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A theory of strategic auditing: How should we select one member from a homogenous group?

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Abstract

This paper theoretically analyzes an audit rule that selects a taxpayer for an audit based on the reported income profile and creates strategic interdependence. Such strategic auditing contrasts with the random auditing rule. This paper proposes the lowest-reporter-audited rule. This rule ensures that the taxpayer with the lowest reported income is inspected from a group of taxpayers that are categorized according to factors such as social status, income level, occupation, and place of residence. We show that, under a realistic penalty rate condition, the lowest-reporter-audited rule is superior to the random audit rule.

1 Introduction

The securing of government tax revenues is a persistent and fundamental problem for all nations (Webber and Wildavsky, 1986). Incentive is high for individuals and companies to avoid excessive tax payments, which leads to tax avoidance, tax evasion, and payment delay. The results of a well-known audit program (the National Research Program, conducted by the US Internal Revenue Service (IRS)) showed that the tax gap (i.e., tax that is due but not paid in a voluntary or timely manner (US Department of the Treasury, IRS, 2006)) in 2001 was estimated to be 345 billion dollars, and this amount represented approximately 3.2% of nominal GDP for that year (Slemrod, 2007). Although the analyses of the tax gaps in other countries are limited for several reasons

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(such as resource constraints, non-publication of surveys), the gaps are estimated or speculated to be considerable (see Slemrod (2007) for details). Thus, the research on policy devices to enhance tax compliance is increasingly significant.

The basic model of tax evasion following Allingham and Sandmo (1972) and Yitzhaki (1974) suggest that a taxpayer chooses the extent of tax evasion by comparing the expected benefit to the expected cost of the evasion, based on the subjective probability of detection and the penalty rate. The basic model assumes that the probability of detection is irrespective of the extent of the evasion and other taxpayers' reporting behavior. However, audit authorities in the majority of countries utilize several factors, including taxpayers' past and current reporting behavior and social status, to select the taxpayers to audit.[1] The probability of audit for a taxpayer depends on that taxpayer's relative position among taxpayers in the same audited class (Collins and Plumlee, 1991; Alm and McKee, 2004). Therefore, the auditing rule itself generates strategic interdependence among taxpayers.

This paper employs a game theoretic framework to formulate and analyze a taxpayer's decision under the "strategic" auditing rule, which intentionally or unintentionally causes strategic interdependence among taxpayers. We suggest that current audit rule practices in the majority of countries result in a suspicious group of taxpayer audits. Taxpayers within the same group are homogeneous a priori, or, taxpayers are separated into groups to create homogeneity. Therefore, a suspicious taxpayer is one with the lowest reported income. We focus on the lowest-income-reporter-audited (LIRA) rule.

We model a taxpayer decision under the LIRA rule with the following consideration. We recognize that two asymmetries exist within the actual auditor and taxpayer context.[2] The first asymmetry exists between an auditor and a taxpayer; the auditor does not know the true taxable income of the taxpayer. This justifies the LIRA rule that insists that the lowest reporter is the most suspicious given a priori homogeneity. A second information asymmetry exists between taxpayers; one taxpayer does not know the actual true income of other taxpayers within the same audit class. This implies that taxpayer coordination within the same category is less significant for our model. This

[1] For instance, the IRS in the US estimates a DIF (discriminant index function) score for each return based on a statistical method and applies this score to the audit consideration process.

[2] Jung (1991) considers a situation where taxpayers have some uncertainty with respect to their true taxable incomes.

is in contrast to Alm and McKee (2004) who formulate taxpayer behavior using the LIRA rule with a symmetric complete information game and analyze a coordination problem among taxpayers.

2 Random auditing

This section introduces a basic random auditing model following the canonical model of Allingham and Sandmo (1972) and Yitzhaki (1974). A taxpayer decides whether and to what extent to evade taxes in the same way an individual would weigh a risky gambling decision. The taxpayer (individual or firm) has a true taxable income of $Y, Y > 0$. Let t be the basic tax rate. The taxpayer pays tY as tax if he reports his true income. However, if the income is under-reported, the taxpayer should pay tr, where $r, r < Y$ represents the under-reported income and $Y - r$ represents the amount of evasion or concealed income.[3] However, detailed auditing is randomly executed in probability p and the tax evasion is detected. In our model, the tax evasion is revealed if the tax authority inspects the under-reporting taxpayer. In the case of inspection, the individual must pay $qt(Y - r)$ as a penalty for the tax evasion, where $q, q > 1$, represents the penalty rate for the illegal activity. Therefore, the penalty is proportional to the concealed income.[4] The expected payoff for an individual reporting his income as $r, 0 \leqq r < Y$, is[5]

$$\begin{aligned} U &= Y - tr - pqt(Y - r) \\ &= (1 - t)Y + t(Y - r)(1 - pq) \qquad (1) \end{aligned}$$

Thus, we assume risk-neutrality of an individual.

The rational behavior of an individual who faces a random inspection depends on the sign of $(1 - pq)$.

Proposition 1. *The following holds true;*

[3] There are other types of reporting decisions such as nonfiling and late payment of taxes owed. However, according to the 2001 IRS estimate of the tax gap, under-reporting represents approximately 82% of the gap and nonfiling and late payment represent 8% and 10% of the gap, respectively (see Slemrod (2007)).Thus, the major source of the tax gap is under-reporting.

[4] This represents typical treatment in the majority of countries. However, if we model the penalty to include imprisonment or the disclosure of actual firms, a fixed penalty or a penalty proportional to the whole amount of the true income is possible.

[5] This paper does not consider firms that report a deficit.

(i) If $pq > 1$, the sincere reporting (reporting true income Y) is a rational behavior, and

(ii) If $pq < 1$, full cheating (reporting 0 as income) is a rational behavior.

Proof. From (1), the utility is linear in r in the domain $[0, Y]$ and the slope is $pq - 1$. Thus, when $pq > 1$, the utility is maximized at $r = Y$, and when $pq < 1$, the utility is maximized at $r = 0$. □

The evasion decision does not depend on the basic tax rate t or the true income Y because of the risk neutrality assumption. If risk aversion is assumed, the extent of evasion varies according to the basic tax rate and true income.[6] A comprehensive review of the theory is seen in Andreoni, Erand, and Feinstein (1998).

This implies that the actual detection probability p and penalty rate q are less significant and $pq < 1$ appears to be realized.[7] The deterrent effect of the current random audit rule is, therefore, weak. Consequently, we focus on an alternative auditing rule that enhances tax compliance.

3 A motivating strategic auditing example

This section demonstrates the improvement in tax compliance as a result of strategic auditing. We assume two taxpayers, taxpayer 1 and taxpayer 2, with true incomes of Y. An audit is conducted on one individual only; thus, the probability of inspection of an individual is 0.5 if random inspection is adopted. We assume a penalty rate of q is 1.5. Therefore, according to Proposition 1, full cheating represents rational behavior for both taxpayers when the audit is randomly conducted.

We now consider an inspection rule where an audit is conducted for the taxpayer with the lowest reported income. When the two reported incomes coincide, the inspected taxpayer is randomly selected. What is the rational outcome for this case? We model this using a one-shot static game and consider a Nash equilibrium of this game. Let (r_1, r_2) be a pair of the reported income. Assume that r_i is a non-negative integer.

[6]Yitzhaki (1974) showed that, under the assumption of decreasing absolute risk aversion, the extent of evasion decreases as the basic tax rate increases, and the extent of evasion increases as income increases.

[7]However, the majority comply with tax law, which is a phenomenon known as the puzzle of tax compliance (Alm, 1991; Feld and Frey, 2002).

Then, the pair $(0,0)$ is not an equilibrium because taxpayer 1 can improve his payoff by choosing 1 instead of 0. In fact, his payoff improves by

$$(1-t)Y + t(Y-1)(1-0\times q) - \left((1-t)Y + t(Y-0)(1-\frac{1}{2}q)\right)$$
$$= \frac{1}{2}qY - 1.$$

The payoff must be positive when Y is sufficiently large. We explore a symmetric Nash equilibrium (r,r) with $r \leqq Y$. In equilibrium, there is no profitable deviation from (r,r) and it is sufficient to verify the one-unit deviation from r because the utility of a deviant taxpayer is linear in r' when a reported income r' is not another's income. Thus, the following two conditions must hold:

$$(1-t)Y + t(Y-r)(1-\frac{1}{2}q) \geqq (1-t)Y + t(Y-r-1)(1-0\times q), \quad \text{and}$$
$$(1-t)Y + t(Y-r)(1-\frac{1}{2}q) \geqq (1-t)Y + t(Y-r+1)(1-1\times q).$$

These are equivalent to

$$\frac{1}{2}q(Y-r) \leqq 1, \quad \text{and} \quad 1-q \leqq (Y-r)\frac{1}{2}q.$$

Because $1-q$ is negative, the second condition holds for any r. Thus, the restriction of r comes from the first condition; it states $Y - r \leqq \frac{2}{q} = \frac{4}{3}$. Both (Y,Y) and $(Y-1, Y-1)$ become equilibria. Therefore, the game theory predicts that tax compliance substantially improves if strategic auditing is adopted.[8]

In the literature, some studies have noted the usefulness of strategic auditing. Collins and Plumlee (1991) consider a model wherein an individual must choose a labor supply decision and a tax evasion decision and experimentally verify that a strategic auditing rule enhances tax compliance when compared to random auditing. Alm and McKee (2004) noted that the strategic interdependence between taxpayers exists based on the DIF auditing rule and formulate a tax evasion game as a coordination problem with Pareto-ranked multiple equilibria exists. Although both studies experimentally show the usefulness of strategic auditing, neither considers a significant hurdle that must be overcome in the use of strategic auditing.

One problem associated with the use of strategic auditing is that the process cannot distinguish between two cases; a case with a low level of reported income because of

[8]A similar observation is reported in Kamijo, Nihonsugi, Takeuchi, and Funaki (2014) in the context of a public goods provision game with punishment rule.

a low level of true income, and a case with a low level of reported income because of substantial tax evasion. The auditor cannot ascertain the true income of the taxpayer (information asymmetry between the auditor and the taxpayer). This challenges the success of previous examples with respect to strategic auditing. Consider the case where the income of taxpayer 1, Y_1, is less than the income of taxpayer 2, Y_2. In this case, $(Y_1, Y_1 + 1)$ becomes an equilibrium. Thus, a taxpayer with greater income will under-report his income in equilibrium. However, this result seems unrealistic because taxpayer 2 is unlikely to know the exact taxable income of taxpayer 1. Information asymmetry exists between the auditor and the taxpayer and, additionally, the asymmetry among taxpayers should be considered in the examination of strategic auditing. The next sections analyze strategic auditing by modeling a static game with incomplete information.[9]

4 Income reporting game with strategic auditing

Let $N = \{1, 2, ..., n\}$ with $n \geqq 2$ as a set of taxpayers (individuals or firms) that should report their income to a tax authority. We consider that taxpayers in N belong to the same social-economic category, or possess the same social or economic attribution to assume initial homogeneity. For $i \in N$, true income is denoted by $Y_i \in [Y_\ell, Y_h]$, where Y_ℓ and Y_h are lower and upper bounds of income normalized to 0 and 1, respectively. Each i with income Y_i report $r_i \in [0, Y_i]$ to the tax authority.[10] In an income reporting game (IRG), taxpayers report their incomes simultaneously. Let $(r_1, r_2, ..., r_n) \in [0, 1]^n$ be a profile of reported incomes. A tax authority observes the profile and inspects the individual with the lowest reported income. If there is a tie, a random selection is made from the tied members.

We assume that the true income of each individual is a random variable. Thus, we model IRG with a strategic inspection as a normal form game with incomplete information (Harsanyi, 1967). We assume that a true income Y_i of an individual is

[9] Beck and Jung (1989) and Cronshaw and Alm (1995) provide a game theoretic model of incomplete information where taxpayers possess private information concerning true income. However, the model assumes that taxpayer true income is either high or low and is not generalized for a continuum of income types.

[10] Thus, we do not consider the case of over-reporting. This assumption simplifies the analysis. If we do not assume this condition, the over-reporting never represents rational behavior and does not constitute an equilibrium.

identically and independently distributed according to a continuous distribution function F on $[0,1]$. Let f be a density function of F. Because the IRG with strategic auditing is a normal form game with incomplete information, the strategy of player i is a function that associates his realized true income Y_i with reporting income r_i. Let γ_i be the strategy of player i.

We adopt the symmetric Bayesian Nash equilibrium (BNE) $(\gamma, \gamma, ..., \gamma)$, where every player uses the same strategy γ as a solution criterion to evaluate strategic auditing. We assume the following differentiability condition.

Assumption 1. A Bayesian equilibrium strategy γ is a continuous, differentiable, and increasing function with $\gamma(0) = 0$.

We explore the conditions that should be satisfied by γ. Suppose $n-1$ individuals, with the exception of player i with income Y (type Y player), follow the strategy γ. The expected payoff of the type Y player reporting $r \leqq Y$ is

$$U(r, Y) = Y - tr - \left(1 - F(\gamma^{-1}(r))\right)^{n-1} qt(Y - r). \tag{2}$$

Note that $\left(1 - F(\gamma^{-1}(r))\right)^{n-1}$ is the probability of r being the lowest reported income among n reported incomes. This is a continuous function in the domain $[0, Y]$ when γ is a continuous function.

By differentiating $U(r, Y)$ in r, we have the following:

$$\begin{aligned} \frac{\partial U}{\partial r} = -t - (n-1)\left(1 - F(\gamma^{-1}(r))\right)^{n-2} \left(-f(\gamma^{-1}(r))\right) \frac{qt(Y - r)}{\gamma'(\gamma^{-1}(r))} \\ + \left(1 - F(\gamma^{-1}(r))\right)^{n-1} qt \end{aligned} \tag{3}$$

For $(\gamma, \gamma, ..., \gamma)$ to constitute a BNE, this must be a local maximum at $r = \gamma(Y)$. Thus, the following FOC condition should be satisfied:

$$\frac{\partial U}{\partial r}(\gamma(Y), Y) \begin{cases} \geqq 0 & \text{if } \gamma(Y) = Y \\ = 0 & \text{if } 0 < \gamma(Y) < Y \\ \leqq 0 & \text{if } \gamma(Y) = 0 \end{cases}$$

$$\iff \frac{\left(\frac{1}{q} - (1 - F(Y))^{n-1}\right)}{(n-1)(1 - F(Y))^{n-2} f(Y)} \gamma'(Y) \begin{cases} \leqq & Y - \gamma(Y) \text{ if } \gamma(Y) = Y \\ = & Y - \gamma(Y) \text{ if } 0 < \gamma(Y) < Y \\ \geqq & Y - \gamma(Y) \text{ if } \gamma(Y) = 0 \end{cases} \tag{4}$$

Let Y^* be defined as follows:

$$Y^* = F^{-1}\left(1 - \left(\frac{1}{q}\right)^{1/(n-1)}\right) \tag{5}$$

Then, for $Y < Y^*$, $\frac{1}{q} - (1 - F(Y))^{n-1} < 0$. Because $\gamma' > 0$ from Assumption 1 and $Y - \gamma(Y) \geqq 0$, $Y = \gamma(Y)$ must hold for $Y < Y^*$. Therefore, a type Y taxpayer for $Y \leqq Y^*$ sincerely reports his income.

Next, consider Y satisfies $Y > Y^*$. The differential equation can be reduced to

$$\gamma'(Y) + A(Y)\gamma(Y) = A(Y)Y \tag{6}$$

where

$$A(Y) = \frac{(n-1)(1 - F(Y))^{n-2} f(Y)}{(\frac{1}{q} - (1 - F(Y))^{n-1})} \tag{7}$$

and $A(Y) > 0$ for $Y > Y^*$. A general solution of the above differential equation is

$$\gamma(Y) = e^{-\int A(Y)dY}\left(\int A(Y)Ye^{\int A(Y)dY}dY + C\right)$$

with an initial condition $\gamma(Y^*) = Y^*$. By using partial integration,

$$\begin{aligned}\gamma(Y) &= e^{-\int A(Y)dY}\left(Ye^{\int A(Y)dY} - \int e^{\int A(Y)dY}dY + C\right)\\ &= Y - e^{-\int A(Y)dY}\left(\int e^{\int A(Y)dY}dY - C\right)\end{aligned}$$

Let $a(Y) = \int A(Y)dY$, that is, an indefinite integral of $A(Y)$. Then, considering the initial condition,

$$\gamma(Y) = Y - \frac{\int_{Y^*}^{Y} e^{a(z)}dz}{e^{a(Y)}} = Y - \int_{Y^*}^{Y} e^{a(z)-a(Y)}dz \quad \text{for } Y > Y^*.$$

Therefore, we have a candidate of an equilibrium strategy as follows:

$$\gamma(Y) = \begin{cases} Y & \text{for } Y \leqq Y^* \\ Y - \int_{Y^*}^{Y} e^{a(z)-a(Y)}dz & \text{for } Y > Y^* \end{cases} \tag{8}$$

The next theorem states that γ constitutes a BNE.

Theorem 1. *Let γ be defined in (8). Strategy profile $(\gamma, \gamma, ..., \gamma)$ is a BNE.*

Proof. The payoff of type Y reporting r is given by (2) and reduced to

$$U(r, Y) = (1-t)Y + t(Y - r)\left(1 - q\left(1 - F(\gamma^{-1}(r))\right)^{n-1}\right). \tag{9}$$

We consider the following two cases separately: (i) $Y < Y^*$ and (ii) $Y \geqq Y^*$.

Case (i) $Y < Y^*$. Because $r \leqq Y < Y^*$ and $\gamma(r) = r$, the payoff described by (9) is re-written as follows:

$$(1-t)Y + t(Y-r)\left(1 - q\left(1 - F(r)\right)^{n-1}\right) \tag{10}$$

Because $r \leqq Y < Y^*$ and Y^* satisfies (5), $1 - q(1-F(r))^{n-1}$ is negative. Therefore, the taxpayer payoff is maximized at $r = Y$.

Case (ii) $Y \geqq Y^*$. When $r \leqq Y^*$, the payoff is given by (10) and is maximized at $r = Y^*$ in the domain $[0, Y^*]$. Next, suppose $r > Y^*$. The first derivative of $U(r, Y)$ given by (3) is rewritten as follows:

$$\frac{\partial U}{\partial r} = -t + (n-1)\left(1 - F(\gamma^{-1}(r))\right)^{n-2} f(\gamma^{-1}(r)) \frac{qt(\gamma^{-1}(r) - r)}{\gamma'(\gamma^{-1}(r))}$$

$$+ \left(1 - F(\gamma^{-1}(r))\right)^{n-1} qt + (n-1)\left(1 - F(\gamma^{-1}(r))\right)^{n-2} f(\gamma^{-1}(r)) \frac{qt(Y - \gamma^{-1}(r))}{\gamma'(\gamma^{-1}(r))}$$

Because $Y^* < \gamma^{-1}(r) < r$ and from (5), γ must satisfy the following

$$\frac{(\frac{1}{q} - (1-F(Y))^{n-1})}{(n-1)\left(1-F(Y)\right)^{n-2} f(Y)} = \frac{Y - \gamma(Y)}{\gamma'(Y)}.$$

Using this, the first derivative is reduced to

$$\frac{\partial U}{\partial r} = -t + (n-1)\left(1 - F(\gamma^{-1}(r))\right)^{n-2} f(\gamma^{-1}(r)) t \left(\frac{(1 - q\left(1 - F(\gamma^{-1}(r))\right)^{n-1})}{(n-1)\left(1 - F(\gamma^{-1}(r))\right)^{n-2} f(\gamma^{-1}(r))}\right)$$

$$\begin{aligned}
&+ \left(1 - F(\gamma^{-1}(r))\right)^{n-1} qt + (n-1)\left(1 - F(\gamma^{-1}(r))\right)^{n-2} f(\gamma^{-1}(r)) \frac{qt(Y - \gamma^{-1}(r))}{\gamma'(\gamma^{-1}(r))} \\
&= -t + t\left(1 - q\left(1 - F(\gamma^{-1}(r))\right)^{n-1}\right) + \left(1 - F(\gamma^{-1}(r))\right)^{n-1} qt \\
&\quad + (n-1)\left(1 - F(\gamma^{-1}(r))\right)^{n-2} f(\gamma^{-1}(r)) \frac{qt(Y - \gamma^{-1}(r))}{\gamma'(\gamma^{-1}(r))} \\
&= (n-1)\left(1 - F(\gamma^{-1}(r))\right)^{n-2} f(\gamma^{-1}(r)) \frac{qt(Y - \gamma^{-1}(r))}{\gamma'(\gamma^{-1}(r))}.
\end{aligned}$$

This is positive for $r \in [Y^*, \gamma(Y))$, negative for $r \in (\gamma(Y), Y]$, and zero if $r = \gamma(Y)$. Thus, U is maximized at $r = \gamma(Y)$.

Therefore, the proof ends. □

An intuition is as follows. Because the lowest reporter is audited, the risk of punishment when cheating is high for low income taxpayers. This implies that sincere reporting is more likely to occur among low income taxpayers. Assuming that every

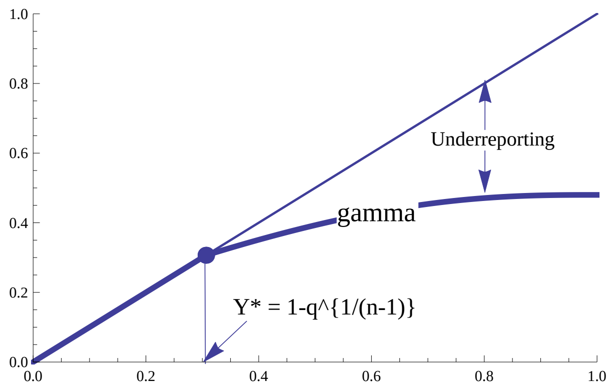

Figure 1: The equilibrium strategy for IRG with strategic auditing. F is a uniform distribution on $[0, 1]$, $n = 4$ and $q = 3$.

taxpayer with an income less than Y honestly reports their true income, the payoff for a taxpayer with income Y when he reports r is given by (10). Therefore, as long as $1 - q\,(1 - F(r))^{n-1}$ is negative, the preferred action is to honestly report. The critical value of honestly reporting is obtained when $1 - q\,(1 - F(Y))^{n-1} = 0$, i.e., $Y = Y^*$. For a taxpayer whose income exceeds Y^*, honest reporting is never a preferred action. The extent of tax evasion is captured by $\frac{\int_{Y^*}^{Y} e^{a(z)}dz}{e^{a(Y)}}$. The slope of γ in the domain $[Y^*, 1]$ is

$$\gamma'(Y) = 1 - \frac{1}{(e^{a(Y)})^2}\left((e^{a(Y)})^2 - e^{a(Y)}A(Y)\int_{Y^*}^{Y} e^{a(z)}dz\right)$$
$$= \frac{A(Y)}{e^{a(Y)}}\int_{Y^*}^{Y} e^{a(z)}dz > 0.$$

Thus, the reported income itself is an increasing function and Assumption 1 is fulfilled.

Figures 1 and 2 demonstrate the equilibrium strategy when $n = 4$ and F is a uniform distribution. From Theorem 1 and Figures 1 and 2, we obtain the following proposition.

Proposition 2. *The following statements hold true;*

(i) Tax behavior does not depend on the basic tax rate t.

(ii) Y^* represents an increasing function on q and decreasing function on n. Thus, under-reporting increases as the penalty rate decreases and as the number of taxpayers increases (see Figure 2).

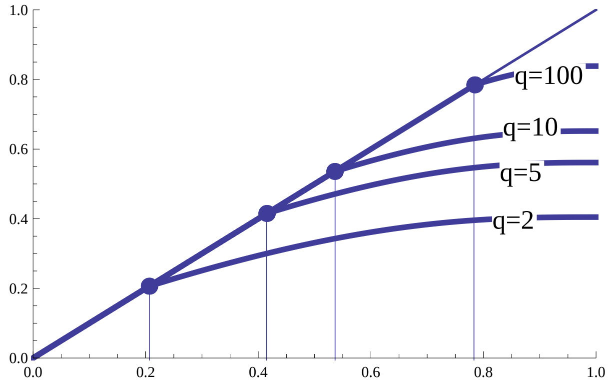

Figure 2: The equilibrium strategies for IRG with strategic auditing varying the value of q. F is a uniform distribution on $[0,1]$ and $n=4$.

(iii) The ratio of taxpayers that sincerely reports is $F(Y^*) = 1-(1/q)^{1/(n-1)}$*; thus, it does not depend on the distribution function* F*.*

(iv) It is impossible that every type of taxpayer sincerely reports under the finite value of q*, although as* q *increases to infinity,* $F(Y^*)$ *goes to* 1*. Moreover, the speed of the increase in* $F(Y^*)$ *as* q *increases is slow (see Figure 2).*

(v) Strategic auditing is not always preferable to random auditing. (see Figure 2). In Figure 2, if $q>4$*, everyone complies with the tax rule from Proposition 1 in a random auditing context.*

(vi) Let γ^q *be the equation defined in (8) when the penalty rate is* q*. For any* q *and* q' *with* $q>q'$*,* $\gamma^q(Y) \geqq \gamma^{q'}(Y)$ *holds for any* $Y \in [0,1]$*.*

Proof. (i) to (v) are obvious. Therefore, we prove (vi). Because Y^* is an increasing function of q, $\gamma^q(Y) \geqq \gamma^{q'}(Y)$ holds for any $Y \in [0, Y^{*q}]$, where Y^{*q} is the Y^* for q. Consider $Y > Y^{*q}$. Then, $A^q(Y) > A^{q'}(Y) > 0$ holds, where $A^q(Y)$ is $A(Y)$ defined in (7) when the penalty rate is q. Because the differential equation is (6), we have

$$\gamma'^p(Y) = A^p(Y)(Y-\gamma^p(Y)) \quad \text{and} \quad \gamma'^{p'}(Y) = A^{p'}(Y)(Y-\gamma^{p'}(Y))$$

Thus, when $\gamma^{p'}(Y)$ is sufficiently close to $\gamma^p(Y)$,

$$\gamma'^p(Y) > \gamma'^{p'}(Y)$$

holds. Because γ is a continuous function, $\gamma^q(Y) > \gamma^{q'}(Y)$ holds for any $Y \in [Y^{*q}, 1]$. Thus, the proof ends. $\square$

The first statement of the proposition is similar to the observation by Yitzhaki (1974) . In a random auditing context, the basic tax rate does not affect the evasion decision when the penalty rate is multiplied by the concealed tax amount, not the concealed income. The second statement suggests that the income range of tax compliance increases as the audit becomes stronger, similar to the basic random auditing prediction model. Surprisingly, the third statement suggests that the distribution function does not affect the compliance rate. However, the critical income that changes compliance behavior varies because of the distribution function. The fourth and fifth statements demonstrate the limits of strategic auditing. Strategic auditing never accomplishes full compliance but attains an intermediate level of tax compliance although the penalty rate is not high. The sixth statement shows the relation among the equilibrium strategies for different values of q, implying that the expected tax revenue increases as q increases without considering the increase of the fined tax amount.

5 The combination of random and strategic auditing

This section considers an inspection rule where, for a probability w, a random inspection is adopted and, for a probability $1 - w$, the strategic inspection (inspection of the lowest reporter) is adopted. The motivation for this analysis stems from random inspections still used in some countries. Alm and McKee (2004) showed that occasional random inspections enhance the DIF rule if taxpayers can coordinate reported income and avoid inspection.

Consider a combination of random and strategic auditing is employed. Suppose $n-1$ individuals, with the exception of one player i with income Y, follow the strategy γ. The expected payoff of a firm Y firm reporting r is

$$U(r, Y) = Y - tr - \left(\frac{w}{n} + (1-w)\left(1 - F(\gamma^{-1}(r))\right)^{n-1}\right) qt(Y - r)$$

for any $r \leqq Y$.

Thus, by simple calculation, FOC is as follows:

$$\frac{(\frac{1}{q}-\frac{w}{n}-(1-w)\left(1-F(\gamma^{-1}(r))\right)^{n-1})}{(n-1)(1-w)\left(1-F(Y)\right)^{n-2}f(Y)}\gamma'(Y)\begin{cases}\leqq & Y-\gamma(Y) \text{ if } \gamma(Y)=Y\\ = & Y-\gamma(Y) \text{ if } 0<\gamma(Y)<Y\\ \geqq & Y-\gamma(Y) \text{ if } \gamma(Y)=0\end{cases}$$

Let Y^* be defined as follows:

$$Y^{**}=F^{-1}\left(1-\left(\frac{n-wq}{nq(1-w)}\right)^{1/(n-1)}\right)$$

This is maximized at $w=0$. Thus, a pure strategic inspection is preferable to a combined random and strategic inspection from the perspective of the proportion of taxpayers complying with tax law.

Proposition 3. *The ratio of the tax observance is maximized at $w=0$. Thus, pure strategic inspection is optimal among any mixture of strategic and random auditing.*

Thus, in contrast to Alm and McKee (2004), we find that a combination of random with strategic auditing does not improve tax compliance. This finding does not originate from Alm and McKee (2004) who find that when taxpayers succeed in coordinating reported income, they avoid inspection. The prediction of our theoretical analysis is not affected by the change in the tie-breaking rule of multiple minimum reporters. Therefore, the contrasting result to Alm and McKee (2004) stems from the information and the taxable income asymmetry among taxpayers.

6 Conclusion

The contributions of this paper are as follows. First, we provide a game theoretic framework to analyze strategic auditing in a general context, which implies that the number of taxpayers is not restricted but finite, taxpayer asymmetry is considered, and the taxpayer type is infinite. Second, we analyze the problem of choosing one individual from a homogenous group. We show that under realistic penalty rate conditions, the lowest-reporter-audited (LIRA) rule is superior to a random selection rule. This implies that improvements in tax compliance can be achieved by simply changing the audit choice strategy from a random rule to a strategic rule process, which incurs no extra cost. However, we also prove that the LIRA rule has a limitation. Although the penalty rate is high, complete tax compliance is never achieved, which is in contrast to the random auditing rule. Third, we show that, in contrast to the findings of Alm and McKee (2004), the combination of LIRA and a randomized rule will not improve tax compliance. Thus, the result relies on a symmetric, complete information setting.

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