statistics of the Effect of Quorums on Turnout

Note: In each cell, the upper figure is the F-value of the analysis of variance, and the lower figure is the Chi-square of the Friedman test. P-values are in parentheses.

	Model 1		Model 2	
Variable	Coefficient	Standard error	Coefficient	Standard error
s=0.6 dummy	0.688**	0.244	0.717**	0.248
Quorum	-0.464**	0.292		
Quorum 3 dummy			-0.056	0.259
Quorum 5 dummy			-0.481*	0.243
Quorum 7 dummy			-1.509**	0.227
Quorum 9 dummy			-2.031**	0.235
Quorum 11 dummy			-2.254**	0.237
Quorum 13 dummy			-2.337**	0.246
Group B dummy	-1.041**	0.123	-1.028**	0.124
s=0.6 dummy x group B dummy	-1.022**	0.178	-1.095**	0.182
Round	0.030**	0.007	0.032**	0.008
Session 2 dummy	0.511*	0.226	0.518*	0.230
Session 3 dummy	-0.063	0.223	-0.062	0.228
Session 4 dummy	-0.163	0.228	-0.171	0.232
Session 5 dummy	0.132	0.235	0.133	0.240
Female dummy	-0.283	0.208	-0.289	0.212
Constant	2.341**	0.219	2.225**	0.289
Observations	1560		1560	

Table 3. Results of Random-Effects Regressions

Note: Superscripts * and ** represent significance at 5% and 1%, respectively. There were no significant variables at 10%.

s	Ex-post majority's win	Ex-post minority's win	Tie	Invalid	Sum
0.51	30	2	4	24	60
0.6	40	1	0	19	60
Sum	70	3	4	43	120

Quorum	Ex-post majority's win	Ex-post minority's win	Tie	Invalid	Sum
13	0	0	0	12	12
11	1	0	0	17	18
9	5	0	1	12	18
7	18	2	2	2	24
5	16	1	1	0	18
3	18	0	0	0	18
1	12	0	0	0	12
Sum	70	3	4	43	120

Table 4. Number of Results according to the Value of *s* and Quorums

Session	s	Round	Quorum	# of Subjects		Votes	
				A	B	A	B
1	0.51	15	5	6	7	6	4
3	0.51	1	7	6	7	5	2
5	0.6	19	7	6	7	6	2

Table 5. Cases of the Ex-Post Minority’s Win

	Session 1, Subject 5				Session 1, Subject 8				Session 2, Subject 7			
Group	A		B		A		B		A		B	
Quorum	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain
13	0	0	0	2	1	0	0	1	1	0	1	0
11	3	0	0	1	2	0	0	1	0	2	0	1
9	2	0	0	0	0	0	0	3	1	0	1	1
7	2	0	1	1	1	0	0	3	0	0	2	2
5	1	0	1	1	2	0	1	0	0	2	1	0
3	2	0	1	0	2	0	1	0	2	0	1	0
1	2	0	0	0	1	0	1	0	0	0	2	0
Behavior	Vote		Randomize/Abstain		Vote		Vote/Abstain		Randomize		Vote/Randomize	

Table 6. Examples of Individual Behaviors

Note: Each number expresses how many times each subject voted or abstained in each group under each quorum.

s=0.51

Group A/B	Vote	Vote/ Randomize	Randomize	Randomize/ Abstain	Vote/ Abstain	Sum
Vote	4	8	1	2	9	24
Vote/Randomize	0	0	1	0	3	4
Randomize	1	2	0	0	1	4
Randomize/Abstain	0	0	0	0	0	0
Vote/Abstain	0	0	0	0	5	5
Sum	5	10	2	2	18	37

s=0.6

Group A/B	Vote	Vote/ Randomize	Randomize	Randomize/ Abstain	Vote/ Abstain	Sum
Vote	2	2	1	5	21	31
Vote/Randomize	0	1	1	1	2	5
Randomize	0	0	0	0	0	0
Randomize/Abstain	0	0	0	1	0	1
Vote/Abstain	0	0	0	1	0	1
Sum	2	3	2	8	23	38

Table 7. Classification of Individual Behaviors

Note: Two subjects for s=0.51 and one subject for s=0.6 were not classified in any of these behaviors; they are not included here.

Number of Group A Members	Number of Group B Members	Probability (%)
0	12	0.02
1	11	0.24
2	10	1.37
3	9	4.75
4	8	11.13
5	7	18.53
6	6	22.50
7	5	20.08
8	4	13.06
9	3	6.04
10	2	1.89
11	1	0.36
12	0	0.03

Table A1. Group Divisions and Probabilities of Twelve Participants (Except You)

	Required Number of Participants	Number of Participants Who Have Chosen 1			Earnings (Yen)	
		Group		In Total	Group	
		A	B		A-members	B-members
Round 1	**5**	**4**	**3**	**7**	**200**	**0**
Round 2						
-- omitted --	-- omitted --	-omitted-	-omitted-	-- omitted --	-- omitted --	-- omitted --
Round 20						

Table A2. Screen 1

	Required Number of Participants	Number of Participants Who Have Chosen 1			Earnings (Yen)	
		Group		In Total	Group	
		A	B		A-members	B-members
Round 1	**5**	**4**	**3**	**7**	**200**	**0**
Round 2	**4**	**1**	**2**	**3**	**100**	**100**
-- omitted --	-- omitted --	-omitted-	-omitted-	-- omitted --	-- omitted --	-- omitted --
Round 20						

Table A3. Screen 2

Figure 1(a). Quorums and Turnout for s=0.51

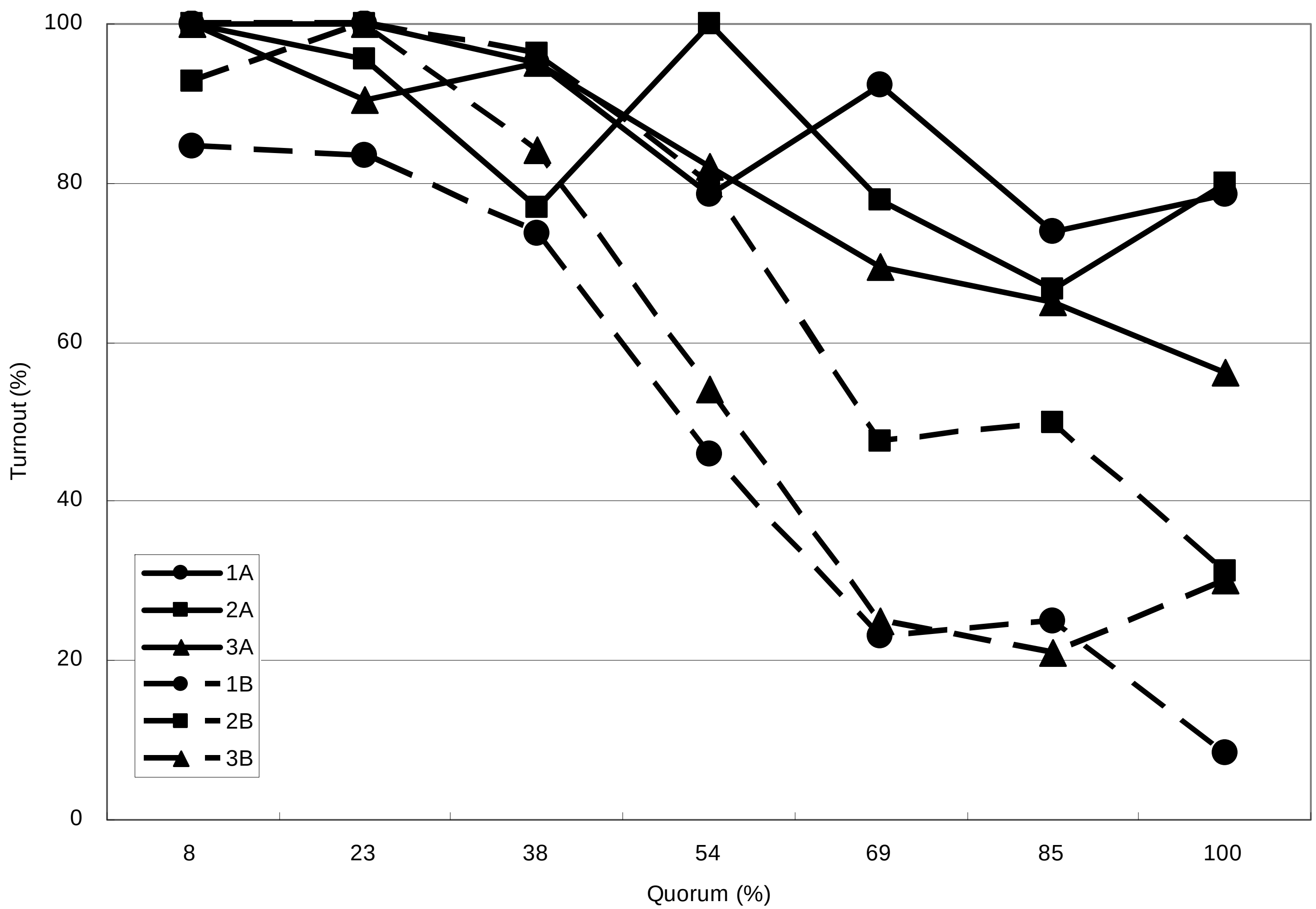

Figure 1(b). Quorums and Turnout for s=0.6