KOCHI UNIVERSITY OF TECHNOLOGY

Social Design Engineering Series SDES-2015-23

Some implications of environmental regulation on social welfare under learning-by-doing of eco-products

Koji Kotani
Kochi University of Technology
Research Center for Social Design Engineering, Kochi University of Technology

Makoto Kakinaka
International University of Japan

29th September, 2015

School of Economics and Management
Research Center for Social Design Engineering
Kochi University of Technology

Some implications of environmental regulation on social welfare under learning-by-doing of eco-products*

Koji Kotani[†] and Makoto Kakinaka[‡]

September 29, 2015

Abstract

This paper examines the significance of environmental regulation in an economy where an eco-product supplied by a single producer is differentiated from a conventional product generating negative externalities. We develop two types of the model: one is a static model without learning effect of eco-product planning, and the other is a dynamic model with learning effect. We show that the regulation should be adopted when the marginal cost of the eco-product production is high enough in a static setting. In a dynamic model, however, whether the regulation improves social welfare is dependent not only on current marginal costs of the eco-product but also on the degree of dynamic learning effect. Particularly, the regulation could improve social welfare when learning effect is either small or large enough, while it could deteriorate social welfare in an intermediate case. Although intuitions tell us that the value of the regulation appears to be monotonically increasing in learning effect, our results suggest that the value possesses a nonmonotone U-shaped feature with respect to learning effect. The optimal decision of the regulation in a dynamic setting could be converse to that of a static setting, providing important policy implications of learning potentials.

Key Words: eco-product, environmental regulation, product differentiation, learning-by-doing

*We are responsible for any remaining errors.

[†]Professor, School of Economics and Management, Kochi University of Technology, 2-22 Eikokuji-cho, Kochi-shi, Kochi 780-0844, Japan (e-mail: kotani.koji@kochi-tech.ac.jp). Correspondence pertaining to this manuscript should be made to Koji Kotani.

[‡]Professor, Graduate School of International Relations, International University of Japan, 777 Kokusai-cho, Minami-Uonuma, Niigata 949-7277, Japan (e-mail: kakinaka@iuj.ac.jp).

1 Introduction

"An important objective of environmental regulation is to induce the substitution of nonpolluting products for polluting ones (Jorgenson and Wilcoxen (1990))."

"Eco-products," "green markets," or "environmental-friendly commodities," etc... Many terminologies have been coined and commonly used in a society to represent an impure public good that jointly plays the two roles of both private and public goods.[1] This is due to the fact that a variety of eco-products has been introduced into markets, and the importance on the promotion of the eco-products is widely recognized to improve the environment (see, e.g., Conrad, 2005). At the same time, various environmental regulations on conventional polluting products generating environmental damages have been implemented by government authorities. Since such regulations affect the output of not only the conventional product but also the eco-product, understanding how the regulations influence the resulting outcome would be crucial for policymakers who intend to promote the eco-product for social welfare improvement. The objective of this study is to clarify the role of the environmental regulation that could affect the promotion of eco-products as well as the resulting social welfare by deriving some important policy implications related to the regulation. More specifically, the result of this paper attempts to answer the following question: under what circumstances should the regulation be implemented?

This paper focuses on two critical features of the eco-product: the first is that the production technology of the eco-product is evolved through process innovation associated with learning-by-doing; and the second is that firms producing eco-products, which generate no negative externalities from production and consumption, compete with the producers of conventional polluting goods due to their substitutability.[2] Concerning the first feature, it has been widely acknowledged that current production enhances future productivity through

[1] This paper uses these terminologies interchangeably. However, we mainly employ an "eco-product."

[2] In reality, however, eco-products may also generate a certain level of negative externalities, but in this paper we assume that the adverse effects on the environment or humans are negligible for simplicity.

accumulation of experience, and such dynamic learning effect in the production process is an essential engine of technological progress in individual firm's as well as national levels.[3] Although research and development is admitted as another important source of innovation, there are sufficient evidences that learning-by-doing plays a crucial role for cost reduction (see, e.g., Bellas, 1998; Argote et al., 1990; Argote and Epple, 1990; Besanko et al., 2014). Since the nature of learning is dynamic, analysis in the framework of a dynamic setting is worthwhile to examine how such learning effect affects the outcomes and to find some important implications.

To capture the second feature of the eco-product in a simple manner, we consider a situation where an eco-product supplied by a single producer is differentiated from a conventional product supplied in a perfectly competitive market. As a first approximation to reality, the analysis assumes the single producer of an eco-product, since a newly-introduced eco-product is usually considered an innovative product, which deserves some degree of economic rent for the survival in a market. Moreover, in this study, learning effect can be internalized in the technology of the single producer without any spill-over. These specifications might be consistent with a situation where a single producer first invents and introduces an eco-product into a market and then competes with a pre-existing conventional product due to their substitutability.

Much work on environmental-friendly products has been done in the literature (see, e.g., Conrad (2005) and Bansal and Gangopadhyay (2003)). Employing a model with product differentiation originated from consumers' environmental awareness, they discuss firms' incentives in choosing the quality or technology that sets the environmental-friendliness of polluting products. However, there is still unsettled dispute on how much environmental awareness incites real consumer choices for eco-products. Recent marketing researches reveal that although some people express high consciousness on being a green consumer in surveys, the portion of those people is still small and such consciousness over environmen-

[3]For early work on learning-by-doing, see, e.g., Arrow (1962), Spence (1981), Fudenberg and Tirole (1983), Romer (1986), and Stokey (1988).

tally sound practices does not necessarily reflect actual consumers' choices (see, e.g., Loureiro et al. (2002) and Grankvist et al. (2004)). Given the evidences, this study takes a different angle from past literature. Product differentiation between eco-products and conventional products is assumed to derive from the distinctions of "purely" perceived quality in this paper, while we admit that consumers' environmental awareness may be another important source of product distinctiveness. For example, organic products in agriculture have "cosmetic defects" relative to the conventional ones, but healthier.[4] The same type of product differentiation can also be seen in the other industries, such as paper and pulp, where eco-products and conventional products co-exist. Instead of a usual assumption that producers set the environmental technology or the degree of the environmental-friendliness of their products, this study explicitly distinguishes the eco-product from the conventional product generating negative externalities.

In terms of environmental regulations, there has been a series of literature which examines the implications of various policies on products generating negative externalities (see, e.g., Arora and Gangopadhyay, 1995; Cremer and Thisse, 1999; Goulder and Mathai, 2000; Moraga-Gonzalez and Padron-Fumero, 2002; Bansal and Gangopadhyay, 2003; Eriksson, 2004; Zhu and Ruth, 2015). Whereas their focus is mainly on the incentive schemes of government policies such as tax and subsidy, the analysis in this paper only consider standards or targets for environmental quality as the government policy tool for environmental regulation, such as emission standards or best available control technology standard. Cropper and Oates (1999) and Kolstad (2010) state that since in reality there are some problems of measurement or other informational obstacles to adopt the first-best incentive policies, the determination of actual environmental policies consists of two steps: the first is that standards or targets for the environmental quality is decided; and the second is that a regulatory system with incentive schemes is arranged so that these standards are efficiently satisfied. From a practical point of view, this paper pays attention to the first stage of setting the

[4]See Loureiro et al. (2002) for an evidence that cosmetic issues are salient in consumer choices.

targets or standards as a government policy tool, while the exploration of an equivalent basis in the second step is left for future researches.[5]

It should also be noted that this study examines the case where the government makes the commitment to a new environmental standard as the regulation for a sufficiently long time, once it is set. Certainly, one may suggest that there are strategic interactions between the government and the industry as in the cases with tax/subsidy scheme. While it could be admitted that such strategic interactions may be present, we still restrict ourselves to the commitment case for the two reasons. One reason is that the commitment case yields a set of interesting results in itself and could be a benchmark analysis. It would be possible that the basic structure of the benchmark model introduced in this paper is extended to the one where strategic interactions exists, and such an extension should be addressed in future researches. The other reason is based on the historical fact that once targets or standards are devised, they tend to sustain for a sufficiently long time such as 10 years or much longer in many countries.[6] Therefore, the analysis based on the commitment seems plausible or a good approximation to a real world.

This paper first develops a static model of the single firm producing an eco-product as a base case, where its production technology, represented by the constant marginal cost, is not associated with any learning effect. The environmental regulation is assumed to raise the production cost of the conventional product and to reduce environmental damages from the conventional product. With this approach, we show the possibility that a tighter environmental standard on the conventional product induces social welfare improvement through promoting the substitutable eco-product when its technology level is low enough. This is due to the fact that when the marginal cost of the eco-product is high enough, the

[5] There are many studies on the role of environmental standards, like emission standards, in the discussions of various contexts, such as abatement technology and compliance. See, e.g., Arora and Gangopadhyay (1995) and Stranlund (1997).

[6] For example, environmental standards for various chemicals have not changed for ten years or much longer in many countries (See Ministry of Environment Government of Japan (2006b) and Ministry of Environment Government of Japan (2006a)) for the Japanese case, and Greenstone (2003) and many others for the case of United States.

conventional product is prevailing in the market, and environmental damages associated with the conventional product is relatively large. In this case, the regulation on the conventional product could reduce environmental damages significantly even though it decreases consumer surplus.

Building upon the static model, this study next develops a dynamic model for the eco-product production with learning effect.[7] We keep the basic structure, but the main difference is that the marginal cost for an eco-product changes over time due to accumulation of experience. In this setting, whether or not the regulation improves social welfare is highly dependent not only on the initial marginal cost of an eco-product, but also on the degree of dynamic learning effect. In particular, the regulation could improve social welfare when the learning effect is either small enough or large enough, while it could deteriorate social welfare in an intermediate case. Since the regulation enhances learning through shifting the demand toward the eco-product, the intuition tells us that the value of the environmental regulation appears to monotonically increase in the degree of dynamic learning effect. Our results, however, suggest that this is not always the case. It is shown that the value of the regulation possesses a nonmonotone U-shaped feature with respect to the degree of dynamic learning effect, i.e., the regulation is likely to improve social welfare in a dynamic sense if the degree of dynamic learning effect is either small enough or large enough. On the one hand, if the degree of learning takes some intermediate values, the regulation could worsen social welfare. This implies that the optimal decision of the regulation in a dynamic case could be converse to that in a static case, given the same condition except the presence of learning, providing important policy implications of learning potentials.

These results in this paper have some connection with a contentious debate of whether or not tightening environmental standards is good for a society (Ambec et al., 2013). The claim made by Porter (1991) and Porter and van der Linde (1995) is that tightening the standards may trigger the firm's innovation so that the regulation could cause everyone better off in the

[7] Miravete (2003) develops a similar type of the model to ours for a single producer with learning-by-doing in the context of international trade.

long run. On the other hand, several studies including Palmer et al. (1995) question their viewpoint. Although the present study somewhat deals with a specific situation compared to the general cases they describe, our result characterizes a situation where the regulation on polluting products brings about the promotion of eco-products and enhances the process innovation associated with learning in the production technology of eco-products. As a result, social welfare could be increased under certain conditions. We believe that this study provides one of the first exemplary models that support the conjecture that tightening environmental standards makes a society better off, and this point could also be counted as one of the main contributions.

In the next section, we elaborate on the basic elements of the model which are common both in the static and dynamic models. The section is followed by presenting the static model without considering learning and provides several main results. In the third section, the dynamic model with learning effect is developed, and the steady state is analytically characterized as a long-run outcome. We then examine the optimal decision rule of the regulation using a numerical analysis. The results are compared with those derived in the static model to illustrate some important implications. In the final section, we offer some conclusions.

2 The Model

We consider an economy with a numeraire sector and an industry consisting of two sectors, an eco-product and a conventional product. The model is written in continuous time. The conventional product is produced in a perfectly competitive market with a constant marginal cost, which is fixed over time. In contrast, the eco-product is a new innovative product, which deserves some degree of monopoly rent. As a first approximation of the analysis, the eco-product is assumed to be produced by a single producer without new entry. Technology of the eco-product is characterized with instantaneous constant returns to scale, but its marginal cost declines with output due to dynamic economies of scale through learning-by-doing, as explained later. In this study, the only state variable is the level of marginal cost of an eco-product. These two products are substitutes. The substitutability is assumed to come from differences of pure perceived characteristics as a product, as mentioned in the previous section (see, e.g., Loureiro et al. (2002) and Grankvist et al. (2004) for empirical evidences).

The other difference between the conventional product and the eco-product is that the former entails negative externalities from consumption or production, while the latter does not. To control pollution, the government may impose the environmental regulation on the production of the conventional product. In order to keep the model simple and obtain clear policy implications, it is assumed that the government decides whether or not to adopt the regulation. The regulation precludes the negative externalities but causes the producers to incur an additional marginal cost. Let $\phi \in \{0, 1\}$ denote the binary choice such that $\phi = 1$ if the government adopts the regulation, and $\phi = 0$ otherwise. Specifically, we assume that the constant marginal cost for the conventional product is given by $\tau(\phi)$ such that $\tau(0) = \underline{\tau}$ and $\tau(1) = \underline{\tau} + \xi$, and the pollution level per unit of the output of the conventional product is given by $\epsilon(\phi)$ such that $\epsilon(0) = \varepsilon$ and $\epsilon(1) = 0$, where $0 < \underline{\tau} < \underline{\tau} + \xi < 1$ and $\varepsilon > 0$. As ξ becomes larger, the regulation causes a larger additional increase in the marginal cost for the eco-product. As ε becomes larger, the production of the conventional product induces a larger amount of pollution under no regulation. Thus, this specification captures the trade-off relationship between an increase in the marginal cost for the conventional product under the regulation and an increase in negative externalities through pollution under no regulation.

Notice that if the regulation is adopted, there is no pollution, i.e., $\epsilon(1) = 0$.[8] The regulation in this study may be interpreted as the adoption of environmental standards on the conventional product which reduce pollution emission to a sufficiently low level at which negative externalities become negligible. This setting can be justified by the fact that new standards for environmental quality are usually determined by the criterion of

[8]The results in what follows will not be affected significantly even if there is a positive level of pollution under the environmental regulation, i.e., $\epsilon(1) \in (0, \varepsilon)$.

"permissible concentration" or "acceptable" (see, e.g., Oates et al. (1989), Baumol and Oates (1988) and Kolstad (2010)). Thus, in our model, both the conventional product and the eco-product are environmental-friendly under the regulation. However, since their substitutability does not come from environmental awareness of consumers, it is not affected by the government decision of whether or not to adopt the regulation. This implies that the demand for both products is totally independent of the environmental performance of the conventional product.

Let x and y denote the consumption of an eco-product sold by the single producer and the consumption of a conventional product in a perfectly competitive market, respectively. Following Singh and Vives (1984), we assume that there is a representative consumer whose preference is described by a quasi-linear utility with respect to a numeraire, $q+u(x,y)$, where q is the consumption of the numeraire and $u(x,y)$ is a quadratic sub-utility with symmetric cross-effects with $u(x,y) = (x+y) - (x^2+y^2+2\gamma xy)/2$. The parameter $\gamma \in [0,1)$ measures the degree of product differentiation between the eco-product and the conventional product. The sufficient condition $\Delta = 1-\gamma^2 > 0$ ensures that the sub-utility function $u(x,y)$ is strictly concave. If $\gamma = 0$ the conventional product and the eco-product are viewed as independent, but as $\gamma \to 1$ these products become closer to perfect substitutes.

At each instantaneous time, the representative consumer maximizes the utility subject to his budget constraint $I = q + px + \tau(\phi)y$, where p is the price of the eco-product decided by the single producer. Notice that the price of the conventional product is equal to its marginal cost $\tau(\phi)$ due to the perfect competition. The first-order conditions yield the following demand functions:

$$x^*(p,\phi) = \frac{1-\gamma(1-\tau(\phi))}{1-\gamma^2} - \frac{1}{1-\gamma^2}p; \qquad y^*(p,\phi) = \frac{(1-\tau(\phi))-\gamma}{1-\gamma^2} + \frac{\gamma}{1-\gamma^2}p. \tag{1}$$

The demand for the eco-product is decreasing in p while the demand for the conventional product is increasing in p, i.e., $x^*_p < 0$ and $y^*_p \geq 0$ with equality if $\gamma = 0$. Since

$\tau(0) = \underline{\tau} < \underline{\tau} + \xi = \tau(1)$, the environmental regulation increases the demand for the eco-product but decreases the demand for the conventional product, i.e., $x^*(p, 0) < x^*(p, 1)$ and $y^*(p, 0) > y^*(p, 1)$. Using demands (1), we obtain instantaneous consumer surplus, $S(p, \phi) \equiv u(x^*(p, \phi), y^*(p, \phi)) - px^*(p, \phi) - \tau(\phi)y^*(p, \phi)$, and the instantaneous profit for the single producer, $\pi(p, \phi, c) \equiv (p-c)x^*(p, \phi)$, where c is the marginal cost for the eco-product.[9]

For simplicity, we assume that the instantaneous negative externalities from the conventional product depends only on the total pollution, which is represented by the pollution level per unit of the output of the conventional product times the conventional product consumption, $\epsilon(\phi)y^*(p, \phi)$. Specifically, the instantaneous negative externalities are given by $E(p, \phi) = \Gamma(\epsilon(\phi)y^*(p, \phi))$, where Γ is strictly increasing and strictly convex with $\Gamma(0) = 0$. Notice that to focus on the dynamics of technological progress, we do not consider the evolution of the concentration of pollution.[10] Then, instantaneous social welfare is simply defined as $T(p, \phi, c) \equiv \pi(p, \phi, c) + S(p, \phi) - E(p, \phi)$.[11]

The central assumption is that technology of an eco-product exhibits instantaneous constant returns to scale, but its marginal cost, c, is reduced over time as the single producer accumulates output, i.e., the level of marginal cost is reduced due to the accumulation of experience. Unlike the standard learning curve specification as in Spence (1981), we also consider the depreciation of experience, or in an alternative interpretation, the existence of potential adjustments costs in the accumulation of such experience, so that some minimum production level is required at every period to ensure a net reduction of the single producer's marginal cost. Several authors claim that such depreciation of knowledge is also important in reality and empirically show some evidence (see, e.g., Argote et al. (1990) and Benkard (2000)).[12] The existence of such a knowledge depreciation allows us to identify a

[9] For just a explanatory reason, consumer surplus in our definition does not include the negative externalities that come from pollution associated with the consumption and production of the conventional product. Such negative externalities are captured separately in this paper.

[10] For the model setting with the evolution of technology and the concentration of pollution, see, e.g., Goulder and Mathai (2000).

[11] Instantaneous social welfare could be described by a more general form of $T(\pi(p, \phi, c), S(p, \phi), E(p, \phi))$. However, the main results in this study are not changed in general.

[12] For a similar setup of the model using the learning curve with knowledge depreciation, see, e.g., Miravete

steady state, which could be important to analyze a long-run outcome in an economy. For analytical tractability, the reduction in the marginal cost is assumed to be described as the following state equation:

$$\dot{c} = -\lambda[x^*(p, \phi) - \sigma(c)], \tag{2}$$

where $\lambda \geq 0$ represents the marginal cost reduction effect per unit of output, or simply the degree of dynamic learning effect, and $\sigma(c)$ captures a situation where the value of experience depreciates over time.

For the consistency with empirical regularity, it is assumed that $\sigma(c) = \eta - \delta c$ with $\eta > 0$ and $\delta > 0$ and that the marginal cost of the eco-product in the initial period is not large enough so that $c_0 < \eta/\delta$.[13] With this specification of the state equation (2), the speed of technological improvement is increased with higher output, lower knowledge depreciation and higher learning effect, as long as $x^*(p, \phi) > \sigma(c)$. In addition, it may be consistent with a real observation in terms of knowledge accumulation. If the learning parameter λ is large, likewise the speed of knowledge accumulation as well as that of knowledge depreciation. Therefore, it agrees with a situation where "knowledge or skills that can be quickly obtained tends to be forgetful very shortly if they are not in practice." Conversely, for small λ, the speed of knowledge accumulation is slow, likewise knowledge depreciation. Contrary to the high learning, the skills that are once obtained would be forgetful very slowly. Finally, notice that when there is no dynamic learning effect, i.e., $\lambda = 0$, our problem is reduced to a static single producer's problem with two differentiated products, in which one is produced by a single producer and the other is produced in a perfectly competitive market.

To analyze the pricing of the eco-product and the environmental regulation, we consider an economy which extends over the following two steps. In step 1, the government decides whether or not to adopt the regulation on the production of the conventional product.[14]

(2003).

[13]This assumption excludes a case in which the knowledge depreciation rate is negative, i.e., $\sigma(c) < 0$.

[14]As mentioned in the previous section, the government is assumed to make the commitment to a newly-

The decision affects the constant marginal cost for the conventional product, $\tau(\phi)$, and the pollution level per unit of the output of the conventional product, $\epsilon(\phi)$. In step 2, taking the government's decision, ϕ, as given, the single producer producing the eco-product decides the pricing schedule over time so that the present value of the profits is maximized. Then, consumption and production of the eco-product and the conventional product take place in each time.

3 Static Model

This section examines a static case where the single producer chooses the optimal price of the eco-product taking the government policy ϕ and the marginal cost c as given. A static case is equivalent to the one in which there is no dynamic learning effect, i.e., $\lambda = 0$. In this case, since the marginal cost for the eco-product is constant over time, the single producer simply maximizes its instantaneous profit $\pi(p, \phi, c)$ with respect to p, taking ϕ and c as given. The first-order condition yields the optimal price of the eco-product:

$$p_M(\phi, c) = \frac{c + \tau(\phi)\gamma + 1 - \gamma}{2}. \tag{3}$$

Using equation (3), we obtain the consumptions of the eco-product and the conventional product and consumer surplus, the single producer's profit and the negative externalities: $x_M(\phi, c) \equiv x^*(p_M(\phi, c), \phi)$, $y_M(\phi, c) \equiv y^*(p_M(\phi, c), \phi)$, $S_M(\phi, c) \equiv S(p_M(\phi, c), \phi)$, $\pi_M(\phi, c) \equiv \pi(p_M(\phi, c), \phi, c)$, and $E_M(\phi, c) \equiv E(p_M(\phi, c), \phi)$. In order to make our analysis simple, in the rest of this paper, we assume that the solution is interior in the sense that the consumption levels of the eco-product and the conventional product are positive.

Based on the above discussions, we derive the impact of technological improvement of the production of the eco-product, which exhibits a decline in its marginal cost. As in the

introduced environmental standard for a sufficiently long time. Although we admit that there may be the precommitment issues with strategic interactions between the government and the single producer of the eco-product, we believe that the commitment could be a first approximation to a real world.

standard static monopoly theory, it is directly shown that, for any $\phi \in \{0, 1\}$, technological progress in the production of the eco-product decreases the optimal eco-product price; increases the output of the eco-product; decreases the output of the conventional product and the negative externalities; and increases consumer surplus as well as the profit for the single producer (see Claim 1 in the Appendix for the proof).[15] Technological progress in the eco-product not only substitutes the conventional product into the eco-product through a decline in the price of the eco-product but also improves social welfare through the increase in consumer surplus and the profit for the single producer and the decrease in the negative externalities.

3.1 Environmental Regulation

In this subsection, we consider the impact of the environmental regulation without dynamic learning effect ($\lambda = 0$). The regulation affects the optimal price of the eco-product through raising the market price of the conventional product. Then, we deduce the following results (see the Appendix for the proof):

Proposition 1 (Environmental Regulation) *Suppose $\lambda = 0$. Then, the environmental regulation* (1) *increases the optimal price of the eco-product;* (2) *increases the output of the eco-product but decreases the output of the conventional product; and* (3) *decreases consumer surplus but increases the profit for the single producer.*

The regulation allows the single producer of the eco-product to take advantage through increasing the production cost of the conventional product. Similar to the case of technological improvement, the regulation promotes the eco-product, reduces the conventional product, and vanishes its negative externalities. However, the effects on the price of the eco-product and consumer surplus have the opposite direction. In this case, promoting the eco-product with preferable environmental performance can be achieved at the expense of consumer surplus. In contrast to the case where technological progress always improves social welfare,

[15] For $\phi = 1$, there are no negative externalities for any technology level.

whether the regulation improves social welfare depends on the degree of positive impacts on the negative externality and the single producer's profit and the degree of the negative impact on consumer surplus.

3.2 Government's Decision Problem

We now find the optimal government's decision regarding the regulation in step 1 when there is no dynamic learning effect. The government chooses to adopt the regulation ($\phi = 1$) if social welfare is larger under the regulation than under non-regulation, i.e., $\Delta T_M(c) \equiv T(p_M(1,c),1,c) - T(p_M(0,c),0,c) > 0$, where $\Delta T_M(c)$ represents the value of the regulation or the effect of the regulation on social welfare. Then, we obtain the following result related to the optimal policy as a possible case (see the Appendix for the proof):

Proposition 2 (Environmental Policy) *Suppose $\lambda = 0$. Then, there may exist a unique value $c_M > 0$ such that the optimal policy is to impose the environmental regulation on the conventional product if $c > c_M$; and not to impose the regulation if $c < c_M$.*

The conditions for the uniqueness of $c_M > 0$ are $\Delta T_M(0) < 0$ and $\Delta T_M(1) > 0$. These require that if the marginal cost of the eco-product is small enough so that the eco-product is diffused enough in the market, the regulation cannot be justified from the standpoint of social efficiency, and that if the marginal cost of the eco-product is large enough so that the conventional product is diffused enough in the market, the regulation can be justified.

The logic behind Proposition 2 is as follows. The value of the regulation or the effect of the regulation on social welfare can be divided into the three elements and is rewritten by $\Delta T_M(c) = \Delta S_M(c) + \Delta\pi_M(c) + \Gamma(\varepsilon y_M(0,c))$, where $\Delta S_M(c) \equiv S_M(1,c) - S_M(0,c)$ represents the value of the regulation related to consumer surplus, $\Delta\pi_M(c) \equiv \pi_M(1,c) - \pi_M(0,c)$ the value of the regulation related to the profit for the single producer, and $\Gamma(\varepsilon y_M(0,c))$ the value of the regulation related to environmental damages. As shown in Proposition 1, the introduction of the regulation reduces consumer surplus ($\Delta S_M(c) < 0$), increases the profit for the single producer ($\Delta\pi_M(c) > 0$), and vanishes environmental damages ($\Gamma(\varepsilon y_M(0,c)) > 0$).

When the cost of the eco-product is low, the sales of the eco-product is already high and those of the conventional product is already low due to their substitutability. In this case, the reduction in environmental damages from the conventional product, associated with the regulation, may not make up for the increased loss in consumer surplus, even though the regulation still induces a demand shift from the conventional product toward the eco-product, i.e., $\Delta S_M(c)+\Delta\pi_M(c)+\Gamma(\varepsilon y_M(0,c)) < 0$ for a low c. That is, since the negative externalities under non-regulation is relatively small due to relatively small demand for the conventional product, the merit of the regulation from vanishing the negative externalities cannot offset the negative effect on consumer surplus. Thus, the regulation cannot be justified.

In contrast, when the cost of the eco-product is high, the sales of the eco-product is low and those of the conventional product is high. In this case, the reduction in environmental damages associated with the regulation may make up for the increased loss in consumer surplus, i.e., $\Delta S_M(c)+\Delta\pi_M(c)+\Gamma(\varepsilon y_M(0,c)) > 0$ for a high c. Since the negative externalities under non-regulation are relatively large due to relatively large demand for the conventional product, the merit of the regulation from vanishing the negative externalities dominates the negative effect on consumer surplus. Thus, adopting the regulation can be justified even though consumer surplus is reduced as a result.

Examination of the impact of a change in ξ and ε is important for policy makers to decide whether or not the regulation should be adopted. At the critical value, it must hold that $\Delta T_M(c_M) = 0$. Our specification of the model implies that the critical value c_M is decreasing in the pollution level per unit of the output of the conventional product, ε (see Claim 2 in the Appendix). A rise in the pollution level ε increases environmental damages under non-regulation, which implies that the value of the regulation related to environmental damages becomes large or the regulation becomes more attractive. Thus, the region of c, in which adopting the regulation is optimal ($c > c_M$), becomes larger as ε increases. Furthermore, our specification also states that the critical value c_M is increasing in the effect of the regulation on the marginal cost of the conventional product, ξ (see Claim 2 in the Appendix). A rise in ξ raises the price of the conventional product, which reduces consumer surplus. Thus, when ξ is higher, the regulation becomes less attractive and the region of c, in which adopting the regulation is optimal ($c > c_M$), becomes smaller.

4 Dynamic Model under Learning-by-Doing

The previous section has examined the static model under the assumption that there is no dynamic learning effect. We now assume a positive dynamic learning effect, i.e., $\lambda > 0$. One important distinction between the static and the dynamic settings is that the marginal cost of the eco-product is exogenously given in the static setting, while it is evolved through learning process in the dynamic setting. This section first characterizes the steady state as a long-run outcome under the optimal price schedule over time decided by the single producer of the eco-product at step 2. We then explain the government's decision problem at step 1 in which the government can decide whether or not to adopt the regulation $\phi \in \{0, 1\}$ at time $t = 0$ without possibility of changing the policy after the decision.

The single producer's problem is to maximize the present value of her profits, while considering the environmental regulation and the learning effect induced by current production through her pricing decisions. This problem can be stated as $\max_p \int_0^\infty \pi(p, \phi, c)e^{-rt}dt$ subject to $\dot{c} = -\lambda[x^*(p, \phi) - \sigma(c)]$ and $c(0) = c^0$, where $r > 0$ represents the discount rate. The value of $x^*(p, \phi)$ is the demand for the eco-product derived in equation (1). Applying the dynamic optimal control with the Hamiltonian, $H^F = \pi(p, \phi, c) - \mu_f \lambda[x^*(p, \phi) - \sigma(c)]$, the equilibrium of our model solves the following set of generalized Hamilton-Jacobi conditions: $x^*(p, \phi) + (p - c - \lambda\mu_f)x_p^*(p, \phi) = 0$ and $\dot{\mu}_f = (r + \lambda\delta)\mu_f + x^*(p, \phi)$. This problem yields a linear differential system:

$$\begin{bmatrix} \dot{p} \\ \dot{c} \end{bmatrix} = M \begin{bmatrix} p \\ c \end{bmatrix} + \begin{bmatrix} \frac{\lambda\eta-(r+\lambda\delta)[1-\gamma(1-\tau(\phi))]}{2} \\ \frac{(1-\gamma^2)\lambda\eta-\lambda[1-\gamma(1-\tau(\phi))]}{1-\gamma^2} \end{bmatrix} \quad \text{where } M = \begin{bmatrix} r+\lambda\delta & -\frac{r+2\lambda\delta}{2} \\ \frac{\lambda}{1-\gamma^2} & -\lambda\delta \end{bmatrix}. \tag{4}$$

The matrix M is independent of the government's decision.

For any $\phi \in \{0,1\}$, a pair of the marginal cost and the price of the eco-product, $(\bar{c}(\phi), \bar{p}(\phi))$, at which $\dot{c} = \dot{p} = 0$ in the linear differential system (4), is called a steady state. A steady state is stable if c and p converge to $\bar{c}$ and $\bar{p}$, respectively (see, e.g., Kamien and Schwartz (2012)). The specification of our dynamic model implies that for any $\phi \in \{0,1\}$, there exists a unique steady state $(\bar{c}(\phi), \bar{p}(\phi))$, and that the unique steady state is stable if the degree of product differentiation between the eco-product and the conventional product is small enough so that $\det(M) < 0$ or $\gamma < \hat{\gamma}$, where $\hat{\gamma} \equiv (1 - \frac{r+2\lambda\delta}{2\delta(r+\lambda\delta)})^{1/2}$ (see Claim 3 in the Appendix).[16] In the rest of the paper, we assume $\gamma \in [0, \hat{\gamma})$. Under this assumption, for any $\phi \in \{0,1\}$, the equilibrium path of state variable c is described by $\hat{c}_t(c_0, \phi) = [c_0 - \bar{c}(\phi)]e^{kt} + \bar{c}(\phi)$, where $k = \frac{r}{2} - \frac{r+2\lambda\delta}{2}[1 - \frac{2\lambda}{(1-\gamma^2)(r+2\lambda\delta)}]^{\frac{1}{2}} \in (-\lambda\delta, 0)$.

Given a state variable $\hat{c} = \hat{c}_t(c_0, \phi)$, the optimal price of the eco-product decided by the single producer is represented by:

$$\hat{p}(\phi, \hat{c}) = \alpha_1 \hat{c} + \alpha_2(\phi), \tag{5}$$

where $\alpha_1 = \frac{(1-\gamma^2)(k+\lambda\delta)}{\lambda} > 0$ and $\alpha_2(\phi) = -\frac{k(1-\gamma^2)}{\lambda}\bar{c}(\phi) - \eta(1-\gamma^2) + 1 - \gamma(1-\tau(\phi))$.[17] Then, given ϕ and $\hat{c}$, the equilibrium levels of the outputs of the eco-product and the conventional product, consumer surplus, the profit for the single producer, and the negative externalities can be respectively represented by $\hat{x}(\phi, \hat{c}) \equiv x^*(\hat{p}(\phi, \hat{c}), \phi)$, $\hat{y}(\phi, \hat{c}) \equiv y^*(\hat{p}(\phi, \hat{c}), \phi)$, $\hat{S}(\phi, \hat{c}) \equiv S(\hat{p}(\phi, \hat{c}), \phi)$, $\hat{\pi}(\phi, \hat{c}) \equiv \pi(\hat{p}(\phi, \hat{c}), \phi, \hat{c})$, and $\hat{E}(\phi, \hat{c}) \equiv E(\hat{p}(\phi, \hat{c}), \phi)$.[18] It is assumed that

[16]If $\gamma < \hat{\gamma}$, the steady state is a saddlepoint and is stable. In contrast, if $\gamma > \hat{\gamma}$, the state variable never converges to the steady state.

[17]Since k is independent of ϕ, α_1 is also independent of ϕ, but $\alpha_2(\phi)$ is dependent of ϕ.

[18]Our specification implies that for given $\phi \in \{0,1\}$ with $c_0 > \bar{c}(\phi)$, the marginal cost of the eco-product

any variable is interior in the equilibrium path in the sense that the production and the consumption of the eco-product and the conventional product are always positive. Figure 1 indicates the general direction of movement that (c, p) would take from any location. c is momentarily stationary along a path as the $\dot{c} = 0$ locus is crossed and k is stationary as the $\dot{k} = 0$ locus is crossed.

4.1 Steady State as Long-Run Outcome

This subsection characterizes the steady state as a long-run outcome. For any ϕ, by applying the Cramer's Rule to the system (4) with $\dot{p} = \dot{c} = 0$, we obtain the price and the marginal cost of the eco-product in the steady state, $(\bar{c}(\phi), \bar{p}(\phi))$, which derive the corresponding outputs of the eco-product and the conventional product, consumer surplus, the profit for the single producer, and negative externalities in the steady state: $\bar{x}(\phi) \equiv x^*(\bar{p}(\phi), \phi)$, $\bar{y}(\phi) \equiv y^*(\bar{p}(\phi), \phi)$, $\bar{S}(\phi) \equiv S(\bar{p}(\phi), \phi)$, $\bar{\pi}(\phi) \equiv \pi(\bar{p}(\phi), \phi, \bar{c}(\phi))$, and $\bar{E}(\phi) \equiv E(\bar{p}(\phi), \phi)$. Notice that all instantaneous variables above are a function of the government's binary choice variable ϕ. By the state equation (2), it must hold that $\bar{x}(\phi) = \sigma(\bar{c}(\phi))$ in the steady state, which yields:

$$\sigma(\bar{c}(\phi)) = \frac{(r + \lambda\delta)[1 - \bar{c}(\phi) - \gamma(1 - \tau(\phi))]}{2(r + \lambda\delta)(1 - \gamma^2) - \lambda}. \tag{6}$$

The condition $\bar{x}(\phi) = \sigma(\bar{c}(\phi))$ states that the output of the eco-product is inversely related to the level of knowledge depreciation in the steady state. If a large degree of knowledge depreciation is associated with the eco-product production, the single supplier would keep a higher level of the production to maintain the knowledge related to the production process.

Notice that in this study, the dynamic learning effect is only the source of technological

is monotone decreasing over time. Thus, the output of the eco-product and consumer surplus are monotone increasing over time, while the price of the eco-product, the output of the conventional product and the negative externatilities are monotone decreasing over time. In an infinite horizon autonomous problem with just one state variable, if there is an optimal path to a steady state, the state variable is monotonic over time, and the steady state must be a saddlepoint. This is illustrated in Figure 1.

progress, and the degree of dynamic learning effect, λ, influences the steady state through the state equation $\dot{c} = -\lambda[x^*(p,\phi) - \sigma(c)]$. Concerning the effect of a change in λ on the steady state, our specification implies that for any ϕ, if $\lambda > 0$ and $\gamma < \hat{\gamma}$, in the steady state, an increase in λ reduces the marginal cost and the price of the eco-product; raises the output of the eco-product; reduces the output of the conventional product and the negative externalities; and increases consumer surplus (see Claim 4 in the Appendix). These results imply that the larger learning effect induces a decline in the price of the eco-product through technological progress. This encourages people to shift the demand from the conventional product to the eco-product, which reduces environmental damages and improves consumer surplus in the long-run.

4.2 Environmental Regulation

We now examine the effect of the environmental regulation on the steady state as a long-run outcome. Solving equation (6) yields the marginal cost of the eco-product in the steady state, $\bar{c}(\phi)$. The impact of the regulation on the marginal cost of the eco-product in the steady state is described by:

$$\bar{c}(0) - \bar{c}(1) = \frac{\gamma}{2\delta(\hat{\gamma}^2 - \gamma^2)}(\tau(1) - \tau(0)) > 0. \tag{7}$$

Then, we deduce the following results related to production technology of the eco-product (see the Appendix for the proof).

Proposition 3 (Environmental Regulation and Marginal Cost in Steady State) *Suppose $\lambda > 0$ and $\gamma < \hat{\gamma}$. Then, the environmental regulation reduces the marginal cost of the eco-product in the steady state, i.e., $\bar{c}(0) > \bar{c}(1)$. Furthermore, the impact of the regulation on the marginal cost, $\bar{c}(0) - \bar{c}(1)$, is increasing in the degree of dynamic learning effect, λ.*

This proposition consists of two important implications. First, the regulation would trigger the firm's innovation (in this case, not product innovation but process innovation) on the eco-product production technology. Second, the impact of the regulation is more significant when the technology entails a larger learning effect. These are due to the fact that the regulation encourages the demand to shift toward the eco-product, which enhances the learning effect more effectively. This discussion may be similar to the one in the infant industry protection arguments, where the government supports targeted infant industries for future returns at the expense of the current distortion.

Based on Proposition 3 with equation (7), we examine the impact of the regulation on the price of the eco-product in the long-run, which is written by:

$$\bar{p}(1) - \bar{p}(0) = [p_M(1, c_0) - p_M(0, c_0)] - \frac{1}{2}\left[1 + \frac{\lambda\delta}{r + \lambda\delta}\right](\bar{c}(0) - \bar{c}(1)). \tag{8}$$

This impact can be divided into two sub-effects. The first sub-effect, $p_M(1, c_0) - p_M(0, c_0) = \frac{\gamma}{2}(\tau(1) - \tau(0)) > 0$, may be regarded as the 'substitution' effect in the sense that this term is associated with a rise in the marginal cost of the conventional product under the regulation. This induces a rise in the price-cost margin of the eco-product through the demand shift from the conventional product toward the eco-product. This sub-effect is consistent with the effect of the regulation in the static setting in Proposition 1. The second sub-effect, $\frac{1}{2}[1 + \frac{\lambda\delta}{r+\lambda\delta}](\bar{c}(0) - \bar{c}(1)) > 0$, may be considered as the 'cost-reduction' effect of the regulation in the sense that this term is associated with a decline in the marginal cost of the eco-product, which is caused by enhancing learning effect through the regulation, as in Proposition 3.

The impact of the regulation on the price of the eco-product is ambiguous since an increase in the demand for the eco-product associated with a rise in the price of the conventional product ambiguously affects the optimal price of the eco-product for the single producer. In fact, which sub-effect dominates the other determines the direction of the impact. Proposition 3 showed that a large degree of learning effect, λ, makes the regulation more effective in the sense that $\bar{c}(0) - \bar{c}(1)$ is increasing in λ. Thus, if learning effect is large enough so that the cost-reduction effect dominates the substitution effect, the regulation reduces the price of the eco-product. This result is in contrast to the result that the regulation always raises the price of the eco-product in the static setting. The crucial difference between the static and the dynamic settings is that the cost-reduction effect associated with learning effect exists in the dynamic setting while it does not exist in the static setting.

Concerning how the regulation affects the promotion of the eco-product, we consider the impact of the regulation on the outputs of the eco-product and the conventional product in the steady state. Use equations (1) and (8), we obtain:

$$\bar{x}(1) - \bar{x}(0) = [x_M(1, c_0) - x_M(0, c_0)] + \frac{1}{2(1-\gamma^2)}\left[1 + \frac{\lambda\delta}{r + \lambda\delta}\right](\bar{c}(0) - \bar{c}(1)) > 0; \qquad (9)$$

$$\bar{y}(1) - \bar{y}(0) = [y_M(1, c_0) - y_M(0, c_0)] - \frac{\gamma}{2(1-\gamma^2)}\left[1 + \frac{\lambda\delta}{r + \lambda\delta}\right](\bar{c}(0) - \bar{c}(1)) < 0, \qquad (10)$$

where $x_M(1, c_0) - x_M(0, c_0) = \frac{\gamma(\tau(1)-\tau(0))}{2(1-\gamma^2)} > 0$ and $y_M(1, c_0) - y_M(0, c_0) = -\frac{(2-\gamma^2)(\tau(1)-\tau(0))}{2(1-\gamma^2)} < 0$. The regulation promotes the eco-product but reduces the output of the conventional product, which in turn reduces the marginal cost of the eco-product through learning effect. This continued mechanism further strengthens the promotion of the eco-product, i.e., $\bar{x}(1) > \bar{x}(0)$ and $\bar{y}(1) < \bar{y}(0)$. Similar to the previous analysis, the impact can be divided into the substitution and the cost-reduction effects. The cost-reduction effect associated with learning effect amplifies the impact compared to the static case in Proposition 1. Furthermore, a rise in λ intensifies the impact of the regulation on the marginal cost of the eco-product (as shown in Proposition 3), which in turn increases the impacts of the regulation on the outputs of the eco-product and the conventional product in equations (9) and (10).

We now discuss the relation between the regulation and the long-run effect on social welfare. As in the static case, the impact of the regulation on social welfare in the steady state can be described by the value of the regulation in the steady state, which is written by $\Delta\bar{T} = \Delta\bar{S} + \Delta\bar{\pi} + \Gamma(\varepsilon\bar{y}(0))$, where $\Delta\bar{S} \equiv \bar{S}(1) - \bar{S}(0)$ and $\Delta\bar{\pi} \equiv \bar{\pi}(1) - \bar{\pi}(0)$ represents the impact of the regulation on consumer surplus and the profit for the single producer in the steady state, respectively. If the value of the regulation is positive, adopting the regulation improves social welfare in the steady state. In contrast, if the value is negative, the regulation reduces social welfare in the steady state. The expression of the value implies that the value can be divided into the three sub-values: the values related to the profit for the single producer, consumer surplus, and negative externalities.

First, the value of the regulation related to the profit is described by $\Delta\bar{\pi} = \bar{x}(1)\bar{m}(1) - \bar{x}(0)\bar{m}(0)$, where $\bar{m}(\phi) \equiv \bar{p}(\phi) - \bar{c}(\phi)$ is the price-cost margin under the government policy ϕ. By equations (7) and (8), the impact of the regulation on the price-cost margin is:

$$\bar{m}(1) - \bar{m}(0) = [p_M(1, c_0) - p_M(0, c_0)] + \frac{\gamma}{2}\left[1 + \frac{1-\delta(1-\gamma^2)}{\delta(\hat{\gamma}^2 - \gamma^2)}\right](\tau(1) - \tau(0)). \tag{11}$$

Assuming that $1 - 1/\delta < \gamma^2 < \hat{\gamma}^2$, the impact of the regulation on the price-cost margin is positive and increasing in λ. Noticing that the impact of the regulation on the output of the eco-product, $\bar{x}(1) - \bar{x}(0)$, is positive and increasing in λ by the previous discussion, the value of the regulation related to the profit, $\Delta\bar{\pi}$, is also positive and increasing in λ. The regulation enhances the monopolistic power by raising the cost of the conventional product, and this tendency is strengthened as the learning effect is more significant.

Second, concerning the value of the regulation related to consumer surplus in the steady state, whether or not the regulation increases consumer surplus is in general ambiguous, i.e., the sign of $\Delta\bar{S}$ is ambiguous. This is due to the fact that the increased price of the conventional product has a negative impact on consumer surplus, while the reduced price of the eco-product due to the learning effect has a positive impact. Furthermore, differentiating $\Delta\bar{S}$ with respect to λ yields:

$$\frac{\partial(\Delta\bar{S})}{\partial\lambda} = \frac{r\delta(\hat{\gamma}^2 - \gamma^2)}{2(r + \lambda\delta)^2(1-\gamma)^2}[\bar{x}(1)^2 - \bar{x}(0)^2] > 0,$$

which implies that $\Delta\bar{S}$ is increasing in λ. A larger degree of dynamic learning effect intensifies the reduction of the price of the eco-product, which makes the regulation more attractive from the standpoint of consumer surplus. This suggests the possibility that the regulation could reduce consumer surplus in the steady state when λ is small, while it could increase consumer surplus when λ is large.

Third, the value of the regulation related to environmental damages in the steady state, $\Gamma(\varepsilon\bar{y}(0))$, is always positive, but is decreasing in λ. A large λ intensifies the demand shift from the conventional product toward the eco-product under non-regulation. This in turn reduces environmental damages associated with pollution from the conventional product $\bar{y}(0)$. That is, if the learning effect is more significant, the regulation becomes less attractive from the perspective of environmental issues in the steady state.

The above discussions of the three sub-values show that the entire value of the regulation in the steady state, $\Delta\bar{T}$, is closely dependent on λ. In general, the relationship between the value of the regulation and the degree of dynamic learning effect is complex. In Figure 2, the graph of the value of the regulation related to environmental damages, $\Gamma(\varepsilon\bar{y}(0))$, is described by the thick curve that is decreasing in λ, and the graph of the value related to total surplus (defined as consumer surplus plus the profit for the single producer), $\Delta\bar{S}+\Delta\bar{\pi}$, by the dotted curve that is increasing in λ. For the better understanding of some insights of the dynamics different from those of the static setting, Figure 2 illustrates an interesting, plausible case where the graph of the value of regulation in the steady state (vertical sum of the thick and dotted curves) is described by the convex and U-shaped curve. The sufficient condition for this is that environmental damages are relatively sensitive to the output of the conventional product such that the convexity of Γ is large enough. Moreover, in Figure 2, it is assumed that a larger learning effect causes the regulation to improve consumer surplus in the steady state such that $\Delta\bar{T} > 0$ with λ large enough, and that a small learning effect causes environmental damages under non-regulation to be significantly large due to small demand shift from the conventional product such that $\Delta\bar{T} > 0$ with λ small enough.

A distinguished feature in this plausible case is that the graph of the value of the regulation, $\Delta\bar{T}$, determines the region of λ in which the regulation improves or reduces social welfare in the steady state. In this case, a region of $\lambda > 0$ can be divided into the following three sub-regions: $\Delta\bar{T} > 0$ in the sub-region $\lambda < \lambda_1$ and in the sub-region $\lambda > \lambda_2$; and $\Delta\bar{T} < 0$ in the intermediate sub-region $\lambda \in (\lambda_1, \lambda_2)$. Focusing on this situation, we summarize the above results:

Proposition 4 (Environmental Regulation and Steady State) *Suppose* $\lambda > 0$ *and* $1 - 1/\delta < \gamma < \hat{\gamma}$*. Then, there may exist a unique pair of values,* λ_1 *and* λ_2 *with* $0 < \lambda_1 < \lambda_2$*, such that the environmental regulation reduces social welfare in the steady state if* $\lambda \in (\lambda_1, \lambda_2)$*, while it improves social welfare in the steady state if* $\lambda < \lambda_1$ *or* $\lambda > \lambda_2$*.*

To understand the intuition behind this, suppose first that λ is small enough so that $\lambda < \lambda_1$. In this case, learning effect is so small that the demand shift toward the eco-product is not so large, and thus environmental damages in the steady state is still relatively large under non-regulation, i.e., the value of the regulation related to environmental damages is high. Even though the regulation reduces total surplus (consumer surplus plus the profit for the single producer) in the steady state, or the value of the regulation related to total surplus is negative, the regulation could improve social welfare in the steady state.

Suppose next that λ is large enough so that $\lambda > \lambda_2$. In this case, learning effect causes the regulation to increase total surplus through a decline in the price associated with a significant cost reduction of the eco-product, i.e., the value of the regulation related to total surplus is positive. In addition, the value of the regulation related to environmental damages is still positive even though large learning effect makes environmental damages in the steady state to be relatively small under non-regulation. As a result, the regulation improves social welfare in the steady state.

It should be important to understand the difference in the reason why the regulation improves social welfare in the steady state between the above two polar cases. When λ is large, the regulation improves social welfare due to its role in increasing total surplus (consumer surplus plus the profit for the single producer) through promoting technological progress, not in vanishing environmental damages. In contrast, when λ is small, the regulation improves social welfare due to its role in vanishing environmental damages, not in promoting technological progress.

More importantly, Proposition 4 shows an intermediate case of $\lambda \in (\lambda_1, \lambda_2)$, where the value of the regulation in the steady state is negative. In contrast to the two polar cases mentioned above, there may be a range of λ in which the regulation clearly has a negative impact on social welfare in the long-run. In this case, the value of the regulation related to total surplus is negative due to the reduction in consumer surplus caused by the regulation. This negative effect of the regulation cannot be offset by the positive value of the regulation related to environmental damages.

Note that Proposition 4 examined the impact of the regulation only on the long-run outcome in the steady state, but did not say anything about the government's optimal policy in a dynamic setting. The optimal decision rule of the regulation depends on whether the current value of social welfare, which must be evaluated over all the path, is increased or decreased by the regulation. In other words, it depends on whether the current value of the regulation over time is positive or negative. In a later section, it will be shown that the non-monotone U-shaped property in the value of the regulation at the steady state implies the non-monotone U-shape in the current value of the regulation.

Given this U-shaped property in the current value of the regulation under dynamic settings, we can deduce some important policy implications by making comparisons of the static and dynamic cases with the government decision. For example, suppose that an initial marginal cost c_0 is larger than c_M, i.e., $c_0 > c_M$. In the absence of learning, the situation simply becomes static and the optimal decision of the government is to adopt the regulation, as shown in Proposition 2. However, the above U-shaped feature in the dynamic setting implies that the optimal decision could be reversed, especially when the degree of dynamic learning effect is in some intermediate range. Therefore, the optimal decision of the regulation in the dynamic case can be in stark contrast to that of the static case. Unfortunately, the current value of the regulation is too complex to be analytically characterized. Numerical analysis will be employed to illustrate the U-shaped property of the current value of the regulation and to elaborate on various intuitions in a dynamic setting for the plausible range of parameters. For this purpose, we next introduce the concept of the current value of the regulation and explain the optimal decision of the government in a dynamic setting before we go to numerical analysis.

4.3 Government's Decision Problem

Until the previous subsection, we have examined an economy with dynamic learning effect in step 2, taking the government policy ϕ as given, and have analyzed the impact of the regulation mainly on the steady state outcomes. This subsection discusses the government's decision problem of whether or not to adopt the regulation at the initial period and formally introduce the current value of the regulation in step 1, which is the basis for numerical analysis in a later section.

The government's problem is to maximize the present value of social welfare over time, consisting of consumer surplus, the profit for the single producer and the negative externalities, taking into account the pricing decision rule made by the single producer of the eco-product. Since instantaneous social welfare is described by $\hat{T}(\phi, c) = \hat{S}(\phi, c) + \hat{\pi}(\phi, c) - \hat{E}(\phi, c)$ for any state variable c, the government's decision problem in time $t = 0$ can be restated as $\max_{\phi \in \{0,1\}} W(\phi, c_0) \equiv \int_0^\infty \hat{T}(\phi, c_t(c_0, \phi)) e^{-rt} dt$, where $c_t(c_0, \phi)$ is the equilibrium path, and $W(\phi, c_0)$ is the present value of social welfare, taking ϕ and c_0 as given.

By simple calculations, $W(\phi, c_0)$ can be rewritten by $W(\phi, c_0) = \hat{\alpha}_0(\phi) + \hat{\alpha}_1(\phi) c_0 + \hat{\alpha}_2(\phi) c_0^2$, where $\hat{\alpha}_l(\phi)$ is constant for $l \in \{0, 1, 2\}$ and $\phi \in \{0, 1\}$. Then, given c_0, the government's optimal decision is to adopt the regulation if $W(1, c_0) > W(0, c_0)$ or

$$\Delta W = W(1, c_0) - W(0, c_0) = \int_0^\infty [\hat{T}(1, c_t(c_0, 1)) - \hat{T}(0, c_t(c_0, 0))] e^{-rt} dt > 0, \tag{12}$$

and not to adopt the regulation if $\Delta W < 0$, where $\hat{T}(1, c_t(c_0, 1)) - \hat{T}(0, c_t(c_0, 0))$ is the difference in instantaneous social welfare between under the regulation and under non-regulation. The value of ΔW can be considered the current value of the regulation.

It should be noted that the present paper considers only a situation where there are two feasible choices by the government: to adopt the regulation and not to adopt it. In reality, however, there are more than two feasible choices. When the government has a finite or an infinite number of feasible policies, $\{\phi_n\}$, instead of two feasible choices, it can find the optimal policy ϕ_{n^*} such that $W(\phi_{n^*}, c_0) = \max_n W(\phi_n, c_0)$. However, for our primary purpose in discussing the relations between dynamic learning effect and the regulation, we believe that analyzing our simple setting with two feasible choices suffices as a simple approximation of the assessment of the government policy.

5 Numerical Analysis

In this section, we explore some numerical examples of both the static model and the dynamic model to illustrate the results and their intuitions. The focus is on how dynamic learning effect can affect the optimal decision of the government compared to that in a static case. We also show that the current value of the regulation (12) could possess a non-monotone U-shaped feature and elaborate on the connections with the U-shaped result in Proposition 4. For these purposes, we assume that instantaneous negative externalities are represented by $E(p, \phi) = \frac{\epsilon y^*(p,\phi)^2}{M - y^*(p,\phi)}$, where M sets the upper bound to which extent a society can accept the negative externalities generated from the conventional product. Throughout this section, we employ $M = 0.61$. For other parameters, we choose a coefficient of the negative externalities, $\varepsilon = 0.005$; the degree of substitution between the eco-product and the conventional product, $\gamma = 0.25$; the marginal cost of the conventional product, $\underline{\tau} = 0.4$; and the additional marginal cost of the conventional product under the regulation, $\xi = 0.1$.

Figure 3 shows the value of the regulation denoted by $\Delta T_M(c)$ as a function of the marginal cost, c, in our static model. The critical marginal cost, c_M, in Proposition 2 is determined by the point at which $\Delta T_M(c) = 0$. In this case, $c_M = 0.6816$. Thus the optimal policy of the government is to adopt the regulation if $c > c_M$ and not to adopt it otherwise.

The dynamic model needs additional parameters: $(\eta, \delta) = (0.90, 1.15)$ is set such that $\dot{c} = -\lambda[x^* - (0.90 - 1.15c)]$. In addition, we choose a discount factor of $r = 0.05$; the initial marginal cost of $c_0 = 0.77$; and three values of the degree of dynamic learning effect $\lambda = \{\lambda_H, \lambda_M, \lambda_L\} = \{2.00, 0.18, 0.01\}$. We consider six scenarios that are respectively represented by the notations of $\{(R, H), (NR, H), (R, M), (NR, M), (R, L), (NR, L)\}$, where R and NR represent 'Regulation' and 'Non-regulation,' respectively, while H, M and L correspond to the case of high learning effect λ_H, moderate λ_M, and low λ_L, respectively. For example, the notation (NR, H) corresponds to the outcome when the economy has high learning effect under non-regulation, keeping the other parameters fixed.

The above six scenarios are assumed for clarity and could be sufficient to exhaustively present our numerical illustration in a dynamic setting. A series of Figures 4 to 9 shows the six trajectories of the equilibrium paths, which respectively represent the six scenarios. Figures 4 to 9 respectively correspond to the marginal cost of the eco-product; the eco-product price; consumer surplus; the profit for the eco-product producers; the negative externalities; and social welfare, as shown in the captions.

We now characterize the impact of the regulation on social welfare along the equilibrium path converging to the steady state (Figure 9) by examining each of the two components of social welfare: total surplus (consumer surplus plus the profit for the single producer) and the negative externalities.[19] Concerning total (consumer) surplus as the first component, notice that the regulation triggers innovation of the eco-product technology so that it lowers the equilibrium path of the marginal cost (Figure 4). When the degree of dynamic learning effect is high, the regulation enhances learning so that it lowers the equilibrium path of the

[19]In our numerical examples, the profit for the single producer is relatively small compared to consumer surplus and can be considered negligible. Thus, the discussions on consumer surplus is almost equivalent to those on total surplus.

price of the eco-product (Figure 5). In this case, under the regulation, the negative impact on consumer surplus associated with the rise in the conventional product price would be recovered over time as the price of the eco-product falls over time (see the trajectories of $\{(NR, H), (R, H)\}$ in Figure 6). However, the regulation does not always cause total (consumer) surplus in later periods to go over the level achieved under non-regulation. Indeed, when the degree of dynamic learning effect is small, the regulation fails to make the pricing schedule sufficiently low and total (consumer) surplus in later periods cannot reach the level under non-regulation (see the trajectories of $\{(NR, M), (R, M)\}$ and $\{(NR, L), (R, L)\}$ in Figures 5 and 6).[20] Taking into account that the current value of the regulation over time is highly concerned with the dynamic trade-off between the steady-state outcome and the initial loss in early periods, the above discussions could imply that the current value of the regulation over time related to total (consumer) surplus is likely to be increasing in the degree of dynamic learning effect. This feature is the same as the one associated with total surplus in the steady state as proven in Proposition 4, however, it is again in sharp contrast with the static case.

Regarding the negative externalities as the second component, the current value of regulation over time related to the negative externalities would be decreasing in the degree of learning as illustrated in Figure 8. Since we assume that the regulation removes the negative externalities, only three trajectories of $\{(NR, L), (NR, M), (NR, H)\}$ are drawn in that figure. A higher degree of dynamic learning effect intensifies the promotion of the eco-products and thus reduces the negative externalities along all the equilibrium path under non-regulation. Thus, the current value of the regulation over time related to the negative externalities also becomes less as the degree of dynamic learning effect is larger.

[20] Figures 4 reveals that for any degree of dynamic learning effect, the equilibrium path of the marginal cost under the regulation, denoted by $(R, j), j = \{L, M, H\}$, is lower than that under non-regulation (NR, i) over all the equilibrium path. (Here it must be noted that although two trajectories of (NR, L) and (R, L) cannot be distinguished, (R, L) is lower than (NR, L) as in the other case). However, by observing the trajectories of (R, L) and (NR, L) in Figure 5, we notice that the equilibrium path of the eco-product price under the regulation may be higher than that under non-regulation if the degree of dynamic learning effect is small. In this case, the situation is very close to the static case, and thus the pricing schedule under the regulation may be higher.

We can summarize the impacts of the regulation on each component of social welfare at this point. There are potentially two different consequences on total (consumer) surplus and the negative externalities between the static and the dynamic models. First, as in the static model, the regulation in the dynamic model also has a potential adverse effect on consumer surplus especially in early periods for any degree of dynamic learning effect (see Figure 6). However, the crucial distinction between the static model and the dynamic model is that in the dynamic model, the regulation might offset such a negative impact with a potential future gain through a decline in the price of the eco-product when the degree of dynamic learning effect is sufficiently high. This implies that learning has a positive impact on the current value of the regulation. Second, as the degree of dynamic learning effect is larger, the negative externalities are endogenously reduced more effectively even without the regulation. Thus, learning has a negative impact on the current value of the regulation, in contrast to the first. However, such two dynamic impacts do not exist at all in the static model.

Combining the impacts of the regulation on the above two components of social welfare, we can conclude that the current value of the regulation over time related to the negative externalities works in the opposite direction with that related to total (consumer) surplus as the degree of dynamic learning effect changes. This situation is quite similar to that in the steady state outcome as shown in Proposition 4, where the value of the regulation related to total (consumer) surplus in the steady state is increasing and that related to environmental damages in the steady state is decreasing in the degree of dynamic learning effect. Thus, the U-shaped property could also be expected for the current value of the regulation.

Figure 10 illustrates the graph of the current value of the regulation as a function of the degree of dynamic learning effect with an initial marginal cost $c_0 = 0.77$ as given. It confirms that the current value of the regulation holds a non-monotone U-shaped feature, and that there may be a certain range of λ such that the current value of the regulation is negative. This result implies that the optimal decision of the regulation in a dynamic setting could be converse to that of a static setting. In our nemerical example, if the degree of dynamic learning effect is moderate, i.e., $\lambda_M = 0.18$, the current value of the regulation is negative and the optimal decision of the government is not to adopt the regulation. On the other hand, if there is no learning, the optimal decision is to adopt the regulation due to $c_0 = 0.77 > c_M = 0.6816$. Therefore, the optimal decision in fact becomes converse.

6 Conclusions

This paper has analyzed the role of the environmental regulation in a situation where an eco-product newly invented and introduced by a single producer into a market competes with a pre-existing product supplied under perfect competition due to their substitutability. Both a static model and a dynamic model have been examined analytically or numerically when a social planner has an option of tightening the environmental regulation to improve social welfare. An essential feature of our dynamic model is learning-by-doing technology in an eco-product planning. We characterize the situation under which the government should adopt the regulation and discussed how learning-by-doing affects the optimal decision of the government.

The research shows that in the static setting without learning effect, the regulation should be adopted if the marginal cost of the eco-product production is high enough, and otherwise it should not. On the other hand, in the dynamic model with learning effect, whether or not the regulation improves social welfare is highly dependent not only on the current marginal cost of the eco-product but also on the degree of dynamic learning effect. These bear some resemblance with the general result from Baumol and Oates (1988) that the environmental regulation should be less stringent under imperfect competition.

In particular, using numerical analysis, this study illustrates an interesting and plausible case where the regulation improves social welfare when learning effect is either small or large enough, while it reduces social welfare in an intermediate range. The current value of the regulation could possess a nonmonotone U-shaped feature with respect to learning effect.

Furthermore, the comparison of static and dynamic cases offers some insight on the optimal decision made by the central authority and suggests that the optimal decision in a static case could be in stark contrast to that in a dynamic case under certain conditions.

This provides important policy implications since the existence of learning potentials could significantly affect the optimal deicision as well as the resulting social welfare. The regulation enhances technological progress and partially plays the similar role of a rise in the degree of learning effect. However, it does not always guarantee that such technological progress induced by the regulation brings about favorable outcomes on social welfare. This implies that if the government does not consider learning in an eco-product planning, the regulation may yield unexpected loss in social welfare. Therefore, the government needs to carefully evaluate learning potentials when the regulation is determined.

There are several additional topics that were not addressed in this paper but can be examined in the future. First, one potential issue is how incentive schemes such as subsidy/tax imposed by the government affect the promotion of the eco-product as well as social welfare in an economy where the eco-product whose technology entails learning is differentiated from the conventional polluting product. Second, the role of research and development should also be examined in the similar situation since it has been widely considered another engine of technological progress in terms of not only process innovation but also product innovation. Third, more importantly, loosening some restrictions on the choice of the regulation placed in this paper would enable us to derive some important results. For example, instead of a binary choice of the regulation, analyzing a continuous standard could help us examine how the optimal standard depends on the degree of dynamic learning effect.

As another future research topic, we are especially interested in the case where the government can change the standard within his discretion. This seems to be somewhat concerned with the problems of time-consistency that has been actively addressed in the other field of economics. In this case, the dynamic strategic interaction between the government and the industry would be crucial in characterizing the equilibrium outcomes. These caveats notwithstanding, we believe that the basic structure of the model introduced in this paper would be a first step towards exploring some fuller models to addressing further issues as listed above. We are hopeful that the results of our research clarify the potential effects of tightening environmental regulation so as to promote eco-products in the presence of learning effect, and stimulate the further research questions.

7 Appendix

For some preliminary purposes, we first derive some important variables in the case of static pricing of the eco-product. Using (3), we obtain $x_M(\phi,c) = \frac{1-c-\gamma(1-\tau(\phi))}{2(1-\gamma^2)} > 0$, $y_M(\phi,c) = \frac{(1-\tau(\phi))(2-\gamma^2)-\gamma(1-c)}{2(1-\gamma^2)} > 0$, and $\pi_M(\phi,c) = \frac{[1-c-\gamma(1-\tau(\phi))]^2}{4(1-\gamma^2)}$. Moreover, using (3), we obtain $E_M(\phi,c) = \Gamma(\varepsilon y_M(0,c))$ if $\phi = 0$ and $E_M(\phi,c) = 0$ if $\phi = 1$.

Claim 1 *For any $\phi \in \{0,1\}$, technological progress in the production of the eco-product* (1) *decreases the optimal eco-product price;* (2) *increases the output of the eco-product;* (3) *decreases the output of the conventional product and the negative externalities; and* (4) *increases consumer surplus as well as the profit for the single producer.*

Proof of Claim 1 Differentiating p_M, x_M and y_M with respect to c yields $\frac{\partial p_M}{\partial c} = \frac{1}{2} > 0$; $\frac{\partial x_M}{\partial c} = -\frac{1}{2(1-\gamma^2)} < 0$; and $\frac{\partial y_M}{\partial c} = \frac{\gamma}{2(1-\gamma^2)} > 0$, which are the desired results in (1), (2) and (3). Noticing that $\frac{\partial S}{\partial p} = (u_x - p)\frac{\partial x^*}{\partial p} + (u_y - \tau(\phi))\frac{\partial y^*}{\partial p} - x^* = -x^* < 0$ and that $x_M(\phi,c) > 0$ or $1-c-\gamma(1-\tau(\phi)) > 0$, we obtain $\frac{\partial S_M}{\partial c} = \frac{\partial S}{\partial p}\frac{\partial p_M}{\partial c} = -x_M(\phi,c)\frac{\partial p_M}{\partial c} < 0$ and $\frac{\partial \pi_M}{\partial c} = -\frac{1-c-\gamma(1-\tau(\phi))}{2(1-\gamma^2)} < 0$, which are the desired results in (4). □

Proof of Proposition 1 Let $\tilde{p}(\xi,c) \equiv p_M(1,c:\xi)$, $\tilde{x}(\xi,c) \equiv x_M(1,c:\xi)$, $\tilde{y}(\xi,c) \equiv y_M(1,c:\xi)$, $\tilde{S}(\xi,c) \equiv S_M(1,c:\xi)$, and $\tilde{\pi}(\xi,c) \equiv \pi_M(1,c:\xi)$. Notice that $p_M(0,c) = \tilde{p}(0,c)$, $x_M(0,c) = \tilde{x}(0,c)$, $y_M(0,c) = \tilde{y}(0,c)$, $S_M(0,c) = \tilde{S}(0,c)$, and $\pi_M(0,c) = \tilde{\pi}(0,c)$. Since $\frac{\partial \tilde{p}}{\partial \xi} = \frac{\gamma}{2} > 0$, it must hold that $p_M(0,c) = \tilde{p}(0,c) < \tilde{p}(\xi,c) = p_M(1,c:\xi)$ for any $\xi > 0$, which is the desired result in (1). Similarly, since $\frac{\partial \tilde{x}}{\partial \xi} = \frac{\gamma}{2(1-\gamma^2)} > 0$ and $\frac{\partial \tilde{y}}{\partial \xi} = -\frac{2-\gamma^2}{2(1-\gamma^2)} < 0$, it must hold that $x_M(0,c) = \tilde{x}(0,c) < \tilde{x}(\xi,c) = x_M(1,c:\xi)$ and $y_M(0,c) = \tilde{y}(0,c) > \tilde{y}(\xi,c) = y_M(1,c:\xi)$ for any $\xi > 0$, which are the desired results in (2). Moreover, notice that $\frac{\partial \tilde{S}}{\partial \xi} = [u_x - \tilde{p}]\frac{\partial \tilde{x}}{\partial \xi} + [u_y - (\underline{\tau} + \xi)]\frac{\partial \tilde{y}}{\partial \xi} - \tilde{x}\frac{\partial \tilde{p}}{\partial \xi} - \tilde{y} = -\tilde{x}\frac{\partial \tilde{p}}{\partial \xi} - \tilde{y} < 0$. Thus, it must hold that $S_M(0,c) = \tilde{S}(0,c) > \tilde{S}(\xi,c) = S_M(1,c:\xi)$ for any $\xi > 0$, which is the desired result in the first part of (3). Since $\frac{\partial \tilde{\pi}}{\partial \xi} = \frac{\gamma[1-c-\gamma(1-\underline{\tau}-\xi)]}{2(1-\gamma^2)} > 0$, it must hold that $\pi_M(0,c) = \tilde{\pi}(0,c) < \tilde{\pi}(\xi,c) = \pi_M(1,c:\xi)$ for any $\xi > 0$, which is the desired result in the second part of (3). □

Proof of Proposition 2 Note $\Delta T_M(c) = T(p_M(1,c),1,c) - T(p_M(0,c),0,c) = \Gamma(\varepsilon y_M(0,c)) + \Phi(\xi,c)$, where $\Gamma(\varepsilon y_M(0,c)) = \Gamma(\varepsilon\frac{(1-\underline{\tau})(2-\gamma^2)-\gamma(1-c)}{2(1-\gamma^2)})$ and $\Phi(\xi,c) = (S_M(1,c) + \pi_M(1,c)) - (S_M(0,c) + \pi_M(0,c))$. For any ε, $\Gamma(\varepsilon y_M(0,c))$ is increasing and convex in c since $\frac{\partial \Gamma}{\partial c} =$

$\Gamma'(\varepsilon y_M)\varepsilon\frac{\partial y_M}{\partial c} > 0$ and $\frac{\partial^2 \Gamma}{\partial c^2} = \varepsilon[\Gamma''(\varepsilon y_M)\varepsilon\left(\frac{\partial y_M}{\partial c}\right)^2 + \Gamma'(\varepsilon y_M)\frac{\partial^2 y_M}{\partial c^2}] > 0$. Moreover, $\Phi(\xi, c)$ is strictly decreasing in c with linearity since $\frac{\partial \Phi(\xi,c)}{\partial c} = -\frac{3\gamma\xi}{4(1-\gamma^2)} < 0$. Taking into account that $\Delta T_M(0) < 0$ and $\Delta T_M(1) > 0$ and that Γ is increasing and convex, it must hold that there exists a unique critical value $c_M > 0$ such that $\Phi(\xi, c) > -\Gamma(\varepsilon y_M(0, c))$ or $T(p_M(1, c), 1, c) > T(p_M(0, c), 0, c)$ if $c > c_M$ and $\Phi(\xi, c) < -\Gamma(\varepsilon y_M(0, c))$ or $T(p_M(1, c), 1, c) < T(p_M(0, c), 0, c)$ if $c < c_M$. $\square$

Claim 2 *The critical value c_M is decreasing in the pollution level per unit of the output of the conventional product, ε. Furthermore, the critical value c_M is increasing in the effect of the regulation on the marginal cost of the conventional product, ξ.*

Proof of Claim 2 Notice that $\Delta T_M(c) = \Phi(\xi, c) + \Gamma(\varepsilon y_M(0, c)) = 0$ is increasing in c in the neighborhood of $c = c_M(\xi, \varepsilon)$. This implies that at $c = c_M(\xi, \varepsilon)$, $\Gamma'(\varepsilon y_M)\varepsilon\frac{\partial y_M}{\partial c} + \frac{\partial \Phi}{\partial c} > 0$. Differentiating $\Delta T_M(c_M(\xi, \varepsilon)) = \Phi(\xi, c_M(\xi, \varepsilon)) + \Gamma(\varepsilon y_M(0, c_M(\xi, \varepsilon))) = 0$ with respect to ε yields $\frac{\partial c_M(\xi,\varepsilon)}{\partial \varepsilon} = -\Gamma'(\varepsilon y_M)y_M[\Gamma'(\varepsilon y_M)\varepsilon\frac{\partial y_M}{\partial c} + \frac{\partial \Phi}{\partial c}]^{-1} < 0$ since $\Gamma'(\varepsilon y_M)\varepsilon\frac{\partial y_M}{\partial c} + \frac{\partial \Phi}{\partial c} > 0$ and $\Gamma' > 0$. Thus, $c_M(\xi, \varepsilon)$ is decreasing in ε. Furthermore, differentiating $\Delta T_M(c_M(\xi, \varepsilon)) = 0$ with respect to ξ yields that $\frac{\partial c_M(\xi,\varepsilon)}{\partial \xi} = -\frac{\partial \Phi}{\partial \xi}\left[\Gamma'(\varepsilon y_M)\varepsilon\frac{\partial y_M}{\partial c} + \frac{\partial \Phi}{\partial c}\right]^{-1} > 0 \Leftrightarrow \frac{\partial \Phi}{\partial \xi} < 0$. Thus, $c_M(\xi, \varepsilon)$ is increasing in ξ. $\square$

Claim 3 *For any $\phi \in \{0, 1\}$, there exists a unique steady state $(\bar{c}(\phi), \bar{p}(\phi))$. Furthermore, the unique steady state is stable if the degree of product differentiation between the eco-product and the conventional product is small enough so that $det(M) < 0$ or $\gamma < \hat{\gamma}$, where $\hat{\gamma} \equiv (1 - \frac{r+2\lambda\delta}{2\delta(r+\lambda\delta)})^{1/2}$.*

Proof of Claim 3 Since the system (4) is a linear differential system, the desired result in the first part is directly obtained. The proof of the second part follows Kamien and Schwartz (2012). The characteristic equation associated with (4) is $k^2 - rk - \lambda\delta(r + \lambda\delta) + \frac{(r+2\lambda\delta)\lambda}{2(1-\gamma^2)} = 0$ with roots $k_1, k_2 = \frac{r}{2} \pm \frac{1}{2}[(r+2\lambda\delta)^2 - \frac{2\lambda(r+2\lambda\delta)}{1-\gamma^2}]^{1/2} \equiv \frac{r}{2} \pm \frac{1}{2}D^{1/2}$. Then, the solution has the form of $c(t) = Ae^{k_1 t} + Be^{k_2 t}$ for k_1, k_2 real and distinct, or $c(t) = (A + Bt)e^{rt/2}$ if $k_1 = k_2 = r/2$, or $c(t) = e^{rt/2}(A\cos pt + B\sin pt)$, where $p = (-D)^{1/2}$, if p is real. If the roots are real, the larger root must be positive. The smaller root may be either positive or negative. It will be negative if $r < D^{1/2}$; that is if $\delta(r + \lambda\delta) > \frac{r+2\lambda\delta}{2(1-\gamma^2)}$ or $\gamma^2 < 1 - \frac{r+2\lambda\delta}{2\delta(r+\lambda\delta)} \equiv \hat{\gamma}^2$, the roots are real and of opposite signs. Let $k_1 > 0 > k_2$ if $\gamma^2 < \hat{\gamma}^2$ holds. Then, $c(t)$ will converge to zero provided we take $A = 0$. Thus, if $\gamma^2 < \hat{\gamma}^2$ holds, then the roots must be real and the steady state must satisfy the conditions to be a saddlepoint. On the other hand, the roots will be real and nonnegative if $D \geq 0$ and $\gamma^2 < \hat{\gamma}^2$ fails. As long as the roots are both nonnegative, $c(t)$ cannot converge to the steady state. It will move away from it, unless the initial position happens to be the steady state. Finally, the roots will be complex if $D < 0$. Note that since the real part of the complex roots is positive ($= r/2$), the path moves away from the steady state so that the steady state is unstable. In sum, a solution to the system (4) can converge only if $\gamma^2 < \hat{\gamma}^2$ holds. In other cases, all paths diverge. $\square$

Claim 4 *For any ϕ, if $\lambda > 0$ and $\gamma < \hat{\gamma}$, in the steady state, an increase in λ* (1) *reduces the marginal cost and the price of the eco-product;* (2) *raises the output of the eco-product;* (3) *reduces the output of the conventional product and the negative externalities; and* (4) *increases consumer surplus.*

Proof of Claim 4 By equation (6), we obtain $x_M(\phi, \bar{c}(\phi)) = \sigma(\bar{c}(\phi))[1 - \frac{\lambda}{2(r+\lambda\delta)(1-\gamma^2)}]$. We first show $\frac{\partial \bar{c}}{\partial \lambda} < 0$. Differentiating this with respect to λ yields $\frac{\partial \bar{c}(\phi)}{\partial \lambda} = -\frac{r\sigma(\bar{c})}{2(r+\lambda\delta)^2(1-\gamma^2)}[\frac{\partial x_M}{\partial c} - \frac{\partial \sigma}{\partial c}(1 - \frac{\lambda}{2(r+\lambda\delta)(1-\gamma^2)})]^{-1}$. Notice that since $\gamma < \hat{\gamma}$, we obtain $\frac{\partial x_M}{\partial c} - \frac{\partial \sigma}{\partial c}(1 - \frac{\lambda}{2(r+\lambda\delta)(1-\gamma^2)}) = -\frac{\delta}{1-\gamma^2}[\gamma^2 - (1 - \frac{r+2\lambda\delta}{2\delta(r+\lambda\delta)})] > 0$. Thus, we obtain $\frac{\partial \bar{c}}{\partial \lambda} < 0$, which is the desired result in the first part of (1). Note that $\bar{p}(\phi) = p_M(\phi, \bar{c}(\phi)) - \frac{\lambda\sigma(\bar{c}(\phi))}{2(r+\lambda\delta)}$. Differentiating this with respect to λ yields $\frac{\partial \bar{p}(\phi)}{\partial \lambda} = \frac{\partial p_M}{\partial c}\frac{\partial \bar{c}}{\partial \lambda} - \frac{1}{2}[\frac{r\sigma(\bar{c}(\phi))}{(r+\lambda\delta)^2} - \frac{\lambda\delta}{r+\lambda\delta}\frac{\partial \bar{c}}{\partial \lambda}]$. Since $\frac{\partial p_M}{\partial c} > 0$ and $\frac{\partial \bar{c}}{\partial \lambda} < 0$, it must hold that $\frac{\partial \bar{p}(\phi)}{\partial \lambda} < 0$, which is the desired result in the second part of (1). We then show that $\frac{\partial \bar{x}}{\partial \lambda} > 0$ and $\frac{\partial \bar{y}}{\partial \lambda} < 0$, which are the desired results in (2) and (3). By equation (1), the outputs of the eco-product and the conventional product in the steady state are given by $\bar{x}(\phi) = x^*(\bar{p}(\phi), \phi)$ and $\bar{y}(\phi) = y^*(\bar{p}(\phi), \phi)$, which implies that $\frac{\partial \bar{x}(\phi)}{\partial \lambda} = \frac{\partial x^*}{\partial p}\frac{\partial \bar{p}(\phi)}{\partial \lambda} = -\frac{1}{1-\gamma^2}\frac{\partial \bar{p}(\phi)}{\partial \lambda} > 0$ and $\frac{\partial \bar{y}(\phi)}{\partial \lambda} = \frac{\partial y^*}{\partial p}\frac{\partial \bar{p}(\phi)}{\partial \lambda} = \frac{\gamma}{1-\gamma^2}\frac{\partial \bar{p}(\phi)}{\partial \lambda} < 0$. We next show that $\frac{\partial \bar{S}(\phi)}{\partial \lambda} > 0$, which is the desired result in (4). Using $\bar{S}(\phi) = S(\bar{p}(\phi), \phi)$, $\frac{\partial \bar{p}}{\partial \lambda} < 0$ and $\frac{\partial S}{\partial p} = -\bar{x}(\phi)$, we obtain $\frac{\partial \bar{S}(\phi)}{\partial \lambda} = \frac{\partial S}{\partial p}\frac{\partial \bar{p}}{\partial \lambda} = -\bar{x}(\phi)\frac{\partial \bar{p}}{\partial \lambda} > 0$. □

Proof of Proposition 3 Since $\hat{\gamma} > \gamma$, it must hold that $\Delta\bar{c} \equiv \bar{c}(0) - \bar{c}(1) > 0$, which is the desired result of the first part. Moreover, since $\frac{\partial \hat{\gamma}}{\partial \lambda} < 0$, it must hold that $\frac{\partial(\Delta\bar{c})}{\partial \lambda} > 0$, which is the desired result of the second part. □

References

Ambec, S., Cohen, M. A., Elgie, S., and Lanoie, P. (2013). The Porter hypothesis at 20: Can environmental regulation enhance innovation and competitiveness? *Review of environmental economics and policy*, 7:2–22.

Argote, L., Beckman, S. L., and Epple, D. (1990). The persistence and transfer of learning in industrial settings. *Management science*, 36(2):140–154.

Argote, L. and Epple, D. (1990). Learning curves in manufacturing. *Science*, 247(4945):920–924.

Arora, S. and Gangopadhyay, S. (1995). Toward a theoretical model of voluntary overcompliance. *Journal of environmental economics and management*, 28:289–309.

Arrow, K. J. (1962). The economic implications of learning by doing. *Review of economic studies*, 29:155–173.

Bansal, S. and Gangopadhyay, S. (2003). Tax/subsidy policies in the presence of environmentally aware consumers. *Journal of environmental economics and management*, 45:333–355.

Baumol, W. J. and Oates, W. E. (1988). *The theory of environmental policy*. Cambridge university press, 2 edition.

Bellas, A. S. (1998). Empirical evidence of advances in scrubber technology. *Resource and energy economics*, 20:327–343.

Benkard, C. (2000). Learning and forgetting: The dynamics of aircraft production. *American economic review*, 90(4):1034–1054.

Besanko, D., Doraszelski, U., and Kryukov, Y. (2014). The economics of predation: What drives pricing when there is learning-by-doing. *American economic review*, 104:868–897.

Conrad, K. (2005). Price competition and product differentiation when consumers care for the environment. *Environmental and resource economics*, 31:1–19.

Cremer, H. and Thisse, J. (1999). On the taxation of polluting products in a differentiated industry. *European economic review*, 43:575–594.

Cropper, M. L. and Oates, W. E. (1999). Environmental economics. In Stavins, R. N., editor, *Economics of the environment, selected readings*. W. W. Norton & Company, Inc.

Eriksson, C. (2004). Can green consumerism replace environmental regulation?—A differentiated-products example. *Resource and energy economics*, 26:281–293.

Fudenberg, D. and Tirole, J. (1983). Learning-by-doing and market performance. *Bell journal of economics*, 14:522–530.

Goulder, L. H. and Mathai, K. (2000). Optimal CO_2 abatement in the presence of induced technological change. *Journal of environmental economics and management*, 39:1–38.

Grankvist, G., Dahlstrand, U., and Biel, A. (2004). The impcat of environmental labelling on consumer preference: Negative vs. positive labels. *Journal of consumer policy*, 27:213–230.

Greenstone, M. (2003). Estimating regulation-induced substitution: The effect of the clean air act on water and ground pollution. *American economic review*, 93(2):442–448.

Jorgenson, D. W. and Wilcoxen, P. J. (1990). Environmental regulation and U.S. economic growth. *RAND journal of economics*, 22:41–69.

Kamien, M. I. and Schwartz, N. L. (2012). *Dynamic optimization: The calculus of variations and optimal control in economics and management*. Elsevier North Holland.

Kolstad, C. C. (2010). *Environmental economics*. Oxford university press, 2 edition.

Loureiro, M. L., Mccluskey, J. J., and Mittelhammer, R. C. (2002). Will consumers pay a premium for eco-labeled apples? *Journal of consumer affairs*, 36(2):203–219.

Ministry of Environment Government of Japan (2006a). *Kankyo Hakusho*.

Ministry of Environment Government of Japan (2006b). *List of Environmental Qaulity Standards in Japan at http://www.env.go.jp/en/standards/*.

Miravete, E. J. (2003). Time-consistent protection with learning-by-doing. *European economic review*, 47:761–790.

Moraga-Gonzalez, J. L. and Padron-Fumero, N. (2002). Environmental policy in a green market. *Environmental and resource economics*, 22:419–447.

Oates, W. E., Portney, P. R., and McGartland, A. M. (1989). The net benefit of incentive-based regulation: A case study of environmental standard setting. *American economic review*, 79(5):1233–1242.

Palmer, K., Oates, W. E., and Portney, P. R. (1995). Tightening environmental standards: The benefit-cost or the no-cost paradigm. *Journal of economic perspectives*, 9(4):119–132.

Porter, M. E. (1991). America's green strategy. *Scientific american*, 264:168.

Porter, M. E. and van der Linde, C. (1995). Toward a new conception of the environment-competitive relationship. *Journal of economic perspectives*, 9(4):97–118.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of political economy*, 94:1002–1037.

Singh, N. and Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *RAND journal of economics*, 15:546–545.

Spence, M. A. (1981). The learning curve and competition. *Bell journal of economics*, 12:49–70.

Stokey, N. L. (1988). Learning by doing and the introduction of new goods. *Journal of political economy*, 96:701–717.

Stranlund, J. K. (1997). Public technological aid to support compliance to environmental standards. *Journal of environmental economics and management*, 34:228–239.

Zhu, J. and Ruth, M. (2015). Relocation or reallocation: Impacts of differentiated energy saving regulation on manufacturing industries in china. *Ecological economics*, 110:119–133.

Figure 1: Phase diagram in marginal cost-price space

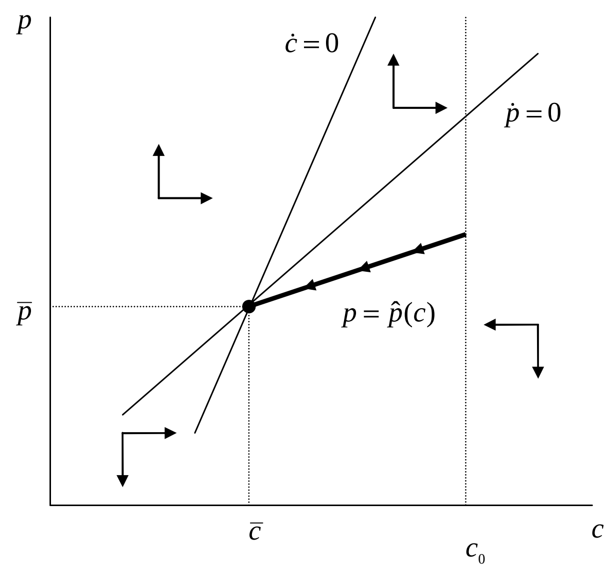

Figure 2: The value of the regulation $\Delta\bar{T}$ and the degree of dynamic learning effect λ

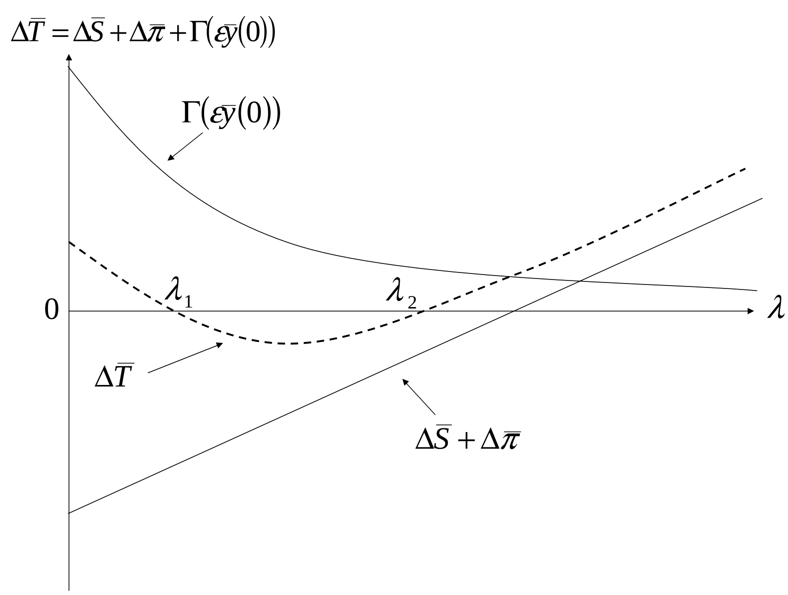

Figure 3: The value of the regulation and the marginal cost of the eco-product (Static Case)

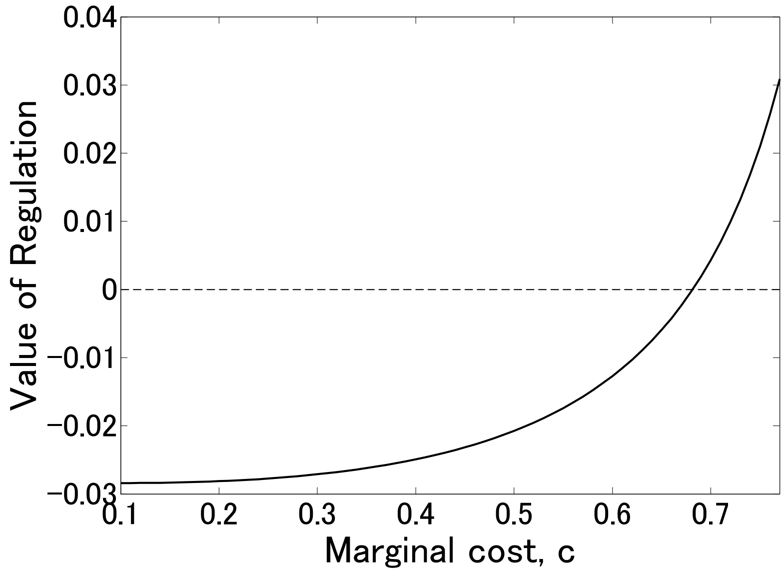

Figure 4: The marginal cost of the eco-product over time

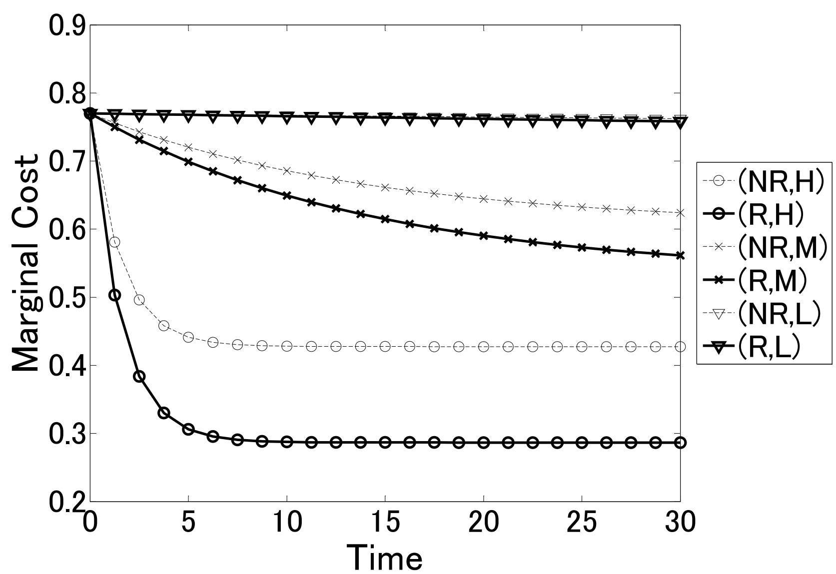

Figure 5: The price of the eco-product over time

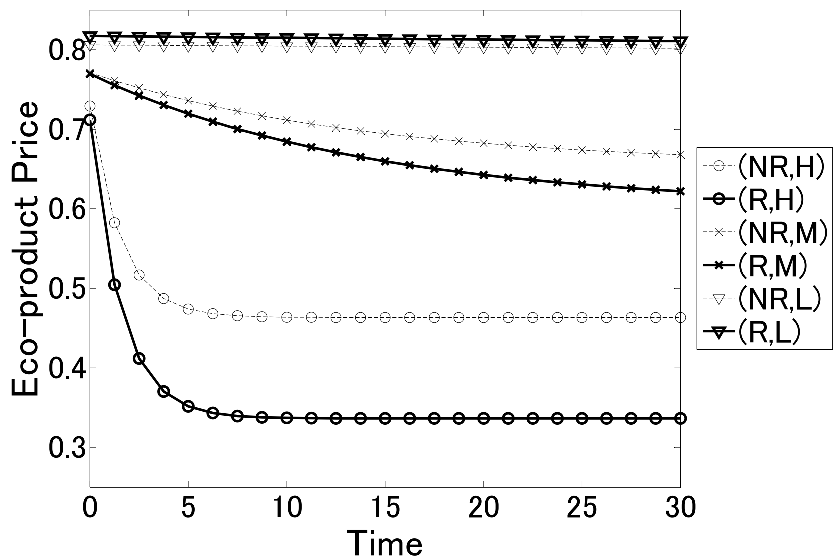

Figure 6: Consumer surplus over time

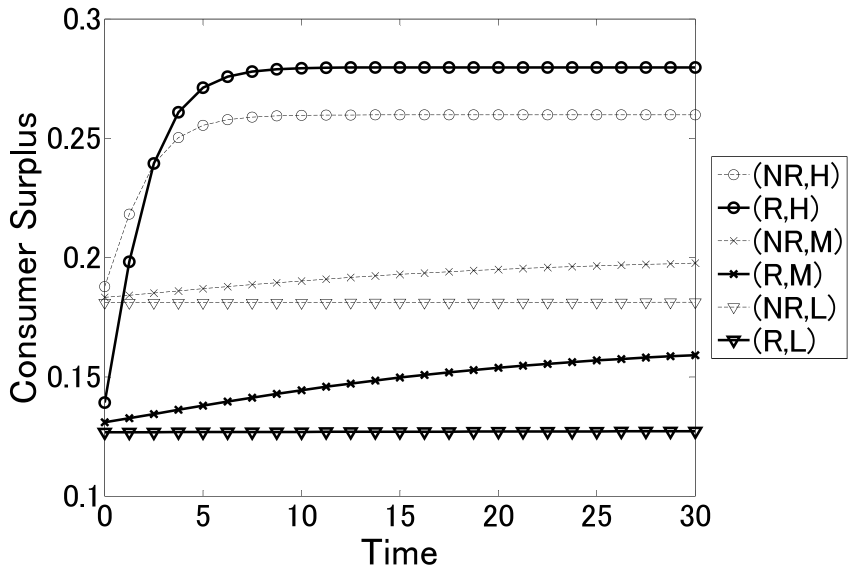

Figure 7: The profit of the single producer of an eco-product over time

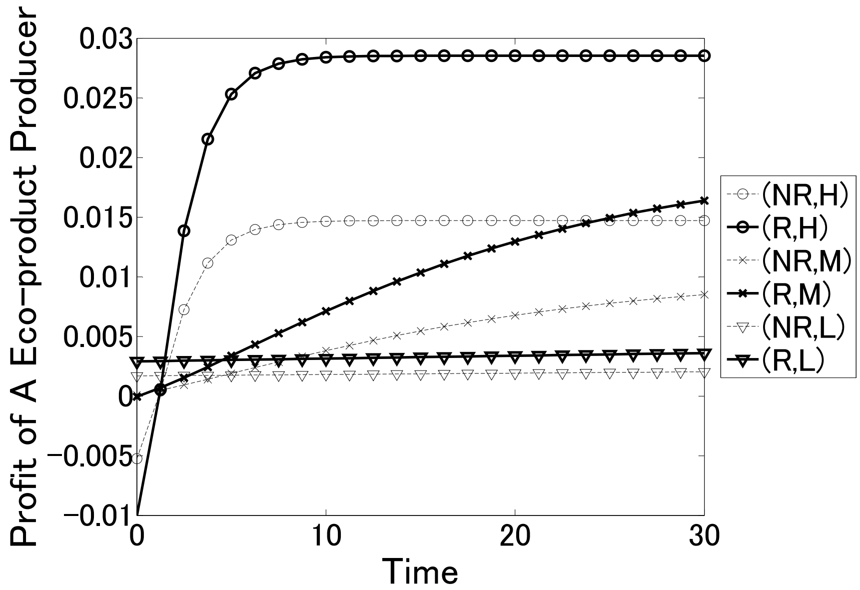

Figure 8: The negative externalities over time

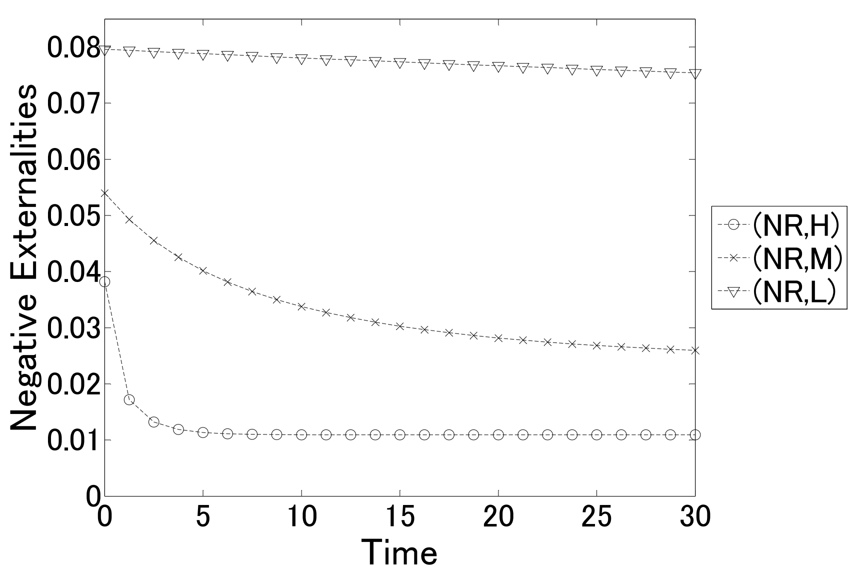

Figure 9: Social welfare over time

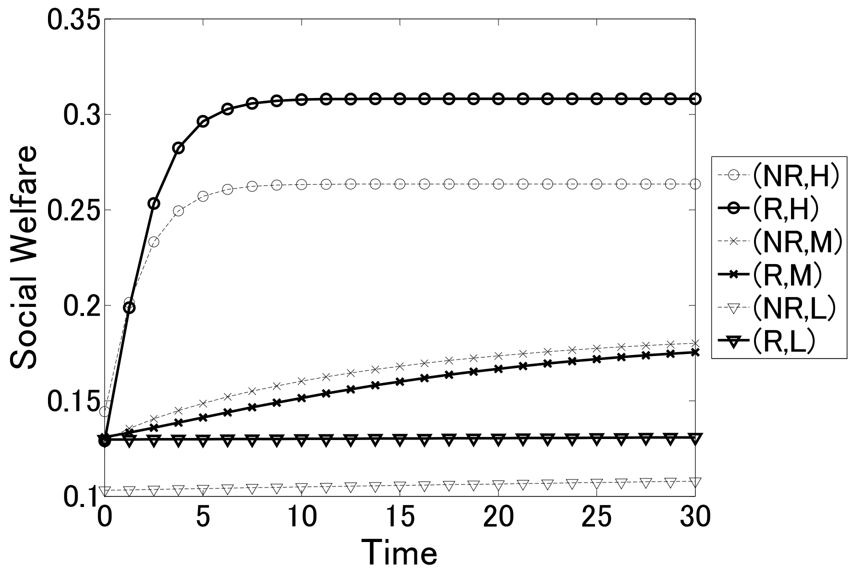

Figure 10: Current value of the regulation over time with the initial marginal cost $c_0 = 0.77$ as a function of the degree of dynamic learning effect λ