Growth and non-regular employment

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Abstract

The share of non-regular employment has been increasing in many developed countries during the past two decades. The objective of this paper is to study a cause of the upward trend in non-regular employment by focusing on productivity growth. Data from Japan shows that productivity growth reduces both unemployment and the proportion of non-regular workers to total employed workers. In order to study the impact of long-run productivity growth on unemployment and non-regular employment, I develop a search and matching model with disembodied technological progress and two types of jobs, regular and non-regular jobs. The numerical analysis demonstrates that faster growth reduces the share of non-regular employment and the unemployment rate, which is consistent with empirical facts.

Keywords: Growth, Unemployment, Non-regular employment, Search and matching model

JEL classification: E24, J64, O40

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1 Introduction

The share of non-regular employees in total employed workers has been increasing in many developed countries during the past two decades.1 In general, non-regular workers have lower wages, lower benefits, and a higher risk of dismissal. Since an increase in non-regular workers has serious consequences in labor markets such as wage differentials, employment instability, and poor working condition, it is important to study the major cause of the upward trend in non-regular employment.

Although it is well known that the Japanese labor market is characterized by full-time, long-term contract and high worker protection, in the past decades, regular employment has declined and non-regular employment has increased.2 Recently, the proportion of non-regular employment exceeds one-third of employment as a whole. While the proportion of non-regular workers has been increasing rapidly, the performance of the Japanese economy has changed dramatically. The growth rate slowed down from an average of 3-4% in the 1980s to just 1% in through 1990s and 2000s. The unemployment rate increased from an average of 2.5% in 1980s to 4.7% in 2000s. The simultaneous slowdown of productivity growth and rise in both the share of non-regular employment and unemployment suggests that there is a close connection among them.

The purpose of the paper is to study an effect of productivity growth on both non-regular employment and unemployment. I first document the fact that slowdown of productivity growth increases the share of non-regular employment in the economy and the unemployment rate in Japan. I then develop a search and matching model with two types of jobs, regular jobs and non-regular jobs. They differ in their separation and costs of creating new jobs. In order to study the impact of long-run productivity growth on non-regular employment and unemployment, I incorporate disembodied technological progress, as in Pissarides (2000) and Pissarides and Vallanti (2007).3

The numerical analysis demonstrates that faster productivity growth reduces the share of non-regular employment in the economy, which is consistent with data. Since faster productivity growth makes to work in a regular job more appealing by reducing costs of being a regular worker, a larger number of job seekers shifts from non-regular jobs to regular jobs. This increases job creation in regular jobs and thus reduces non-regular employed workers.

This paper also finds that the effect of productivity growth on unemployment differs be-

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1The share of temporary jobs in total employment rose in certain European countries, such as France, Italy, the Netherlands, Portugal, and Span. See Bentolilla and Saint-Paul (1992), Wasmer (1999), and OECD (2002).
2See Rebick (2005) for the details of the Japanese employment system.
3An effect of productivity growth on unemployment has been studied in the framework of a search and matching model. See Pissarides (2000), Mortensen and Pissarides (1998), and Pissarides and Vallanti (2007).
between regular and non-regular jobs. Faster productivity growth has several counteracting effects on unemployment. First, due to the well-known capitalization effect, higher productivity growth lowers unemployment in both regular and non-regular job sectors.\(^4\) Second, faster productivity growth may increase or reduce unemployment in each sector through the reallocation of workers. On one hand, an increased worker flows from non-regular to regular jobs facilities firms in the regular job sector to find a new worker, inducing more vacancy creation. On the other hand, it makes more difficult for unemployed workers to find jobs in the regular job sector. The opposite occurs in the non-regular sector. Third, faster growth affects unemployment by changing output prices. It turns out that faster productivity growth reduces output prices in the regular job sector while it increases output prices in the non-regular sector. This induces less job creation and thus higher unemployment in the regular job sector and more job creation and thus lower unemployment in the non-regular job sector.

Under plausible parameter values, faster productivity growth increases unemployment in regular jobs, while it reduces unemployment in non-regular jobs. Since the latter effect dominates the former one, an increase in productivity growth reduces aggregate unemployment, which is consistent with the data.

This paper is related to the literature of non-regular workers in Japan. Asano et al. (2013) empirically investigate factors that drive the secular increase of non-regular employment in Japan. This paper puts forward their analysis by studying the effect of productivity growth on non-regular employment. Similar to this paper, Nosaka (2011) develops a search and matching model with regular and non-regular workers. While his focus is on labor market dynamics over the business cycle, this paper investigates the long-run effect of growth on labor market outcomes. By using a labor search model, Esteban-Pretel et al. (2010) also address the important issue of the non-regular employment. But, their focus is on career choice of young workers.

This paper also adds to a literature that studies the relationship between growth and unemployment. Recently, a number of papers study the effect of growth on unemployment in search and matching models (Pissarides, 2000; Mortensen and Pissarides, 1998, Pissarides and Vallanti, 2007). These studies use one sector matching models when they examine the effect of productivity growth on unemployment. In contrast, this paper develops a search and matching model with two types of jobs, regular and non-regular jobs. Wasmer (1999) also develops a similar model and studies the effect of productivity growth and labor force growth on temporary jobs. While he studies the case in which firms can hire both types of workers, this paper studies the case in which firms choose what type of vacancies to create before searching their employees. Thus, my model can be viewed as a complement to Wasmer (1999).

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\(^4\)For an exposition of the capitalization effect, see Pissarides (2000, Ch.3).
The reminder of the paper is organized as follows. Section 2 presents salient features of the Japanese labor market and discuss the relationship among productivity growth, non-regular employment, and unemployment. Section 3 develops a search and matching model with two types of jobs, a regular job and a non-regular job and disembodied technological progress. In Section 4, I calibrate the model parameters and present the results of quantitative comparative statics exercises. I also discuss the sensitivity of the quantitative results to my choice of parameter values. Section 5 concludes.

2 Japanese labor market facts

This section presents some of the salient features of the Japanese aggregate labor market over the past 30 years. I discuss the relationship between productivity growth and the labor market. I focus on labor productivity growth and two labor market variables: non-regular workers and the unemployment rate.

One of the most important changes that are taking place in the Japanese labor market is an increase in non-regular employment. In order to understand the meaning of non-regular employment in Japan, I first define the complement of this employment status, that is, regular employment. In Japan, regular employment is generally considered as the status of a worker who is hired directly by his employer without a predetermined period of employment and works for scheduled hours. In other words, a regular employee is an employee who holds a permanent and full time job. Non-regular employment is the status of a worker with a job contract that is different from regular employment. Non-regular jobs include part-time, temporary, dispatched, and contract or entrusted workers. I obtain the data from the Special Survey of the Labour Force Survey (LFS) and the Labour Force Survey [Detailed Tabulation]. Both surveys are conducted by the Statistics Bureau and the Director-General for Policy Planning.

Figure 1 presents the number of non-regular workers and the proportion of non-regular workers to the total employed workers from 1984 to 2010. In 1984, there were 600 million non-regular workers. Non-regular workers have increased steadily and exceeded 1,000 million in 1995, and became more than 1,500 million in 2002. The proportion of non-regular workers to total employed workers increased from 15.3% in 1984 to 34.4% in 2010. Thus, recently, the proportion of non-regular employment exceeds one-third of employment as a whole in Japan.

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5 Esteban-Pretel et al. (2011) and Genda et al. (2012) provide a definition and description of both regular and non-regular employment in Japan.

6 There was the change in the survey frame in 2001. The data for 1984 to 2001 is obtained from the SSLFS and the data for 2002-2010 comes from the LFS [Detailed Tabulation].
Figure 1: The share and number of non-regular workers.

*Note:* The solid line indicates the proportion of non-regular workers the total employed workers. The dashed line indicates the number of non-regular workers. Sample covers 1984-2010.
Figure 2 shows the unemployment rate and its trends. I obtain the data from LFS. In order to obtain the trend component of the data, I use the Hodrick-Prescott filter (Hodrick and Prescott, 1997). Following Ball and Mankiw (2002), I consider two different values of the smoothing parameter in the HP filter, $\lambda = 100$ and $\lambda = 1000$. The unemployment rate has been significantly low until the middle of 1990s, with an average of 2.5%. It increased gradually and exceeded 5% in 2001. Then, the unemployment rate declined in the early and middle of 2000s, but it increased after the global financial crisis occurred in 2008.

Figure 3 presents the productivity growth rate and its trends. Productivity is measured as real output per employed workers. The output measure is based on the National Income and Product Accounts, while employment is constructed by Statistics Bureau and Statistics Center. The productivity growth rate is the first differenced logged labor productivity. Similar to the unemployment rate, I use the HP filter to obtain the smoothed series of the productivity growth rate. The productivity growth rate increased until the middle of 1980, and then gradually declined. The productivity growth rate declined sharply in 1990s and was relatively stable in 2000s.

Figure 4 shows smoothed series for the proportion of non-regular workers to total employed workers ($\varphi$), the unemployment rate ($u$), and the productivity growth rate ($g$). Table 1 summarizes the relationship among smoothed series of these three variables.

**Table 1: Summary statistics: Correlation matrix**

<table>
<thead>
<tr>
<th></th>
<th>$\varphi$</th>
<th>$g$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>1</td>
<td>-0.768</td>
<td>0.954</td>
</tr>
<tr>
<td>$g$</td>
<td>-</td>
<td>1</td>
<td>-0.748</td>
</tr>
<tr>
<td>$u$</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: Correlation between the proportion of non-regular workers to total employed workers ($\varphi$), the productivity growth rate ($g$), and the unemployment rate ($u$). All series are smoothed with the Hodrick-Prescott filter with the smoothing parameter $\lambda = 100$. Sample covers 1984-2010.

Table 1 and Figure 4 show that there is a strong negative relationship between productivity growth and the proportion of non-regular workers to total employed workers in their trend terms. The correlation between these series is -0.768. I also find that the productivity growth rate is negatively correlated with the unemployment rate and the correlation is -0.748. Interestingly, there is a positive relationship between the proportion of non-regular workers to total employed workers and the unemployment rate. The correlation between them is 0.954.
Figure 2: Unemployment rate and trends.

Note: the solid line indicates the unemployment rate. The dashed line indicates the trend of the unemployment rate constructed by using the HP-filter with smoothing parameter 100. The dash-dotted line indicates the trend of the unemployment rate constructed by using the HP-filter with smoothing parameter 1000. Sample covers 1980-2010.
Figure 3: Productivity growth rate and trends.

*Note:* the solid line indicates the productivity growth rate. Labor productivity is measured as real output per employed workers, and the productivity growth rate is the first differenced logged labor productivity. The dashed line indicates the trend of the productivity growth rate constructed by using the HP-filter with smoothing parameter 100. The dash-dotted line indicates the trend of the productivity growth rate constructed by using the HP-filter with smoothing parameter 1000. Sample covers 1980-2010.
Figure 4: Productivity growth and labor market variables.

Note: The solid line indicates the trend of the proportion of non-regular workers to total employed workers. The dashed line indicates the trend of the productivity growth rate. The dash-dotted line indicates the trend of the unemployment rate. The trends are HP filters with the smoothing parameter 100. Sample covers 1984-2010.
In order to calculate the effect of productivity growth on labor market variables, following previous studies (for example, Pissarides and Vallanti, 2007; Miyamoto and Takahashi, 2011), I consider the following linear relationship between the long-run labor market variables and long-run productivity growth

\[ y_t = \beta_0 y + \beta_1 g_t + \epsilon_{yt}, \]

where \( y_t \) is the long-run labor market variables (the proportion of non-regular workers to total employed workers \( \varphi \) and the unemployment rate \( u \)), \( g_t \) is the long-run productivity growth rate, \( \beta_0 \) and \( \beta_1 \) are parameters, and \( \epsilon_t \) is a well-behaved stochastic disturbance. By using the trend components of the productivity growth rate and labor market variables, I have the following OLS estimates:

\[ \varphi_t = 31.68 - 4.32 g_t + \hat{\epsilon}_{\varphi t}, R^2 = 0.59, T = 27, \]

and

\[ u_t = 4.70 - 0.65 g_t + \hat{\epsilon}_{ut}, R^2 = 0.56, T = 27, \]

where Newey–West HAC standard errors are reported in parentheses.

Although caution is needed to see the results due to the small sample size, the regressions show that a 1% increase in the long-run productivity growth reduces the proportion of non-regular workers by 4.32%. It is also shown that a 1% increase in the long-run growth reduces the unemployment rate by 0.65%. This is in line with existing empirical studies. For example, Blanchard and Wolfers (2000) estimate that a 1% increase in the growth rate leads to a 0.25%-0.7% increase in the unemployment rate.

3 Model

I consider a continuous time search and matching model with two types of jobs, a regular job and a non-regular job. Regular and non-regular jobs differ in their separation and hiring costs. The difference between the two types of jobs is due to labor laws and institutions that impose different duration, termination and hiring costs on employers. The basic structure of the model follows Pissarides (2000). In order to study the impact of productivity growth on labor market dynamics, I introduce disembodied technological progress, as in Pissarides (2000) and Pissarides and Vallanti (2007).

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7Acemoglu (2001) develops a search and matching model with two types of jobs that differ according to the costs of job creation.
The environment  There is a large measure of firms and a unit measure of workers in an economy. Both firms and workers are infinitely lived and risk neutral. Each firm can have at most one job. There are two types of jobs: a regular (R) and a non-regular (N). A firm must decide *ex ante*, that is, before searching for a worker, whether it will open a regular or a non-regular job. Regular and non-regular jobs differ in their separation and hiring costs. A regular job is terminated with a low exogenous separation rate \( \delta_R \), in which the firm needs to pay a firing cost \( f_t \).\(^8\) A non-regular job is terminated with a high exogenous separation rate \( \delta_N \) (\( \delta_N > \delta_R \)), in which case the firm does not need to pay the firing cost. Furthermore, it is assumed that the cost of posting a new regular job, \( \gamma_R \), is higher than that of posting a non-regular job, \( \gamma_N \).

There are two types of workers: regular workers and non-regular workers. They differ in their skill level. It is assumed that a regular worker has higher skill than a non-regular worker. The proportion of regular workers in the economy is denoted by \( \phi \). I assume that while a non-regular worker cannot change her skill level in the short-run, she can up-grade her skill by paying the cost of training \( C_t \) and become a regular worker in the long-run.

Production technology  Firms with filled regular jobs and firms with filled non-regular jobs produce two intermediated goods that are then sold in a competitive market and immediately transformed into the final consumption good. Workers derive utility from the consumption of the final good and maximize the present discount value of their utility. On other hand, firms maximize the present discount value of their income. Discount rate is denoted by \( r \).

Production of a firm of type \( j \) with \( j \in \{R, N\} \) at time \( t \) is given by

\[
y_{jt} = A_t e_{jt},
\]

where \( A_t \) is a general productivity parameter common to all producing jobs and \( e_{jt} \) is employment in \( j \) type jobs. Suppose that the leading technology in the economy is driven by an exogenous invention process that grows at the rate \( g < r \). Thus, \( A_t = A \exp(g t) \), where \( A > 0 \) is some initial productivity level.

The technology of production for the final good is given by

\[
Y_t = [\alpha y^c_R + (1 - \alpha) y^c_N]^{1/\sigma},
\]

where \( \sigma < 1 \) and the parameter \( \alpha \) is the relative share of the regular job’s input in final production. The elasticity of substitution between \( y_R \) and \( y_N \) is \( 1/(1 - \sigma) \). The price of final goods is normalized to one. Since the two intermediate goods are sold in competitive markets, their prices are

\[
P_{Rt} = \alpha Y_t^{1-\sigma} y^c_R^{-1},
\]

\(^8\)Note that the firing cost is a fixed cost and not a transfer from the firm to the worker.
The labor market  

The labor market is subject to frictions. Firms and workers cannot meet instantaneously but must go through a time-consuming search process. I assume that the search process is directed. Each firm posts either a vacancy for a regular worker or one for a non-regular worker. Free entry determines endogenously the number of firms in each labor market. On the other hand, unemployed workers search for jobs. Specifically, regular workers direct their search towards the labor market for type $R$ jobs, whereas non-regular workers search for type $N$ jobs. The number of matches between type $j$ vacancies and unemployed workers search for type $j$ jobs is determined by the matching function

$$m_{jt} = M(v_{jt}, u_{jt}),$$

where $v_{jt}$ is the number of vacancies posted and $u_{jt}$ is the number of unemployed workers. The matching function $M(v_{jt}, u_{jt})$ is continuous, twice differentiable, increasing in its arguments, and has constant returns to scale. Define $\theta_{jt} = v_{jt} / u_{jt}$ as labor market tightness in the market for type $j$ jobs. The rate at which a firm with a vacancy is matched with a worker is

$$M(v_{jt}, u_{jt}) / v_{jt} = M(1, u_{jt} / v_{jt}) \equiv q(\theta_{jt}).$$

Similarly, the rate at which an unemployed worker is matched with a firm is $M(v_{jt}, u_{jt}) / u_{jt} = \theta_{jt} q(\theta_{jt})$. Since the matching function has constant returns to scale, $q(\theta_{jt})$ is decreasing in $\theta_{jt}$ and $\theta_{jt} q(\theta_{jt})$ is increasing in $\theta_{jt}$.

Since the total number of workers in the economy is one, I have

$$u_{Rt} + u_{Nt} + e_{Rt} + e_{Nt} = 1.$$  

The total unemployed workers are given by $u_t = u_{Rt} + u_{Nt}$. Since the labor force is one, $u_t$ represents an aggregate unemployment rate. The fraction of regular workers in the economy is denoted by $\phi$. Then, I have

$$\phi = u_{Rt} + e_{Rt}, 1 - \phi = u_{Nt} + e_{Nt}.$$  

The evolution of unemployment in the market for type $j$ jobs is given by the difference between the flow into unemployment and flow out of it. Thus,

$$\dot{u}_{jt} = \delta_{jt} e_{jt} - \theta_{jt} q(\theta_{jt}) u_{jt}.$$  

### 3.1 The Value functions

I restrict my attention to stationary equilibrium, and labor market tightness is assumed to be constant over time. The value of a firm with a type $j$ filled job, $\Pi_{jt}$, is characterized by the

$$P_{Nt} = (1 - \alpha) Y_t^{1-\sigma} y_{Nt}^{\sigma-1}.$$  

and

$$\Pi_{jt} = (1 - \alpha) Y_t^{1-\sigma} y_{jt}^{\sigma-1}.$$
following Bellman equation:

\[
 r\Pi_{jt} = P_{jt}A_t - w_{jt} + \delta_j \left[ V_{jt} - \Pi_{jt} - f_{jt} \right] + \dot{\Pi}_{jt} \quad \text{for } j \in \{R, N\},
\]

where \( w_{jt} \) is the wage rate at time \( t \) and \( V_{jt} \) is the value of a firm with a type \( j \) vacant job. The firm receives flow revenues \( P_{jt}A_t - w_{jt} \), which is the productive output of the match minus the wage paid to the worker. The match is destroyed by the exogenous shock at rate \( \delta_j \), in which case the firm loses its asset value of the filled job, pays the firing tax, and obtained the value of a vacant job. It is important to note that job separation rates are different between type \( R \) jobs and type \( N \) jobs, and only firms with type \( R \) jobs pay the firing cost. The asset value of a match is expected to change over time due to exogenous technological progress.

The value of a firm with a type \( j \) vacant job at time \( t \) is given by

\[
 rV_{jt} = -\gamma_{jt} + q(\theta_j) \left[ \Pi^0_{jt} - V_{jt} \right] + \dot{V}_{jt} \quad \text{for } j \in \{R, N\},
\]

where \( \gamma_{jt} \) is the cost of posting a vacancy and \( \Pi^0_{jt} \) is the expected value of a new match to a firm with a type \( j \) job at time \( t \). The expected profit of a new match to a firm with a type \( R \) job is different from \( \Pi^0_{Rt} \), as defined in (1). This is due to the existence of the firing cost that is paid at the moment of job separation. In contrast, the expected value of a new match to a firm with a type \( N \) job is the same to \( \Pi^0_{Nt} \).

Given the starting wage \( w^0_{Rt} \), the initial value of a firm with a type \( R \) filled job satisfies

\[
 r\Pi^0_{Rt} = P_{Rt}A_t - w^0_{Rt} + \delta_R \left[ V_{Rt} - \Pi^0_{Rt} - f_{Rt} \right] + \dot{\Pi}^0_{Rt}.
\]

I now turn to the worker’s side. The value of an employed worker in a firm with a type \( j \) at time \( t \), \( W_{jt} \), is characterized by the following Bellman equation:

\[
 rW_{jt} = w_{jt} + \delta_j \left[ U_{jt} - W_{jt} \right] + \dot{W}_{jt} \quad \text{for } j \in \{R, N\},
\]

where \( U_{jt} \) is the value of an unemployed worker who searches for a job of type \( j \). The value of an employed worker is determined by several factors. The worker receives the wage \( w_{jt} \). The match may be destroyed by the exogenous shock at rate \( \delta_j \), in which case the worker loses the current asset value and obtains the asset value of being unemployed. The asset value of a match is expected to change over time due to technological progress.

The value of an unemployed worker searching for a job of type \( j \) is

\[
 rU_{jt} = z_t + \theta_j q(\theta_j) \left[ W^0_{jt} - U_{jt} \right] + \dot{U}_{jt} \quad \text{for } j \in \{R, N\},
\]

where \( W^0_{jt} \) is the value of an employed worker at the moment of job creation. Given an initial wage \( w^0_{jt} \), the initial value of an employed worker in a type \( R \) job is given by

\[
 rW^0_{Rt} = w^0_{Rt} + \delta_R \left[ U_{Rt} - W^0_{Rt} \right] + \dot{W}^0_{Rt}.
\]
Note that an initial value of employed worker in a type $N$ job is the same to the continuing value of it. Thus, $W_{Nt}^0 = W_{Nt}$. The firm that has a job with value $\Pi_{jt}$ at time $t$ expects to make a capital gain of $\Pi_{jt} - \Lambda \Pi_{jt}$. The same holds for an employed worker and an unemployed worker, where the capital gain is $gW_{jt}$ and $gU_{jt}$, respectively. The value of a vacant job $V_{jt}$ does not change because it is zero by the free entry condition.

I focus on the steady state. This corresponds to a balanced growth path where the economy grows at the rate of productivity growth $g$. To make the model stationary, I assume that all exogenous variables grow at the rate of productivity growth $g$. Thus, I define five positive exogenous parameters $\gamma_j$, $z$, $f$, and $C$ such that $
abla_j = \Lambda \nabla_j$, $z = \Lambda z$, $f = \Lambda f$, and $C = \Lambda C$.

Replacing the capital gain by its steady-state value, the above Bellman equations can be rewritten as follows:

\begin{align*}
(r - g) \Pi_j &= P_j A - w_j + \delta_j \left[ V_j - \Pi_j - \Lambda f \right] \text{ for } j \in \{R, N\}, \\
(r - g) V_j &= -\Lambda \gamma_j + q(\theta_j) \left[ \Pi_j^0 - V_j \right] \text{ for } j \in \{R, N\}, \\
(r - g) \Pi_R^0 &= P_R A - w_R^0 + \delta_R \left[ V_R - \Pi_R^0 - \Lambda f \right] \\
(r - g) W_j &= w_j + \delta_j \left[ U_j - W_j \right] \text{ for } j \in \{R, N\}, \\
(r - g) U_j &= Az_j + \theta_j q(\theta_j) \left[ W_j^0 - U_j \right] \text{ for } j \in \{R, N\},
\end{align*}

and

\begin{align*}
(r - g) W_R^0 &= w_R^0 + \delta_R \left[ U_R - W_R^0 \right].
\end{align*}

The wages are determined through the Nash bargaining between a firm and a worker over the share of expected future joint income. Due to the firing cost, the wage determination mechanism differs between type $R$ jobs and type $N$ jobs market. I first look at the wage determination in the market for type $R$ jobs. When a firm and a worker first meet, the payoff to the firm is $\Pi_R^0 - V_R$ and the payoff to the worker is $W_R^0 - U_R$. Therefore, the starting wage is determined by the following equation

\begin{align*}
(1 - \eta_R) \left( W_R^0 - U_j \right) = \eta_R \left( \Pi_R^0 - V_R \right).
\end{align*}

Once the worker is employed, the firm has to pay the firing tax $Af$ if the firm fails to agree to a continuation wage. Thus, the continuing wage is chosen as

\begin{align*}
(1 - \eta_R) \left( W_R - U_R \right) = \eta_R \left( \Pi_R - V_R + Af \right).
\end{align*}

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9 In the literature, in order to ensure the existence of a balanced growth path, usually all the exogenous variables are assumed to follow the pace of productivity growth. See, for example, Mortensen and Pissarides (1999) and Pissarides and Vallanti (2007).
Since a firm with the type $N$ job does not need to pay the firing cost, there is no difference between a starting wage and a continuation wage. The wage in the type $N$ job is determined by

\[(1 - \eta_N) (W_N - U_N) = \eta_N (\Pi_N - V_N). \tag{10}\]

It is assumed that a non-regular worker can upgrade her skill by paying the training cost $AC$. In equilibrium, the value of staying in the type $R$ job sector is equivalent to that of staying in the type $N$ job sector. Thus, arbitrage by workers between sectors implies that

\[U_R - AC = U_N. \tag{11}\]

Free entry implies that the value of a vacant job is zero in equilibrium. Thus,

\[V_j = 0 \text{ for } j \in \{R, N\}. \tag{12}\]

Finally, the steady-state unemployment is given by

\[u_R = \frac{\delta_R \phi}{\delta_R + \theta_R q(\theta_R)}, \tag{13}\]

and

\[u_N = \frac{\delta_N (1 - \phi)}{\delta_N + \theta_N q(\theta_N)}. \tag{14}\]

### 3.2 Characterization of steady-state equilibrium

A steady-state equilibrium is a profile \(\{u_j, \theta_j, \phi, w_j, w^0_R, \Pi_j, \Pi^0_R, V_j, W_j, W^0_R, U_j\}\) that satisfies the Bellman equations (2)-(7), the wage equations (8), (9) and (10), the workers’ arbitrage condition (11), the free entry condition (12), and the steady-state unemployment rate conditions (13) and (14).

The free entry condition \(V_j = 0\) together with (3) yields

\[\Pi^0_R = \frac{A \gamma_R}{q(\theta_R)} \text{ and } \Pi^0_N = \Pi_N = \frac{A \gamma_N}{q(\theta_N)}. \tag{15}\]

By using (8), (12), and (15), the value of an unemployed worker in sector $j$ can be rewritten as

\[(r - g) U_j = Az_j + \frac{\eta_j \theta_j A \gamma_j}{1 - \eta_j}, \text{ for } j \in \{R, N\}. \tag{16}\]

By using all the value functions (2)-(7), the free entry condition, and the wage sharing rules (8)-(10), I obtain the following equilibrium wages:

\[w^0_j = \eta_j P_j A - \eta_j \delta A f + \left(1 - \eta_j\right) Az_j + \eta_j \theta A \gamma_j, \tag{17}\]
and

\[ w_j = \eta_j P_j A + \eta_j (r - g) Af_j + \left(1 - \eta_j\right) A z_j + \eta_j \theta A \gamma_j. \tag{18} \]

Making use of (11) and (16), I derive the following equilibrium condition

\[ z_R + \frac{\eta_R \theta_R \gamma_R}{1 - \eta_R} = z_N + \frac{\eta_N \theta N \gamma_N}{1 - \eta_N} + (r - g) C. \tag{19} \]

The numbers of employed workers in sector \( R \) and \( N \) are determined as \( e_R = \phi - u_R \) and \( e_N = 1 - \phi - u_N \). Then, the aggregate production in the sector \( R \) and \( N \) are obtained by

\[ y_R = \frac{A \phi \theta_R q(\theta_R)}{\delta_R + \theta_R q(\theta_R)} \quad \text{and} \quad y_N = \frac{A (1 - \phi) \theta_N q(\theta_N)}{\delta_N + \theta_N q(\theta_N)}. \]

Then, the prices of the two inputs can be obtained as

\[
P_R = \alpha \left[ \frac{\phi \theta_R q(\theta_R)}{\delta_R + \theta_R q(\theta_R)} \right]^{-r-1} \left[ \alpha \left( \frac{\phi \theta_R q(\theta_R)}{\delta_R + \theta_R q(\theta_R)} \right)^{r} + (1 - \alpha) \left( \frac{(1 - \phi) \theta_N q(\theta_N)}{\delta_N + \theta_N q(\theta_N)} \right) \right]^{1-r}.
\]

\[ \equiv P_R(\theta_R, \theta_N, \phi), \]

\[
P_N = (1 - \alpha) \left[ \frac{(1 - \phi) \theta_N q(\theta_N)}{\delta_N + \theta_N q(\theta_N)} \right]^{-r-1} \left[ \alpha \left( \frac{\phi \theta_R q(\theta_R)}{\delta_R + \theta_R q(\theta_R)} \right)^{r} + (1 - \alpha) \left( \frac{(1 - \phi) \theta_N q(\theta_N)}{\delta_N + \theta_N q(\theta_N)} \right) \right]^{1-r}.
\]

\[ \equiv P_N(\theta_R, \theta_N, \phi). \]

Substituting the price of goods in the \( R \) sector and (17) into (4) and using the free entry condition \( V_R = 0 \) and (3), I obtain the equilibrium job creation condition

\[ \frac{\gamma_R}{q(\theta_R)} = \frac{(1 - \eta_R) (P_R(\theta_R, \theta_N, \phi) - z) - \eta_R \theta_R \gamma_R - \delta f (1 - \eta_R)}{r + \delta_R - g}. \tag{20} \]

Similarly, by substituting the price of goods in the sector \( N \) and (18) into (2), and using (3) and (12), I have the following job creation condition in the type \( N \) job sector:

\[ \frac{\gamma_N}{q(\theta_N)} = \frac{(1 - \eta_N) (P_N(\theta_R, \theta_N, \phi) - z) - \eta_N \theta_N \gamma_N}{r + \delta_N - g}. \tag{21} \]

The job creation condition states that the expected cost of posting a vacancy, the left-hand side of (20)(21), is equal to the firm’s share of the expected net surplus from a new job match, the right-hand side of (20)(21).

The system of equations (19), (20), and (21) determine endogenous variables \( \theta_R, \theta_N, \) and \( \phi \). Given \( \theta_R, \theta_N, \) and \( \phi \), equations (13) and (14) determine the number of unemployed workers in the sectors \( R \) and \( N \), respectively.

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4 Quantitative analysis

In this section, I calculate the equilibrium of the above model using numerical methods. I first calibrate the model to match several dimensions of the Japanese labor market data. Then, I perform quantitative comparative statics by calculating the steady-state response to an increase in the rate of productivity growth. I also discuss the sensitivity of the results to my choice of parameter values.

4.1 Calibration

I choose the model period to be one-year and set the discount rate at \( r = 0.04 \). This choice of the parameter is somewhat a priori, but is consistent with other studies such as Braun et al. (2006). Since the level of productivity does not influence the steady-state, I normalize \( A = 1 \) without loss of generality. For benchmark case, I set \( g = 0.016 \), the average productivity growth rate in Japan from 1980 to 2014.

For the final goods production function, I choose \( \sigma = 0 \) for the benchmark specification. Thus, the production function is the Cobb-Douglas.

The matching function is Cobb-Douglas, given by \( M(v_j, u_j) = m_j v_j^{1-\xi} u_j^\xi \), where \( m_j \) is the matching constant and \( \xi \) is the matching elasticity with respect to unemployment. Then, the vacancy filling rate is \( q(\theta_j) = m_j \theta_j^{1-\xi} \) and the job finding rate is \( \theta_j q(\theta_j) = m_j \theta_j^{1-\xi} \). I assume that matching constants (\( m_j \)) are different across sectors, while the elasticity parameters (\( \xi \)) are the same. Lin and Miyamoto’s (2014) estimate of the elasticity \( \xi \) for the Japanese labor market is 0.6. This value lies in the plausible range of 0.5-0.7 reported by Petrongolo and Pissarides (2001). I use the Hosios (1990) condition to pin down the worker’s bargaining power, so \( \eta_R = \eta_N = \xi \).

Given this, I target the average monthly job finding rate of 0.142 and the average monthly separation rate of 0.0048, which are reported by Miyamoto (2011) and Lin and Miyamoto (2012). The Report on Employment Service conducted by Ministry of Health, Labour and Welfare (MHLW) reports the job openings to applications ratios (yuko kyujin bairitsu) of both regular and temporary workers, which reflects labor market tightness. Based on the dataset, I target the ratio of labor market tightness for non-regular workers to that for regular workers \( \theta_N / \theta_R = 2.24 \).

I also target the share of non-regular workers in total employed workers. Based on the Labour Force Survey, the mean value of the ratio over the period of 1984-2014 is 0.26. From the Survey on Employment Trends conducted by Ministry of Health, Labour and Welfare, the ratio of the job-finding rate for regular workers to that for non-regular workers is 0.45 and the ratio of the

\[10\] By using the panel property of the monthly Labour Force Survey, Miyamoto (2011) and Lin and Miyamoto (2012) constructed the job-finding rate and the separation rate in Japan.
separation rate for regular workers to that for non-regular workers is 0.49. I target these ratios. I target the unemployment flow utility $z$ to be 40% of the average of employed workers in the economy.\footnote{According to Martin (1998), the replacement rate, the ratio of the unemployment benefit to the average wage, in Japan is about 0.6. Given this, Miyamoto (2011) targets the unemployment flow utility $z$ to be 60% of the average of employed workers in the economy. In contrast, this study uses the lower target value. This is because in the model, wages for non-regular workers are much lower than those for regular workers and it is necessary to make the value of $z$ enough low compared to the average wage.} I also target a wage ratio between regular workers and non-regular workers to be equal to 0.6, based on the Basic Survey on Wage Structure conducted by MHLW. Finally, following Nosaka (2011), I target the firing cost $f$ to be approximately 0.5 of an average wage.\footnote{Hopenhayn and Rogerson (1993) assume that the firing cost is 0.5-1.0 of annual wage. Alonso-Borrego et al. (2005) estimate the value of $f = 0.51\bar{w}$ using Spanish data.} Without loss of generality, I normalize $\theta_R$ to one.

I thus have ten target moments and ten model parameters: $c, f, z, \alpha, m_R, m_N, \delta_R, \delta_N, \gamma_R,$ and $\gamma_N$. I choose the parameter values that most closely match the ten target moments. The parameter values are summarized in Table 2.

Selected model solutions under the calibrated parameter values are reported in Table 3. The unemployment and job finding rates in the economy, the ratio of non-regular workers to total employed workers, and the ratios of labor market tightness and job-finding rates between sectors are equal to their target values. The unemployment rate in the regular-job sector, 2.5%, is much higher than that in the non-regular job sector, 0.8%. The number of vacancies in the regular job sector is 0.025, while that in the non-regular job sector is 0.018. Prices in the regular job sector are about 69% higher than those in the non-regular job sector.

\subsection*{4.2 Effects of growth on the labor market}

I now examine the effects productivity growth on labor market variables of interest by calculating the steady-state response to an increase in the productivity growth rate $g$. The results are shown in Figure 5.

A faster rate of productivity growth increases regular employed workers and reduces non-regular employed workers, increasing in the proportion of non-regular workers in the economy. The negative relationship between productivity growth and the share of non-regular workers is consistent with the empirical finding in Section 2. The channel through which faster productivity growth reduces the share of non-regular workers is as follows. Since an increase in productivity growth makes to work in the regular sector more appealing by reducing costs of working in the regular sector, a larger number of job seekers shifts from the non-regular sector.
Figure 5: Comparative statics for the productivity growth rate $g$
Table 2: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source/target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>A general productivity parameter</td>
<td>1.0</td>
<td>Normalization</td>
</tr>
<tr>
<td>$r$</td>
<td>Discount rate</td>
<td>0.036</td>
<td>Data</td>
</tr>
<tr>
<td>$g$</td>
<td>The rate of productivity growth</td>
<td>0.016</td>
<td>Data</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Elasticity of matching function</td>
<td>0.6</td>
<td>Lin and Miyamoto (2014)</td>
</tr>
<tr>
<td>$\eta_R$</td>
<td>Worker’s bargaining power in the $R$-sector</td>
<td>0.6</td>
<td>$\eta = \xi$ (efficiency)</td>
</tr>
<tr>
<td>$\eta_N$</td>
<td>Worker’s bargaining power in the $N$ sector</td>
<td>0.6</td>
<td>$\eta = \xi$ (efficiency)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>CES-elasticity parameter</td>
<td>0.0</td>
<td>Cobb-Douglas</td>
</tr>
</tbody>
</table>

$\begin{align*}
c &= \text{Cost of being a regular worker} & 10.085 & \text{Job finding rate} \\
f &= \text{Firing cost} & 0.338 & \text{Separation rate} \\
z &= \text{Flow value of unemployment} & 0.271 & \text{Replacement rate} \\
a &= \text{CES-weight} & 0.828 & \text{Share of non-reg. employed workers} \\
m_R &= \text{Scale parameter of matching function in the $R$ sector} & 1.316 & \text{Job find. rate for $R$ workers/job find. rate for $N$ workers} \\
m_N &= \text{Scale parameter of matching function in the $N$ sector} & 2.118 & \text{Sep. rate for $R$ workers/sep. rate for $N$ workers} \\
\delta_R &= \text{Separation rate in the $R$ sector} & 0.046 & \text{Wage for $N$ workers/wage for $R$ workers} \\
\delta_N &= \text{Separation rate in the $N$ sector} & 0.092 & \text{Tightness for $N$ workers/tightness for $R$ workers} \\
\gamma_R &= \text{Vacancy cost in the $R$ sector} & 0.248 & \text{Ratio of firing cost to wage} \\
\gamma_N &= \text{Vacancy cost in the $N$ sector} & 0.039 & \text{Normalization} \\
\end{align*}$

Under my choice of parameter values, faster productivity growth reduces the aggregate unemployment rate, which is also consistent with the data. In order to understand the mechanism behind the effect of productivity growth on unemployment, it is useful to see the effect of productivity growth on vacancies.

The impact of productivity growth on vacancies is ambiguous because there are several counteracting effects. First, a rise in the productivity growth rate increases vacancies in both sectors. Since a higher rate of productivity growth increases the return from creating a job, firms in both sectors have a greater incentive to open vacancies. This is because the cost of creating a vacancy is paid at the start but the profits accrue in the future. When the growth rate rises, all future income flows are discounted at lower rate, so firms are encouraged to create more vacancies. This effect is well-known as the capitalization effect. Second, faster productivity growth tends to increase vacancy creation in the regular job sector but to reduce vacancy creation in
Table 3: Parameter values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_R$</td>
<td>Labor market tightness in the $R$ sector</td>
<td>1.0</td>
</tr>
<tr>
<td>$\theta_N$</td>
<td>Labor market tightness in the $N$ sector</td>
<td>2.24</td>
</tr>
<tr>
<td>$\phi$</td>
<td>The proportion of non-regular employed workers</td>
<td>0.26</td>
</tr>
<tr>
<td>$u_R$</td>
<td>Unemployment rate in the $R$ sector</td>
<td>0.025</td>
</tr>
<tr>
<td>$u_N$</td>
<td>Unemployment rate in the $N$ sector</td>
<td>0.008</td>
</tr>
<tr>
<td>$u$</td>
<td>Unemployment rate</td>
<td>0.033</td>
</tr>
<tr>
<td>$v_R$</td>
<td>Vacancies in the $R$ sector</td>
<td>0.025</td>
</tr>
<tr>
<td>$v_N$</td>
<td>Vacancies in the $N$ sector</td>
<td>0.018</td>
</tr>
<tr>
<td>$e_R$</td>
<td>Employment in the $R$ sector</td>
<td>0.716</td>
</tr>
<tr>
<td>$e_N$</td>
<td>Employment in the $N$ sector</td>
<td>0.252</td>
</tr>
<tr>
<td>$e$</td>
<td>Employment</td>
<td>0.968</td>
</tr>
<tr>
<td>$\theta_R q(\theta_R)$</td>
<td>Job finding rate in the $R$ sector</td>
<td>1.316</td>
</tr>
<tr>
<td>$\theta_N q(\theta_N)$</td>
<td>Job finding rate in the $N$ sector</td>
<td>2.924</td>
</tr>
<tr>
<td>$w_R$</td>
<td>Wage in the $R$ sector</td>
<td>0.679</td>
</tr>
<tr>
<td>$w_N$</td>
<td>Wage in the $N$ sector</td>
<td>0.406</td>
</tr>
<tr>
<td>$P_R$</td>
<td>Price in the $R$ sector</td>
<td>0.691</td>
</tr>
<tr>
<td>$P_N$</td>
<td>Price in the $N$ sector</td>
<td>0.410</td>
</tr>
</tbody>
</table>

the non-regular sector through increasing worker flows from the non-regular job sector to the regular job sector. An increase in worker flows from the non-regular job sector to the regular job sector makes firms in the regular sector find a worker easier and induces more vacancy creation. On the other hand, in the non-regular job sector, since a number of job seekers decreases due to the worker reallocation, firms have less intensive to open vacancies. I call this the worker reallocation effect. Third, faster productivity growth affects output prices and thus vacancy creation in both sectors. An increase in the productivity growth rate reduces the output price in the regular job sector and increases the output price in the non-regular job sector. An increased employment in the regular sector increases the supply of output, lowering the price of goods, while a decreased employment in the non-regular job sector reduces output and thus increases the price of goods. This price effect reduces vacancy creation in the regular job sector by reducing the return from creating a job, while it increase vacancy creation in the non-regular job sector by increasing the return from creating a job.

As seen in Figure 5, under my choice of parameter values, faster productivity growth increases vacancies in both sectors. In the regular job sector, since the capitalization and worker
reallocate reallocation effects dominate the price effect, faster productivity growth increases vacancies. In the non-regular job sector, the capitalization and price effects dominate the worker reallocation effect. As a result, the increase in productivity growth leads to more vacancies.

I now turn to see the effect of productivity growth on unemployment. Since in the non-regular job sector, firms open more vacancies and a number of job seeker decreases due to the worker reallocation effect, the job-finding rate increases. This leads to a lower unemployment rate in the non-regular job sector. On the other hand, in the regular job sector, increased worker flows from the non-regular job sector cause congestion to one another when trying to match with vacancies. Although firms post more vacancies, the congestion effect is strong enough to reduce the job-finding rate and thus increase the unemployment rate.

While faster productivity growth increases the unemployment rate in the regular job sector, it reduces the unemployment rate in the non-regular job sector. Thus, the effect of faster productivity growth on the aggregate unemployment rate depends on which effect dominates. Under my choice of parameter values, since the latter effect dominates the former one, faster productivity growth reduces the aggregate unemployment rate.

Figure 5 shows that faster productivity growth increases the average wage in the economy. This is because faster productivity growth increases the share of regular employed workers whose wages are higher than non-regular employed workers’ wages.

Finally, I study whether the model’s prediction on the response of the proportion of non-regular jobs in the economy to productivity growth is empirically plausible. In Section 2, I find that a 1% increase in the productivity growth rate reduces the proportion of non-regular workers by 4.32%. In my model, a 1% increase in the growth rate reduces the proportion of non-regular workers by 4.7%. Thus, my model can explain the observed response of non-regular workers to growth.

4.3 Sensitivity analysis

In the benchmark case, faster productivity growth reduces the proportion of non-regular workers and the aggregate unemployment rate. I now discuss how these results vary with the value of the firing cost $f$ and the parameter in the final goods production function $\sigma$. When I change these parameters, I also re-calibrate parameters $c, z, \alpha, m_R, m_N, \delta_R, \delta_N, \gamma_R, \gamma_N$ in order to maintain my calibration target values.

First, I consider the impact of the value of the firing cost $f$. Figure 6 reports the relationship between the productivity growth rate and the aggregate unemployment rate. It also shows the relationship between the productivity growth rate and the proportion of non-regular workers in the economy for different values of the firing cost $f$. Although the size of impact is slightly
changed, the sign of the relationship between growth and unemployment does not change. It is also clear that allowing for firing costs \( f \) to vary does not have a significant impact on the relationship between the rate of productivity growth and the proportion of non-regular workers.

Next, I discuss the sensitivity of the results to my choice of the parameter value \( \sigma \). In my benchmark calibration, I set \( \sigma = 0 \) and assume that the final goods production function is Cobb-Douglas. I now consider two different values of \( \sigma \), -1 and -1/3. In the former case, the elasticity of substitution is 0.5 and in the latter case, the elasticity of substitution is 0.75. Figure 7 shows the results. It shows that the negative relationship between growth and unemployment is robust to change in values of \( \sigma \). Figure 7 also shows that the negative relationship between growth and the share of non-regular employment is robust to choice of parameter values of \( \sigma \). decreases.

### 5 Conclusion

This paper studies an effect of productivity growth on both non-regular employment and unemployment. I document the fact that productivity growth reduces both the share of non-regular employment in the economy and the unemployment rate at low frequencies in Japan. To account for these empirical findings, I develop a search and matching model with disembodied technological progress and two types of jobs, regular and non-regular jobs. The numerical analysis demonstrates that under the set of plausible parameter values, faster growth reduces the proportion of non-regular workers and the unemployment rate. Thus, my model succeeds to capture the empirical pattern of the share of non-regular employment and the unemployment rate in response to productivity growth.

A number of important issues remain for future research. Considering an intensive margin for adjusting labor input is an important issue. It is shown that in Japan, the labor input adjustment relies on both working hour adjustment and changing the number of workers. One would expect that an increase in non-regular workers shifts the burden of adjustment from hours to employment. Thus, the increase in non-regular workers affects both intensive and extensive margin for adjustment labor input. Another important line of future research is to incorporate endogenous job separation. I assumed that matches are destroyed solely for exogenous reasons. However, productivity growth may affect employment and worker flows via the destruction of matches. Furthermore, assuming exogenous job separation implies that the effect of firing costs in the model are reduced, since they only affect the hiring decision and not job separation decision. Thus, to incorporate endogenous job separation and study an effect of productivity growth on non-regular employment and unemployment is a fruitful avenue for research.\(^{13}\)

\(^{13}\)Although it is interesting to allow endogenous job separation in my model, doing so is not trivial. This is because
Figure 6: Sensitivity analysis with respect to $f$
Figure 7: Sensitivity analysis with respect to $\sigma$
References


—a matching model with productivity growth and endogenous job separation tends to have a positive relationship between productivity growth and unemployment, which is not consistent with the data. See Prat (2007) and Miyamoto and Takahashi (2011) for the detail.


